Efficient bailouts in markets with adverse selection

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September 28, 2009

Abstract

We study optimal government interventions in markets that collapse because of adverse selection [...]

PRELIMINARY AND INCOMPLETE

An important insight of economic theory from the last forty years is that asymmetric information can lead to market collapse. George Akerlof demonstrated this phenomenon in a classic paper (Akerlof (1970)). Many features of the financial market collapse in the Fall of 2008 suggest a role for asymmetric information. Not only did spread widen (as they would in any case given the increase in counter-party risk), but transaction stopped in many markets. In interbank market, only overnight loans remained. Banks refrained from lending to each other because they were afraid of not being repaid, as the assets that the borrowing bank would put as collateral could be in fact worth nothing (toxic). In the OTC market, the range of acceptable forms of collateral was dramatically reduced "leaving over 80% of collateral in the form of cash during 2008", while the "repo financing of many forms of collateralized debt obligations and speculative-rate bonds became essentially impossible." (Duffie (2009)). Investors and banks were unable to agree on the price for legacy assets or for bank equity.

Governments stepped-in to try to alleviate the problem. In the US, the initial TARP program called for 700 billion to purchase illiquid assets from the banks. Subsequently other proposals were introduced and implemented with varying degrees of success. The main others were equity injection and debt guarantees. As of August 2009, there was 307 billion of outstanding debt issued by financial companies and guaranteed by the FDIC.[1] The original TARP called for 700 billion to purchase illiquid assets from the banks. It was transformed into a Capital Purchase Program (CPP) to invest $250 billion in U.S. banks. Treasury also insured 306 billions of Citibank's assets, and 118 billion of Bank America's.

The objective of this paper is to compare these programs, and to derive the optimal mechanism to prevent interbank lending from freezing, in a simple tractable model where the main friction is the presence of asymmetric information.

This kind of comparison is important and interesting for at least two reasons. The first reason is the scale of government interventions. The second reason is that there is no consensus about which program is better. Soros (2009) and Stiglitz (2008) argue for equity injections, Bernanke (2009) is in favor of assets buy backs and debt guarantee, Diamond, Kaplan, Kashyap, Rajan, and Thaler (2008) view assets buy backs and equity injection as

[1] http://www.fdic.gov/regulations/resources/tlgp/index.html. Citigroup sold another 5 billion of guaranteed debt in September 2009. The program is set to expire at the end of October 2009.

best alternatives, whereas, Ausubel and Cramton (2009) argue for a careful way to 'price the assets, either implicitly or explicitly.'

Our analysis first highlights what are the key ingredients that need to be present in order for asymmetric information to cause a problem. The most important feature is that the banks seeking to borrow should have risky investment opportunities: otherwise they would always choose to carry out a project when it pays more than the cost for sure and hence a loan towards them would be riskless. Second, the lowest possible value for their assets should be below the amount a banks need to borrow: otherwise there is again no risk since the bank can always repay using its assets. In normal times, the non-crisis regime, this is what prevents this market from breaking down. Even in the worst case scenario, banks have enough collateral to pay-off their loan. However, that changed in the Fall of 2008 where it became likely that actually big financial institutions have completely worthless legacy assets.

We seek to find the government intervention that restores efficient financing at the minimum expense of taxpayers' money. We do so from two perspectives: First we look at the case where banks decide to participate before they know the value of their assets and then we look at the case where participation decisions take place when banks know the realized value of their assets. Both these perspectives are empirically relevant. Our two main results are to derive cost-minimizing programs for each one of these two perspectives. Both these mechanism design problems are non-standard because depending on the type of intervention, the payoffs are not quasi-linear in money, and different mechanism work through very different channels: For example for the case of equity injection, the government injects cash in return for equity, which helps because it lowers the amount that a bank need to borrow. On the other hand, in the case of debt guarantees, banks pay a fee giving them access to a specific amount that they can borrow at a low interest rate. One first key insight is analyze this problem in terms of payoffs rather then mechanisms. This gives us an unambiguous bound for the minimum cost (or maximum profit - in the case cost is negative) that the government can guarantee. Using this, we show that the cost-minimizing government program from the ex-ante perspective is actually profitable. The profits equal the net welfare gain due to the program. We also show that the optimal versions of equity injection, asset-buy-back and debt guarantee, all generate the maximal profits.

Then we turn to look at the design of government programs, at the point where banks know the value of their assets when deciding whether to participate or not. This mechanism design problem has an additional complication compared to the ex-ante one: The bank's mere participation decision maybe enough to signal valuable information to the government. For example, if the government could design a program that attracted only banks that can always repay their loans, then all banks would face a fair interest rate, and the government could be able to make money by providing a program that works as a successful signaling devise. Our analysis shows that this is impossible. We show that all the government can do is to either design a program that attracts only the bad banks or all banks. Both these programs are costly. Again we derive the lower bound on costs using our payoffs approach. However, this insight alone is not enough to make things tractable here, as we have to address the following subtle issue: Suppose that the government is contemplating a program that attracts all banks, that is in equilibrium all banks participate. What would the market then infer if a bank opts-out? This inference is crucial because it affects bank's outside options, which ultimately affect how costly the government's program is.[2] We proceed by deriving the cost bounds for some abstract market-response rate and we show that minimum bound is often achieved by using debt-guarantees. We also show that debt-guarantee is always less costly than equity injection and asset-buy-backs. In the extensions, we consider the design of government programs that include menus: that is programs that consists of different options for different types of banks. Interestingly, it turns out that all incentive compatible menus for the case of debt guarantees and asset-buy-backs boil down to the case of a pooling contract. The government can sustain incentive compatible menus, only in the case of equity and then the cost-minimizing menu achieves the lower bound.

1 General Model

1.1 Description

The model has a continuum of financial institutions of measure 1. Financial institutions are financial companies such as commercial banks, investment banks, insurance companies,

[2] The paper of Cramton and Palfrey (1995) formulates a refinement to impose restrictions on out-off-equilibrium beliefs. We chose not to select one particular belief and provide a characterization valid for all conceivable out-off-equilibrium reactions.

or finance companies. For simplicity, we refer to all of them as banks.

The model has three dates $t = 0, 1, 2$. Banks start time 0 with given initial assets and liabilities. At time 1 banks learn the value of the legacy assets on their balance sheets, and they receive new investment opportunities. They can lend to, and borrow from each other and from outside investors. To avoid confusion with inter-bank lending, we use the word "investments" to refer to the new loans that banks make to the non-financial sector at time 1. All returns are realized at time 2, and profits are paid out to investors. We assume risk-neutral investors and we normalize the risk-free rate to 0.

The government announces its interventions at time 0, but the implementation can happen either at date 0 or at date 1. The difference matters because banks learn about the value of their existing assets and about their new investment opportunities at date 1. Interventions at date 1 are therefore subject to adverse selection, while interventions at date 0 are not. The two cases are empirically relevant, and we therefore analyze both.

Initial assets and cash balance

On the asset side, banks have two types of assets: cash and long term assets. Cash is liquid and can be used for investments or for lending at date 1. Let c_t be cash holdings at the beginning of time t. All banks start time 0 with c_0 in cash. Cash holdings cannot be negative:

$$c_t \geq 0 \text{ for all } t.$$

Long-term legacy assets deliver a random payoff $a \in [A_{\min}, A]$ at time 2. We ignore for now the issue of outstanding long term debt and deposits, so it is best to think of a as the payoff net of transfers to senior creditors. We refer the reader to Philippon and Schnabl (2009) for a model where debt overhang is the main friction.

Information and new investment at time 1

At time 1 banks learn their type θ and they receive investment opportunities. Investments cost the fixed amount x at time 1 and deliver a random payoff $v \in [0, V]$ at time 2. At time 2 total bank income y depends on the realization of two random variables, a and v:

$$y = a + c_2 + v \cdot i \tag{1}$$

where i is dummy for the decision to invest at time 1. The conditional distribution of (a, v) depends on the type θ. The conditional distribution is $F(a, v|\theta)$.

Banks can borrow and lend at time 1 on a competitive credit market. Borrowing and lending at time 1 can take place between banks with investment projects and banks without investment projects (interbank lending), or between banks and outside investors. Let l be the amount raised from new lenders at time 1. The budget constraint of the bank at time 1 is:

$$c_2(i) = c_1 + l - x \cdot i, \tag{2}$$

where $c_2(i)$, $i = 0, 1$ is the bank's cash at time 2 as a function of its investment decision at time 1. The type θ is revealed to the bank at time 1. The market does not observe θ, but might observe a signal σ about θ. Let y^l be the amount paid back to investors at time 2. Because the credit market is competitive and investors are risk neutral, in any candidate equilibrium, the participation constraint of investors implies:

$$E\left[y^l|\sigma, i = 1\right] = l. \tag{3}$$

1.2 Symmetric information

We first consider the case where information is symmetric, that is $\sigma = \theta$. Banks raise money at time 1 to finance their investments. Banks maximize total value as of time 1:

$$E[y^e|\theta] = E[a|\theta] + c_2 + E[v|\theta] \cdot i - E\left[y^l|\theta\right] \cdot i,$$

subject to the time 1 budget constraint.

The bank will go ahead with the investment if

$$E[a|\theta] + c_2(1) + E[v|\theta] - E\left[y^l|\theta\right] \geq E[a|\theta] + c_2(0), \tag{4}$$

It is easy to see that if the bank goes forward with the investment, it weakly prefers to spend all of its own cash (the bank is indifferent under symmetric information, and it strictly prefers under asymmetric information). Hence, given Assumption A1, we always have $c_2(1) = 0$ whereas in the case that the bank does not invest we have that $c_2(0) = c_1$. Equation (4) simply becomes $E[v|\theta] \geq c_1 + E\left[y^l|\theta\right]$. The break-even constraint for the lenders (banks without investment opportunities) is $E\left[y^l|\theta\right] \geq l$, which binds as long as the lending market is competitive. Using (2), this implies that $E\left[y^l|\theta\right] = l = x - c_1$, which we can use to write the investment condition as $E\left[v|\theta\right] \geq x$. The following Lemma summarizes the equilibrium under symmetric information:

Proposition 1 *Under symmetric information, investment takes place at time 1 if and only if* $E\left[v|\theta\right] > x$.

It is important to emphasize that under symmetric information, $E\left[a|\theta\right]$ is irrelevant. Investment decisions are independent of the quality of legacy assets on the banks' balance sheet.

1.3 Asymmetric information

Now assume that θ is private information and the market only sees the signal σ, and the decision to invest. This is different from $E\left[y^l|\theta\right] \geq l$ which is the first best rule.

If a bank is going to invest, it is always better to first use its internal cash, therefore $c_2 = 0$ when $i = 1$. In case of investment, the bank borrows $l = x - c_1$ at time 1, and the bank's shareholders receive $a + v - y^l$ at time 2. Investment is profitable for the bank if its payoff net of repayments $E\left[a+v|\theta\right] - E\left[y^l|\theta\right]$ is more than its payoff without investing $E\left[a+c_1|\theta\right]$. We therefore obtain an investment condition which must be satisfied in any equilibrium:

$$i\left(\theta\right) = 1 \Longleftrightarrow E\left[a + v - y^l|\theta\right] > E\left[a|\theta\right] + c_1$$

In theory, the contract with the new investors at time 1 could be debt or equity. It turns out, however, that there is no loss of generality in considering standard debt contracts, with interest rate r. This is because the bank always wants to give priority to the uninformed investors to mitigate adverse selection issues. Under the usual seniority rules, the payoffs to investors are:

$$y^l = \min(y, rl).$$

Then it is optimal to invest, conditional on r and l, if:

$$i\left(\theta\right) = 1 \Longleftrightarrow E\left[\max(a + v - rl, 0)|\theta\right] > E\left[a|\theta\right] + c_1. \tag{5}$$

We now study two particular cases where the efficient outcome obtains despite asymmetric information.

Riskless projects

We now show that uncertainty in v conditional on θ is required for adverse selection, irrespective to the information structure with respect to the legacy assets a.

Lemma 1 *Suppose that* $\theta = (\theta^a, v)$ *where* θ^a *indexes the conditional distribution of* a, $F(a|\theta^a)$. *Then the equilibrium under asymmetric information is the same as under symmetric information.*

Proof. We have

$$E[\max(a + v - rl, 0)|\theta] = \int_{rl-v}^{A} (a + v - rl)\, dF(a|\theta^a).$$

If $rl > v$, then the investment condition (5) is clearly violated. So banks with $v < rl$ would never invest. But if only banks with $v > rl$ invest, then debt is risk free and $r = 1$, and, since $l = x - c$, the investment condition is simply

$$E[a|\theta^a] + v - l > E[a|\theta^a] + c_1 \Longleftrightarrow v > x.$$

This is the first best rule. ■

This Lemma is quite general. In particular, it holds for any signal σ. Special cases include $\theta = (a, v)$ where a and v are known with certainty at time 1 by the bank. It also covers the binary case where $a \in \{0, A\}$ and $\Pr(a = A) = \theta^a$. The important point is that uncertainty in v conditional on θ is critical for adverse selection.

Minimum quality balance sheet

Suppose now that $A_{\min} \geq x - c_0$. This corresponds to the normal state of interbank flows. The scale of the new investment is small relative to the size of the balance sheet, even under pessimistic expectations.

Lemma 2 *If* $A_{\min} \geq x - c_0$, *then the symmetric information allocation is an equilibrium under asymmetric information.*

Proof. $y^l = \min(y, rl)$ and $y = a + v \geq A_{\min} > x - c_0 = l$. So $r = 1$ satisfies the participation constraint of lenders. With $r = 1$, $\max(a + v - rl, 0) = a + v - rl$ and the investment condition becomes

$$E[v|\theta] > l + c_0 = x$$

which is the first best investment rule. ■

This Lemma characterizes non-crisis lending. As long as the balance sheet can be pledged to new lenders, new projects can be financed at a low rate even if there is asymmetric information.

2 Benchmark model

2.1 Payoff and information structure

We now present our benchmark model, which is a special case of the general model of Section 1. The results in Section 1 show that two properties are critical for adverse selection to occur in the credit market. First, there must be risk in the new project v conditional on θ. Second, there must be private information with respect to the legacy assets' ability to cover losses from new investments.

These two insights allow us to construct the simplest model where borrowing and lending is sensitive to information. We therefore assume that:

- Private types learnt at time 1 are binary and fully determine the eventual payoffs of legacy assets at time 2: $a = 0$ when $\theta = B$ (bad type), and $a = A$ when $\theta = G$ (good type). We define the ex-ante (time 0) probability of a good type as:

$$\pi \equiv \Pr(\theta = G).$$

- All new projects are identical. The payoffs are binary $v \in \{0, V\}$ with probability of success

$$q \equiv \Pr(v = V).$$

In addition, we make the following two parameter assumptions:

Assumption A1: $c_0 < x < V < A + c_0$

Assumption A2: $qV > x$

AssumptionsA1 says that banks need to borrow in order to invest, that projects are valuable, and that new projects do not yield more that the face value of the legacy assets on the balance sheet. Assumption A2 says that new projects have positive NPV.

The role of asymmetric information is easily seen in this model. Consider a candidate equilibrium with interest rate r. A bank of high quality, $a > rl$ knows it will always repay its lenders, to it will invest if and only if $qV - rl > c_0$. A bank of low quality $a < rl$ knows that it will not repay in the low state, so it will invest if and only if $q(V + a - rl) > a + c_0$. The potential for adverse selection with respect to a exists because the investment equation is more likely to hold for lower values of a. Consider a type θ who is perceived by the market to be type $\hat{\theta}$ and has cash holdings c. If $\hat{\theta} = B$, the market believes the borrower is risky and the equilibrium lending rate must be $1/q$. If $\hat{\theta} = G$ on the other hand, the interest rate must be lower. Assumptions A1 and A2 ensure that $A > (x - c_0)/q$, so good types could repay their debt in all cases, and the interest rate must be equal to 1 when $\hat{\theta} = G$. The net value of investing for a good type facing an interest rate r is:

$$qV - x - (r - 1)(x - c)$$

The term $(r - 1)(x - c)$ is the informational rent paid by the good type. Conversely, a bad type earns rents because the it only pays back its creditors with probability q:

$$qV - x + (1 - qr)(x - c).$$

Clearly, the rents are zero if the interest rate correctly reflect the risks of the borrower, $r = 1$ for a good type, and $r = 1/q$ for a bad type.

2.2 Equilibria without interventions

Consider first an equilibrium where all banks invest. In such a pooling equilibrium the interest rate would be

$$r_{\mathcal{P}} \equiv \frac{1}{\pi + (1 - \pi) q}. \quad (6)$$

Lemma 3 *In the benchmark model, there exists a pooling equilibrium $\mathcal{P}$ with both types investing if and only if:*

$$c_0 \geq c_{\mathcal{P}} \equiv x - \frac{qV - x}{r_{\mathcal{P}} - 1}. \quad (7)$$

Proof. The interest rate in $\mathcal{P}$ must be $r_{\mathcal{P}}$. It is clearly optimal for the bad types to invest. On the other hand, the good types chose to invest if and only if $qV - x - (r_{\mathcal{P}} - 1)(x - c_0) > 0$. ■

Lemma 4 *In the benchmark model, there exists a separating equilibrium $\mathcal{S}$ with only bad types investing if and only if*

$$c_0 \leq c_{\mathcal{S}} \equiv x - \frac{q}{1-q}(qV - x) \tag{8}$$

Proof. The interest rate in $\mathcal{S}$ must be $r = 1/q$. It is clearly optimal for the bad types to invest since $qV > x$. On the other hand, the good types chose not to invest if and only if $qV - x - (1/q - 1)(x - c) < 0$. ■

Since $1/q > r_{\mathcal{P}}$, we have $c_{\mathcal{S}} > c_{\mathcal{P}}$. We can therefore state the following result:

Proposition 2 *The set of equilibria for the benchmark model are:*

- *If $c_0 \in [0, c_{\mathcal{P}}]$, the unique equilibrium is $\mathcal{S}$;*
- *If $c_0 \in [c_{\mathcal{P}}, c_{\mathcal{S}}]$, there are multiple equilibria, either $\mathcal{S}$ or $\mathcal{P}$.*
- *If $c_0 \in [c_{\mathcal{S}}, x]$, the unique equilibrium is $\mathcal{P}$.*

Proof. We only need to check that there is no separating equilibrium where the good types invest alone. In such an equilibrium, the interest rate would be $r = 1$, and the bad types would always want to invest. ■

Note that there does not exist a separating equilibrium where the good banks invest. In other words, all separating equilibria are inefficient.

Proposition 3 *In the benchmark model, pooling is welfare improving and more cash helps sustain pooling.*

The implication of this last Proposition is that government interventions should seek to establish the pooling equilibrium if and when it fails to happen. We are now going to describe interventions under symmetric information (participation decision at time 0) and under asymmetric information (participation decision at time 1).

3 Government interventions

3.1 General description

If the equilibrium without intervention is efficient (pooling), then the government does not intervene. If the equilibrium is inefficient, the government may chose to intervene to restore efficiency. We can describe all programs in terms of the cash m injected at time 1, and the payments y^g received by the government at time 2. Both m and y^g can be random and type-dependent. The cash at time 1 of a bank that opts in the program is

$$c_1 = c_0 + m.$$

The payments to the shareholders at time 2 become $y_2 - y^l - y^g$. Let Ψ be the expected cost of the government program

$$\Psi = E\left[m - y^g\right]$$

Definition 1 ***Objective of the government*** *is*

$$\min \Psi$$

subject to

$$i(\theta) = 1 \text{ for } \theta = G, B \tag{9}$$

We consider in particular three government interventions: capital injection, asset buy backs, and debt guarantees. Let us briefly describe each intervention. The banks are all ex-ante identical, so the government makes the same offers to all. It is straightforward to extend the model to an heterogenous population by making the offers conditional on observable characteristics, such as size, for instance.

- Capital injection: the government offers cash m_α against a share α of equity returns, $y^g = \alpha\left(y_2 - y^l\right)$
- Asset buy back: the government offers to buy an amount Z of legacy assets for cash m_z. If a bank opts in the program, the face value of its legacy assets decreases by Z

- Debt guarantee: the government offers to guarantee debt issuance up to an amount S for a fee ϕ paid up-front: $m = -\phi S$. Private lenders accept an interest rate of 1 on the guaranteed debt, so the date 1 budget constraint becomes

$$x = c_0 + (1 - \phi) S + l^u,$$

where l^u is the unsecured loan. In case of default, the government will have to make payments, so $y^g \leq 0$.

Let V_{in}^{θ} be the value for type θ to be in the government program:

$$V_{in}^{\theta} = E\left[y_2 - y^l - y^g | \theta\right] \tag{10}$$

Proposition 4 *In any program where all banks participate, the cost is*

$$\Psi = E^{\pi}\left[V_{in}^{\theta}\right] - W$$

where

$$W = \pi A + c_0 + qV - x$$

Proof. By definition

$$V_{in}^{\theta} = E\left[y_2 - y^l - y^g | \theta\right]$$

In any program where $i(\theta) = 1$ for $\theta = G, B$, we must have $E[y_2|\theta] = E[a|\theta] + qV + c_2(1)$. From (2), we get

$$c_2(1) = c_0 + l - x + m$$

Taking unconditional expectations

$$E^{\pi}\left[V_{in}^{\theta}\right] = \pi A + qV + E^{\pi}\left[c_0 + l - x + m - y^l - y^g\right]$$

The break even constraint of investors is

$$E^{\pi}\left[l - y^l\right] = 0$$

and the expected cost of the government is by definition

$$\Psi = E^{\pi}[m - y^g].$$

Therefore

$$E^{\pi}\left[V_{in}^{\theta}\right] = \pi A + c_0 + qV - x + \Psi.$$

■

3.2 Interventions under symmetric information

Let us now study interventions at time 0, i.e. before firms learn their types. The participation constraint then applies in expectation

$$E^{\pi}\left[V_{in}^{\theta}\right] \geq E^{\pi}\left[V_{out}^{\theta}\right] \tag{11}$$

where V_{out}^{θ} is the value of staying outside the government program.

Proposition 5 *When banks choose to participate in the government program before they know the value of their assets, the program is profitable for the government. The maximum profit of a time 0 intervention is* $\Psi = \pi\left(qV - x\right)$

Proof. If the equilibrium without intervention is inefficient, then we have

$$E^{\pi}\left[V_{out}^{\theta}\right] = \pi\left(A + c_0\right) + \left(1 - \pi\right)\left(c_0 + qV - x\right) = W - \pi\left(qV - x\right).$$

From the participation constraint and Proposition 4, we get $\Psi \geq E^{\pi}\left[V_{out}^{\theta}\right] - W$. If the participation constraint binds, we get $\Psi = -\pi\left(qV - x\right)$. ■

We can now state our first main result:

Theorem 1 *The three interventions are equivalent when implemented at* $t = 0$*, and achieve the maximum profits for the government.*

Proof. See appendix ■

The intuition is simple. In the inefficient separating equilibrium $\mathcal{S}$, the good types do not invest. In the efficient pooling equilibrium, the good types do invest. The net welfare gain is equal to $\pi\left(qV - x\right)$. Since the new lenders who come in at time 1 must break even on average, the welfare gains must accrue to the government and the initial shareholders. Since the government makes a take-it-or-leave-it offer at time 0, it can extract all the surplus. The critical point of the Theorem is that the interventions can actually make sure that (9) is satisfied and at the same time the participation constraint (11) holds with equality.

The Theorem provides a useful benchmark for the remaining of the paper, when we analyze interventions at time 1, under asymmetric information. These interventions are more difficult to analyze because banks know how much their assets are worth but the government does not. Not only does this create adverse selection issues for the government, but it also implies that the decision to participate in the government program may signal some information about the value of their assets, and therefore influence the market rates offered to participating and non participating banks.

4 Separating Interventions

Let us now consider interventions at time 1, when banks have private information regarding their legacy assets. We maintain the assumption that $c_0 < c_{\mathcal{P}}$. From Proposition 2, we know that without government interventions, the unique equilibrium is the inefficient separating equilibrium $\mathcal{S}$ where only the bad types invest. We now consider interventions by the government to ensure that good types also invest.

The decision to participate in the government program can signal the type of the bank. We therefore need to consider separating-participation equilibria and pooling-participation equilibria. Notice that here the notion of pooling and separation refers to participation in the program, not to investment. In any case, all banks will invest since otherwise the government intervention is useless.

Consider first a candidate equilibrium where only the good types participate. In this equilibrium, participation would reveal good type, non participation reveals bad type. The interest rate would be 1 for participating banks, and $1/q$ for non-participating banks. Since $qV - x < (1/q - 1)(x - c_0)$, the alternative to participation for a good bank would be not to invest. For a bad bank, it would be to invest with a fair rate.

If this equilibrium existed, it is clear that all banks would invest since their types would be revealed by the participation decision. For that reason, the government would not need to inject any cash into banks, and could even ask for assets or equity. Unfortunately, such an equilibrium cannot exist.

Proposition 6 *There cannot exist equilibria where only the good types participate in the government program.*

Proof. See Appendix ■

Consider now an equilibrium where only bad types participate in the government program. Participation reveals bad type, non participation reveals good type. The non participation value for a good bank is

$$V_{out}^{G} = A + c_0 + qV - x$$

and the non participation value for a bad bank is

$$V_{out}^{B} = c_0 + qV - x + (1-q)(x - c_0).$$

Proposition 7 *The minimum cost for the separating intervention is*

$$\Psi(\mathcal{B}) = (1-\pi)(1-q)(x - c_0)$$

Proof. Calculations similar to the ones done in the previous section show that

$$\Psi(\mathcal{B}) = (1-\pi)\left(V_{in}^{B} - (c_0 + qV - x)\right)$$

The participation constraint of the bad type is $V_{in}^{B} \geq V_{out}^{B}$. Therefore

$$\Psi(\mathcal{B}) \geq (1-\pi)(1-q)(x - c_0)$$

■

The intuition is straightforward. Separating requires paying informational rents to the bad types.

Proposition 8 *Asset buy-backs can always be used to separate bad types at the minimum cost. Equity injections can be used for some range of parameters. Debt guarantees cannot be used to separate types.*

Proof. See Appendix ■

5 Pooling Interventions

In a pooling intervention, all banks participate in the government program. The interest rate conditional on participation must therefore be $r_{\mathcal{P}}$. The outside options depend on the out-of-equilirium belief of investors regarding a bank that would unexpectedly opt out of the program. Let $\tilde{r}$ be the interest rate a bank would face if it decided to opt out of the government program. The outside option of a good bank is

$$V_{out}^{G}\left(\tilde{r}\right)=A+\max\left\{qV-\tilde{r}\left(x-c_{0}\right),c_{0}\right\} \tag{12}$$

and the outside option of a bad bank would be

$$V_{out}^{B}\left(\tilde{r}\right)=q\left(V-\tilde{r}\left(x-c_{0}\right)\right). \tag{13}$$

The inside options depend on the details of the government program. The government must ensure that both types invest in the pooling equilibrium. We summarize these various constraints in the following program:

Definition 2 ***Optimal Pooling Program***. *The optimal pooling program* Γ^{*} *solves*

$$\Gamma_{\mathcal{P}}^{*}\left(\tilde{r}\right)=\arg\min_{\Gamma}\Psi$$

subject to

$$V_{in}^{\theta}\left(r_{\mathcal{P}},\Gamma\right)\geq V_{out}^{\theta}\left(\tilde{r}\right)\ \textit{for}\ \theta=B,G \tag{14}$$

and

$$V_{in}^{\theta}\left(r_{\mathcal{P}},\Gamma,i=1\right)\geq V_{in}^{\theta}\left(r_{\mathcal{P}},\Gamma,i=0\right)\ \textit{for}\ \theta=B,G \tag{15}$$

First we can obtain a lower bound on the cost for any program.

Proposition 9 ***Minimum Cost***. *There exists a lower bound on the cost*

$$\Psi_{\min}=\left(x-c_{0}\right)\left(1-\left(1-\pi\right)q\tilde{r}\right)-\pi\min\left\{\tilde{r}\left(x-c_{0}\right),qV-c_{0}\right\}$$

Proof. Since all banks participate in a pooling equilibrium, we know from Proposition 4 that

$$\Psi=E^{\pi}\left[V_{in}^{\theta}\left(\Gamma\right)\right]-W$$

Using the participation constraints and the outside options (12) and (13).

$$\begin{aligned}
\Psi &\geq E^{\pi}\left[V_{out}^{\theta}\left(\tilde{r}\right)\right]-W\\
&= \pi A+\pi\max\left(qV-\tilde{r}\left(x-c_{0}\right),c_{0}\right)+\left(1-\pi\right)q\left(V-\tilde{r}\left(x-c_{0}\right)\right)-\left[\pi A+c_{0}+qV-x\right]\\
&= \pi\max\left(qV-\tilde{r}\left(x-c_{0}\right),c_{0}\right)-\pi qV+\left(x-c_{0}\right)\left(1-\left(1-\pi\right)q\tilde{r}\right).\\
&= \left(x-c_{0}\right)\left(1-\left(1-\pi\right)q\tilde{r}\right)-\pi\min\left\{\tilde{r}\left(x-c_{0}\right),qV-c_{0}\right\}
\end{aligned}$$

■

Let us know consider capital injections, debt guarantees and assets buy-backs. A general program combining these three instruments is characterized by $\Gamma = \{\alpha, Z, S, m_\alpha, m_z, \phi\}$.

Lemma 5 ***Inside values***. *The inside value of a good bank in a pooling equilibrium*

$$V_{in}^{\theta}(r_{\mathcal{P}}, \Gamma) = (1-\alpha)(A - Z + \Sigma + qV + r_{\mathcal{P}}(c_1 - x)) \tag{16}$$

and the inside value of a bad bank is

$$V_{in}^{\theta}(r_{\mathcal{P}}, \Gamma) = (1-\alpha)q(V + \Sigma + r_{\mathcal{P}}(c_1 - x)) \tag{17}$$

where $c_1 = c_0 + m_\alpha + m_z$*, and the net value of debt guarantee is*

$$\Sigma(\phi, S) = ((1-\phi) r_{\mathcal{P}} - 1) S. \tag{18}$$

After joining the program a bad bank always wants to invest, and a good bank wants to invest if and only if

$$(r_{\mathcal{P}} - 1) c_1 + \Sigma \geq r_{\mathcal{P}} x - qV \tag{19}$$

Proof. By entering the program, the bank receives $m_\alpha + m_z$ in cash. It issues guaranteed debt S at an interest rate of 1 and pays ϕS to the government. Its new cash balance is then $c_1 + (1-\phi) S$. Its unsecured borrowing, at rate $r_{\mathcal{P}}$, is therefore:

$$l^u = x - c_1 + (1-\phi) S.$$

Now consider the total shareholder value at time 2. A good bank always repays all its loans, therefore total shareholder value is

$$A - Z + qV - S - r_{\mathcal{P}} l^u = A - Z + qV - r_{\mathcal{P}}(x - c_1) + (r_{\mathcal{P}}(1-\phi) - 1) S$$

If it does not invest, its shareholder value is $A - Z + c_1$. Comparing with the previous equation leads to condition (19). Since initial shareholders only keep a fraction $1-\alpha$ of the total at time 2, we obtain (16). A bad bank, by contrast, knows it will default with probability q. Total shareholder value at time 2 is then

$$q(V - S - r_{\mathcal{P}} l^u) = qV - q r_{\mathcal{P}}(x - c_1) + q(r_{\mathcal{P}}(1-\phi) - 1) S$$

which leads to (17). If it does not invest, its shareholder value is c_1. ■

We must now try to simplify the program by figuring out which constraints are binding, and which ones are not. We already know that the investment constraint is slack for bad types. Let us now compare the participation constraints across types:

Lemma 6 *For any outside market rate $\tilde{r}$, the participation constraint (14) is always tighter for the good type than for the bad type.*

Proof. Suppose the participation constraint holds for $\theta = G$. Then, $V_{in}^{\theta}(\mathcal{P}, \Omega) \geq V_{out}^{\theta}(\tilde{r}) \geq A + qV - \tilde{r}(x - c_0)$ which we can write as

$$\tilde{r}(x - c_0) - r_{\mathcal{P}}(x - c_1) + \Sigma \geq \alpha(A + \Sigma + qV + r_{\mathcal{P}}(c_1 - x)) + (1 - \alpha) Z.$$

Now notice that $A + c_0 > V$ and $c_0 < x < qV$ implies $A > (1 - q) V$. Therefore, since $0 \leq \alpha \leq 1$ and $Z \geq 0$, we see that the previous equation implies

$$\tilde{r}(x - c_0) - r_{\mathcal{P}}(x - c_1) + \Sigma > \alpha(V + r_{\mathcal{P}}(c_1 - x) + \Sigma)$$

which implies that the participation constraint holds for $\theta = B$. ■

Next, let us compare the participation and investment constraints for good types.

Lemma 7 *For the good type, the participation constraint (14) binds and implies the investment constraint (15).*

Proof. The two constraints for $\theta = G$ are:

$$(15) \quad : \quad (r_{\mathcal{P}} - 1) c_1 + \Sigma \geq r_{\mathcal{P}} x - qV$$
$$(14) \quad : \quad (r_{\mathcal{P}} - 1) c_1 + \Sigma \geq r_{\mathcal{P}} x - qV + Z + \frac{\alpha A + \max(qV - \tilde{r}(x - c_0), c_0)}{1 - \alpha} - c_1$$

In the pure debt guarantee case – characterized by $\alpha = 0$, $Z = 0$ and $c_1 = c_0$ – we can write ((14)) as:

$$(r_{\mathcal{P}} - 1) c_0 + \Sigma \geq r_{\mathcal{P}} x - qV + \max(qV - \tilde{r}(x - c_0) - c_0, 0)$$

So clearly the participation constraint implies the investment constraint in the pure debt guarantee case. In all other cases, the government can always increase α or Z to make the participation constraint binding, without changing the investment constraint, but also lowering its cost Ψ. So clearly the investment constraint is always binding ■

We can now compute the cost of the government intervention. There are two ways to think about the cost. The direct one is to compute the NPV of the cash flows for the government:

Lemma 8 *The expected cost of the government program is*

$$\Psi = x - c_0 + (1-\alpha)\left(\frac{\Sigma}{r_{\mathcal{P}}} - \pi Z + c_1 - x\right) - \alpha\left(\pi A + qV\right)$$

Proof. The government finances $c_1 - c_0$ up-front. The expected default loss on the credit insurance is $(1-\pi)(1-q)$ since bad type defaults when their new projects fail. The net cost of the insurance liability is therefore:

$$((1-\pi)(1-q) - \phi) S = \left(1 - \frac{1}{r_{\mathcal{P}}} - \phi\right) S = \frac{\Sigma}{r_{\mathcal{P}}}$$

From the good type, the government receives $Z + \alpha\left(A - Z + \Sigma + qV + r_{\mathcal{P}}(c_1 - x)\right)$. From the bad type the government receives $\alpha q\left(V + \Sigma + r_{\mathcal{P}}(c_1 - x)\right)$. The net cost to the government is therefore

$$\begin{aligned}
\Psi(\Omega) &= c_1 - c_0 + \frac{\Sigma}{r_{\mathcal{P}}} - (1-\alpha)\pi Z - \alpha\left(\pi A + qV + (\pi + (1-\pi)q)\left(\Sigma + r_{\mathcal{P}}(c_1 - x)\right)\right) \\
&= c_1 - c_0 + \frac{\Sigma}{r_{\mathcal{P}}} - (1-\alpha)\pi Z - \alpha\left(\pi A + qV + \frac{\Sigma}{r_{\mathcal{P}}} + (c_1 - x)\right).
\end{aligned}$$

The formula then follows from the definition of $r_{\mathcal{P}}$ in equation (6) and Σ in (18). ■

Using the fact that $\pi r_{\mathcal{P}} + (1-\pi) q r_{\mathcal{P}} = 1$, we can also rewrite the cost function as

$$\Psi = (1-\alpha)\left(c_1 - c_0 + ((1-\pi)(1-q) - \phi) S - \pi Z\right) - \alpha\left(\pi A + c_0 + N\right).$$

The intuition is clear. The government gets a share α of the equity, whose net value (averaged across types) is $\pi A + c_0 + N$. On the other hand, the government injects cash $c_1 - c_0$, receives assets worth πZ and provides credit guarantee at price ϕ against expected loss $(1-\pi)(1-q)$.

The NPV approach is useful to compare programs, but less useful to gain intuition. Using our previous Lemmas, we can show

Lemma 9 *The cost of the pooling program* Γ *is*

$$\Psi(\Gamma) = \Psi_{\min} + (1-\pi)\left(V_{in}^{B}(r_{\mathcal{P}}, \Gamma) - V_{out}^{B}(\tilde{r})\right)$$

If $qV - \tilde{r}(x - c_0) > c_0$*, then a pure debt guarantee program reaches the lower bound. If* $qV < \tilde{r}(x - c_0) + c_0$*, the pure debt guarantee program exceeds the minimum by* $(1-\pi)\, q\left(\tilde{r}(x - c_0) - qV + c_0\right)$*.*

Proof. We have seen that the participation constraint of good types binds and implies both the investment constraint for good types, and the participation constraint for bad types. So any cost in excess of the minimum comes from the slackness of the participation constraint for bad types. For the pure debt program, the participation constraints are

$$\begin{aligned}
(G) &\ : \ \Sigma + r_{\mathcal{P}}(c_0 - x) \geq \max\{qV - \tilde{r}(x - c_0), c_0\} - qV \\
(B) &\ : \ \Sigma + r_{\mathcal{P}}(c_0 - x) \geq -\tilde{r}(x - c_0)
\end{aligned}$$

If $qV - \tilde{r}(x - c_0) > c_0$, they are equivalent, hence $V_{in}^{B}(r_{\mathcal{P}}, \Gamma) - V_{out}^{B}(\tilde{r}) = 0$ and $\Psi^{debt} = \Psi_{\min}$. If $qV - \tilde{r}(x - c_0) < c_0$ then

$$V_{in}^{B}(r_{\mathcal{P}}, \Gamma) - V_{out}^{B}(\tilde{r}) = q\left(\tilde{r}(x - c_0) - qV + c_0\right)$$

■

WE NEED TO REPHRASE THESE THEOREMS.

We can now state our main theorems. First, let us compare the pooling equilibria.

Theorem 2 *AMONG THE 3 INSTRUMENTS [...] The lowest-cost pooling equilibrium is always achieved by a pure debt guarantee program. If the outside option of the good type is to invest, i.e. if* $qV > \tilde{r}(x - c_0) + c_0$*, then the cost of the program is*

$$\Psi^{debt} = \left(1 - \frac{\tilde{r}}{r_{\mathcal{P}}}\right)(x - c_0)$$

If the outside option of the good type is not to invest, then the cost is

$$\Psi^{debt} = x - c_0 - \frac{qV - c_0}{r_{\mathcal{P}}}$$

In the absence of debt guarantees, a pure equity program always dominates a pure asset buyback program.

Proof. See appendix ■

Finally, we can compare across all equilibria

Theorem 3 *The best government intervention is to create a debt guarantee program in which all banks participate*

Proof. The cost of the separating equilibrium is

$$\Psi(\mathcal{B}) = (1-\pi)(1-q)(x-c_0)$$

Note that the cost of the pooling equilibrium is decreasing in $\tilde{r}$. So for any $\tilde{r} > 1$, we have

$$\frac{r_{\mathcal{P}} - \tilde{r}}{r_{\mathcal{P}}} < \frac{r_{\mathcal{P}} - 1}{r_{\mathcal{P}}} = (1-\pi)(1-q)$$

And therefore $\Psi(\mathcal{P}) < \Psi(\mathcal{B})$. ■

6 Extensions

6.1 Menus of Contracts

When we analyzed pooling interventions we assumed that there was a single government program intended for all types of banks. This assumption was without any loss: The minimum cost for the government when we allow for type-dependent programs is the same as in the case of a single program.

We first establish that in the cases of debt guarantee and asset-buy-backs there do not exist incentive compatible type-dependent menus: The only incentive compatible contract is a pooling one:

Proposition 10 *For the case of asset-buy-backs and the case of debt guarantee the only incentive compatible contract is a pooling one.*

Proof. We first look at asset-buy-back programs. The revelation principle implies that without loss we can assume that each program consists of a option for good banks and an option for bad banks:

$$m_G, Z_G \text{ and } m_B, Z_B.$$

Then, since bad banks have no assets we immediately get that $Z_B = 0$. Then, the incentive constraints for the two types of banks are as follows: The incentive compatibility constraint for good banks is:

$$A - Z_G + m_G + c_0 + N > A + m_B + c_0 + \max\left\{0, N - \frac{(1-q)}{q}(x - c_0 - m_B)\right\},$$

which reduces to:

$$m_G + N - Z_G > m_B \text{ if } 0 = \max\left\{0, N - \frac{(1-q)}{q}(x - c_0 - m_B)\right\} \tag{20}$$

$$m_G q + (1-q)(x - c_0) - qZ_G > m_B \text{ otherwise} \tag{21}$$

The incentive compatibility constraint for bad banks is:

$$\begin{aligned} c_0 + m_B + N &> c_0 + m_G + N + (1-q)(x - m_G - c_0) \\ m_B &> m_G + N + (1-q)(x - m_G - c_0) \\ m_B &> qm_G + N + (1-q)(x - c_0) \end{aligned} \tag{22}$$

But, then it is immediate that (20) (or (21)) and (22) cannot be satisfied simultaneously, so there does not exist an incentive compatible two-option menu asset-buy-back program.

Now to turn to examine debt guarantee programs. Here the two menus are

$$\phi_G, S_G \text{ and } \phi_B, S_B.$$

IC for bad banks:

$$\begin{aligned} qV - qS_B - [x - (c_0 - \phi_B S_B) - S_B] &\geq qV - qS_G - q[x - (c_0 - \phi_G S_G) - S_G] \\ -qS_B - [x - (c_0 - \phi_B S_B) - S_B] &\geq -qS_G - q[x - (c_0 - \phi_G S_G) - S_G] \\ -S_B - \frac{[x - (c_0 - \phi_B S_B) - S_B]}{q} &\geq -S_G - [x - (c_0 - \phi_G S_G) - S_G] \end{aligned} \tag{23}$$

General IC for good banks to take care of investment decision in the event of deviation:

$$A + qV - S_G - [x - (c_0 - \phi_G S_G) - S_G] \geq A + \max\{c_0, qV - S_B - r_B[x - (c_0 - \phi_B S_B) - S_B]\}$$

If $c_0 \leq qV - S_B - r_B[x - (c_0 - \phi_B S_B) - S_B]$, then the IC for the good banks implies:

$$\begin{aligned} A + qV - S_G - [x - (c_0 - \phi_G S_G) - S_G] &\geq A + qV - S_B - r_B[x - (c_0 - \phi_B S_B) - S_B] \\ -S_G - [x - (c_0 - \phi_G S_G) - S_G] &\geq -S_B - r_B[x - (c_0 - \phi_B S_B) - S_B] \end{aligned} \tag{24}$$

But because $r_B = \frac{1}{q}$ can hold simultaneously, only if they hold with equality. Hence two-option menus boil down to one-option menus. Now, if $c_0 \geq qV - S_B - r_B\left[x - (c_0 - \phi_B S_B) - S_B\right]$, the IC for good banks becomes:

$$A + qV - S_G - [x - (c_0 - \phi_G S_G) - S_G] \geq A + c_0$$

but because of $c_0 \geq qV - S_B - r_B\left[x - (c_0 - \phi_B S_B) - S_B\right]$, the inequality (24) must hold in this case as well. Hence two-option menus boil down to one-option menus. ■

Now we move on to examine the optimal menu for the case of equity. Again, the revelation principle tells us that without any loss the program can be taken to consist of an option for each type:

$$\alpha_G, m_G \text{ and } \alpha_B, m_B$$

In order to be feasible each of these options must satisfy the participation and minimum cash constraints as before, which depend on the market response and on the out-of-equilibrium move do not participate. Hence the cost-minimizing Program for the government solves:

$$\min_{\alpha_G, m_G, \alpha_B, m_B} \pi\left\{(1-\alpha_G) m_G - \alpha_G (\pi A + c_0 + N)\right\} + (1-\pi)\left\{(1-\alpha_B) m_B - \alpha_B (c_0 + N)\right\}$$

subject to:

$$\begin{aligned}
IC_B &: (1-\alpha_B)(qV + c_0 + m_B - x) - (1-\alpha_G) q (V + c_0 + m_G - x) \geq 0 \\
IC_G &: \begin{array}{l} (1-\alpha_G) q (A + qV + c_0 + m_G - x) - (1-\alpha_B)(qA + V + c_0 + m_B - x) \geq 0 \text{ if } m_B > \frac{x - q^2 V}{(1-q)} - c_0 \\ (1-\alpha_G)(A + qV + c_0 + m_G - x) - (1-\alpha_B)(A + c_0 + m_B) \geq 0 \text{ otherwise} \end{array} \\
PC_G &: (1-\alpha_G)(A + qV + c_0 + m_G - x) - A - qV + \tilde{r}(x - c_0) \geq 0 \\
PC_B &: (1-\alpha_B)(qV + c_0 + m_B - x) - q(V - \tilde{r}(x - c_0)) \geq 0 \\
A_G &: \alpha_G \geq 0 \\
A_B &: \alpha_B \geq 0
\end{aligned}$$

Note that the constraints that the $\alpha' s$ must be less than one are ignored because they are implied by the participation constraints.

Proposition 11 *The optimal menu is given by*

$$\alpha_G^* = 0 \text{ and } m_G^* = -(\tilde{r} - 1)(x - c_0)$$

and α_B^* and m_B^* are such that

$$(1 - \alpha_B)m_B^* - \alpha_B^*(N + c_0) = (1 - q\tilde{r})(x - c_0),$$

which guarantees that both IC_B and PC_B hold with equality. This menu achieves the same cost as the cost-minimizing debt-guarantee program, namely

$$\Psi^* = \frac{r_{\mathcal{P}} - \tilde{r}}{r_{\mathcal{P}}}(x - c_0).$$

Proof. See appendix ■

6.2 Senior debt and deposits

TBC

7 Conclusion

TBC

COMPARISON WITH DEBT OVERHANG: Philippon and Schnabl (2009).

A Proof of Theorem 1: Equivalence of Time 0 Interventions

A.1 Capital injection

The government cost function is

$$\Psi_0^E = m_\alpha - \alpha E\left[a + c_2 + v \cdot i - y^l\right]$$

To sustain the efficient pooling equilibrium, the government must inject $m_\alpha = c_\mathcal{P} - c_0$. In this equilibrium, all firms invest, so $i = 1$ and $c_2 = 0$ for all types, and the cost function becomes

$$\Psi_0^E = (1-\alpha)\, m_\alpha - \alpha\,(\pi A + c_0 + N)$$

By participating, a bank knows that it will be able to invest irrespective of its type, and that it will receive a cash m_α. In return, it will give up a fraction α of its equity. The participation constraint at time 0 is therefore:

$$\alpha\,(\pi A + c_0 + N) = (1-\alpha)\, m_\alpha + \pi N$$

Therefore we see that the cost is negative:

$$\Psi_0^E = -\pi N.$$

A.2 Asset buyback program

Let $\tilde{r}_\mathcal{P}$ be the pooling rate in the modified equilibrium with assets reduced to $A - Z$. We consider two cases.

Case 1: $A - Z > r_\mathcal{P} l$

Then the good type is not risky and the equilibrium conditions are the same as in the equilibrium without intervention. This means that $\tilde{r}_\mathcal{P} = r_\mathcal{P}$. The cash injection needed is $m_z = c_\mathcal{P} - c_0$. The government cost is

$$\Psi_0^A = m_z - \pi Z$$

The participation constraint at time 0 is simply $\pi N \geq \pi Z - m_z$, therefore

$$\pi Z = \pi N + m_z$$

The cost function is $\Psi_0^A = -\pi N$.

Case 2: $A - Z < r_\mathcal{P} l$

Then the good type is risky and $\tilde{r}_\mathcal{P} > r_\mathcal{P}$. We can find the new rate using $E\left[y^l\right] = l$:

$$\tilde{r}_\mathcal{P} = \frac{l - (1-q)\,\pi\,(A-Z)}{ql}$$

Assuming good types invest, the participation constraint at time 0 is

$$\pi\,(A - Z) + qV - (x - c_0 - m_z) \geq \pi\,(A + c_0) + (1-\pi)\,(qV - (x - c_0))$$

Binding participation constraint means that $\pi Z = \Delta^{NPV} + m_z$, and once again we find $\Psi^a = -\Delta^{NPV}$. Given the rate, the good type wants to invest iff $q\,(V - r_\mathcal{P} l + A - Z) > A - Z + c_0 + m_z$. Using $l = x - c_0 - m_z$, this is equivalent to

$$qV > x + (1-q)\,(1-\pi)\,(A-Z)$$

When this investment constraint is slack, this case achieves the same outcome as the previous case. If the investment constraint is violated, this case cannot sustain the efficient pooling outcome. So there is no loss of generality in considering only the first case.

The cost function is the same as with equity injections:

$$\Psi_0^A = -\pi N$$

A.3 Debt guarantee program

With this program, the firm has a different capital structure at time 1. It has debt on its balance sheet. If this debt is super senior, it can create debt overhang, which would be inefficient. So the government should make sure that the firm can issue new debt l^u which is senior to S. If l^u is senior, then the repayments to new lenders do not depend on S, so the pooling rate is also the same $r_{\mathcal{P}}$. The good type chooses to invest if and only if:

$$q\left(A+V-S-r_{\mathcal{P}}l^u\right)+(1-q)\max\left(A-S-r_{\mathcal{P}}l^u,0\right)>A-S+c_0+(1-\phi)S$$

Once again, there are two cases. If $A>S+r_{\mathcal{P}}l^u$, then the investment condition becomes

$$qV>r_{\mathcal{P}}l^u+c_0+(1-\phi)S \tag{25}$$

So if $m_\alpha=(1-\phi)S$, we get exactly same pooling as in equity injection. The expected cost for the government is

$$\Psi_0^S=(1-\phi)S-\pi S-(1-\pi)\left(q\min\left(V-r_{\mathcal{P}}l^u,S\right)+(1-q)\,0\right)$$

Assume for now that $V>S+r_{\mathcal{P}}l^u$, we get

$$\Psi_0^S=(1-\phi)S-\pi S-(1-\pi)qS$$

The participation constraint at time 0 is

$$\pi A+qV-\left(x-c_0-(1-\phi)S\right)-\pi S-(1-\pi)qS\geq\pi\left(A+c_0\right)+(1-\pi)\left(qV-(x-c_0)\right)$$

So

$$\pi\left(qV-x\right)+(1-\phi)S=\pi S+(1-\pi)qS$$

And we get the same as above, $\Psi^s=-\pi N$. We must now check that we can implement the program with $V>S+r_{\mathcal{P}}l^u$. To sustain pooling, the cash injection must be such that $c_0+(1-\phi)S=c_{\mathcal{P}}=x-\frac{N}{r_{\mathcal{P}}-1}$. So $l=\frac{N}{r_{\mathcal{P}}-1}$ and $r_{\mathcal{P}}l^u=N+l^u$. So we want $V>S+N+l^u$, or $(1-q)V>\phi S-c_0$. This is clearly satisfied as long as $\phi\leq1-q$. This simply means that the credit premium cannot be tougher than the premium the market would charge to a low type. It can be equal, however. This means that the government can always implement with at least a fair premium, and that the constraint $V>S+r_{\mathcal{P}}l^u$ is not binding.

The second case is when $A<S+r_{\mathcal{P}}l^u$. Then investment condition becomes

$$qV>qr_{\mathcal{P}}l^u+(1-q)(A-S)+c_0+(1-\phi)S \tag{26}$$

but since $A<S+r_{\mathcal{P}}l^u$ we know that

$$r_{\mathcal{P}}l^u>qr_{\mathcal{P}}l^u+(1-q)(A-S)$$

therefore if (25) is satisfied, then (26) is satisfied. In other words, the investment condition is easier to satisfy. In expected value, however, we still get same cost, because the participation constraint at time 0 is

$$\begin{aligned} & \pi A + qV - (x - c_0 - (1-\phi) S) - \pi (qS + (1-q)(A - r_{\mathcal{P}} l^u)) - (1-\pi) qS \\ \geq \; & \pi (A + c_0) + (1-\pi)(qV - (x - c_0)) \end{aligned}$$

So

$$\pi (qV - x) + (1-\phi) S = \pi (qS + (1-q)(A - r_{\mathcal{P}} l^u)) + (1-\pi) qS$$

But from $\Psi^s = (1-\phi) S - \pi (qS + (1-q)(A - r_{\mathcal{P}} l^u)) - (1-\pi) qS$ we obtain once again

$$\Psi_0^S = -\pi N$$

B Proof of Proposition 6

B.1 Equity

The participation constraint for good type is $(1-\alpha)(A + c_0 + N) > A + c_0$ which we can write as

$$\alpha < \alpha^G (G) \equiv \frac{N}{A + N + c_0}.$$

Bad types chose not to participate if $(1-\alpha)(c_0 + N + (1-q)(x - c_0)) < c_0 + N$ which we can write as

$$\alpha > \alpha^B (G) \equiv \frac{(1-q)(x - c_0)}{c_0 + N + (1-q)(x - c_0)}.$$

This equilibrium can be sustained if and only if

$$\alpha^B (G) < \alpha^G (G)$$

which is equivalent to

$$N (N + c_0) > (1-q)(x - c_0)(c_0 + A). \tag{27}$$

In order for this equilibrium to be feasible we need to investigate whether condition (27) is compatible with (**??**). From (**??**) we have that the largest and smallest values for q. We can rewrite (27) as

$$(qV - x)(qV + c_0 - x) > (1-q)(x - c_0)(c_0 + A).$$

Clearly, the higher is q, the easier it is to satisfy this equation. At the minimum value of q, when $qA = x - c_0$, $qV - x < 0$ and the condition is violated. At the maximum value of q, when $qV = c_0 + (x - c_0)/q$, (27) becomes:

$$qV > q(c_0 + A) + x - c_0$$

which is impossible since $A + c_0 > V$ and $x > c_0$.

B.2 Assets buyback

Good types participate iff $A - Z + c_0 + m_z + N > A + c_0$ or

$$m_z > Z - N.$$

Bad types participate if and only if $c_0 + m_z + N^{bg}(c_0 + m_z) > c_0 + N$, or $qm_z + (1-q)(x - c_0) > 0$ which is always satisfied. This is immediate to understand: Banks with worthless assets are willing to accept anything to get rid of them.

B.3 Debt guarantee

Good types participate iff $A + c_0 + N - \phi S > A + c_0$ or

$$N > \phi S.$$

Bad types participate iff $q(V - x + c_0 - \phi S) = c_0 + N - \phi S + (1-q)(x + \phi S - c_0) > c_0 + N$ or

$$\frac{1-q}{q}(x - c_0) > \phi S.$$

So it can be sustained if we can find parameters such that

$$N > \phi S > \frac{1-q}{q}(x - c_0),$$

which means

$$N > \frac{1-q}{q}(x - c_0)$$

which is rule out by Assumption A3.

C Proof of Proposition 8

C.1 Equity

The incentive constraints for bad banks is $(1-\alpha)(c_0 + m_\alpha + N) > c_0 + N + (1-q)(x - c_0)$, which we can write as:

$$\alpha < \alpha^B(B) \equiv \frac{m_\alpha - (1-q)(x - c_0)}{N + c_0 + m_\alpha}$$

If good banks participate in this equilibrium they are perceived as bad and face the high interest rate. Then good banks would not participate as long as $(1-\alpha)(A + c_0 + m_\alpha) < A + c_0 + N$, which is equivalent to

$$\alpha > \alpha^G(B) \equiv \frac{m_\alpha - N}{A + c_0 + m_\alpha}$$

The equilibrium is sustainable if one can find α such that bad banks opt in and good ones drop out:

$$\begin{aligned}
\alpha^G(B) \quad < \quad & \alpha^B(B) \Longleftrightarrow \frac{m_\alpha - N}{A + c_0 + m_\alpha} \leq \frac{m_\alpha - (1-q)(x - c_0)}{N + c_0 + m_\alpha} \\
\Longleftrightarrow \quad & [m_\alpha - N][N + c_0 + m_\alpha] \leq [A + c_0 + m_\alpha][m_\alpha - (1-q)(x - c_0)] \\
\Longleftrightarrow \quad & (c_0 + m_\alpha)(1-q)(x - c_0) \leq A(m_\alpha - (1-q)(x - c_0)) + N(N + c_0)
\end{aligned}$$

Note that a sufficient condition to sustain an equilibrium where only bad types participate in the government program is

$$1 - q < \frac{N}{x - c_0} < \frac{1 - q}{q}.$$

If this equilibrium is sustainable, i.e. if $\alpha_{Sb}^g < \alpha_{Sb}^b$, we can compute the expected cost of the government program.

$$\begin{aligned}
\Psi^E(\mathcal{B}) &= (1-\pi)\Big(m_\alpha - \alpha_b^b(c_0 + m_\alpha + N)\Big) \\
&= (1-\pi)\left(m_\alpha - \frac{m_\alpha - (1-q)(x-c_0)}{N + c_0 + m_\alpha}(c_0 + m_\alpha + N)\right) \\
&= (1-\pi)(1-q)(x-c_0).
\end{aligned}$$

C.2 Assets buyback

Participation reveals bad type, non participation reveals good type. A good type who participates would not invest. Therefore, in this case a bank with good assets would not participate in the government program if:

$$\begin{aligned}
A - Z + c_0 + m_z &< A + c_0 + N \\
m_z - Z &< N
\end{aligned}$$

Banks with bad assets participate in the government program if:

$$\begin{aligned}
c_0 + m_z + N &> c_0 + N + (1-q)(x-c_0) \\
m_z &\geq (1-q)(x-c_0).
\end{aligned}$$

In order for this equilibrium to be sustained, the following must be true:

$$Z > (1-q)(x-c_0) - N.$$

It is trivial to show that whenever, this equilibrium is sustained, the government chooses a price for the assets that makes the participation constraint bind, that is

$$m_z = (1-q)(x-c_0).$$

This implies the following cost to the government is $(1-\pi)(1-q)(x-c_0)$.

C.3 Debt guarantee

Suppose that only banks with bad assets participate in the government program. The incentive constraints for bad banks is given by:

$$\begin{aligned}
q(V-S) - l^u &= c_0 + N + (1-q-\phi)S > c_0 + N^B(c_0, 1) = c_0 + N + (1-q)(x-c_0) \\
-\phi S &> (1-q)(x - c_0 - S)
\end{aligned}$$

Only feasible if $\phi = 0$ and $S = x - c_0$ i.e. only if government finances all investment for "free."

D Proof of Theorem 2

D.1 Main result

In the main text, we have implicitly assumed that the solvency constraint is satisfied for the good type. The solvency constraint says that legacy assets are enough to repay the creditors:

$$A - Z > S + r_{\mathcal{P}} l^u. \tag{28}$$

The following Lemma explains why we were justified in ignoring this constraint.

Lemma 10 *The participation constraint is always tighter than the solvency constraint.*

Proof. First, we can write the solvency constraint (28) as:

$$A - Z + \Sigma - r_{\mathcal{P}} (x - c_1) \geq 0$$

Suppose it is violated. Then it must mean that good types get nothing if $v = 0$. The participation constraint of good types would then be $(1-\alpha) q (A - Z + V + r_{\mathcal{P}} (c_1 - x) + \Sigma) \geq V^G_{out} (\tilde{r})$. In particular, it implies:

$$(1-\alpha) q (A - Z + V + r_{\mathcal{P}} (c_1 - x) + \Sigma) \geq A + qV - \tilde{r} (x - c_0)$$

$$q (A - Z + r_{\mathcal{P}} (c_1 - x) + \Sigma) \geq A - \tilde{r} (x - c_0) + \alpha q (A - Z + V + r_{\mathcal{P}} (c_1 - x) + \Sigma).$$

Assumptions A1 and A2 ensures that $A > (x - c_0)/q$ and since $\tilde{r} \leq 1/q$, this means that the RHS is strictly positive, which contradicts the assumption that $A - Z + \Sigma - r_{\mathcal{P}} (x - c_1) < 0$. ■

First note that as long as $\phi > 0$, it is never optimal to use the debt guarantee without investing.

Lemma 11 *No bank wants to issue guaranteed debt without investing as long as $\phi \geq 0$*

Proof. A bank that issue guaranteed debt S gets cash balance $c_1 = c_0 + S - \phi S$. If the bank does not invest, shareholders receive

$$a + c_1 - S = a + c_0 - \phi S.$$

This is decreasing in S irrespective of a. ■

The Lagrangian of the government program is

$$\begin{aligned} L &= x - c_0 + (1-\alpha)\left(\frac{\Sigma}{r_{\mathcal{P}}} - \pi Z + c_1 - x\right) - \alpha (\pi A + qV) \\ &\quad - \lambda_{part} \left((1-\alpha)(A - Z + \Sigma + qV + r_{\mathcal{P}} (c_1 - x)) - V^{\theta}_{out} (\tilde{r})\right) \\ &\quad - \lambda_{inv} ((r_{\mathcal{P}} - 1) c_1 + \Sigma - (r_{\mathcal{P}} x - qV)) \\ &\quad - \lambda_\alpha \alpha - \lambda_z Z - \lambda_\Sigma \Sigma - \lambda_{c_1} c_1 \end{aligned}$$

The first order conditions are

$$\begin{aligned} \frac{\partial L}{\partial \alpha} &= 0 : \lambda_{part} = \frac{\frac{\Sigma}{r_{\mathcal{P}}} - \pi Z + c_1 - x + \pi A + qV + \lambda_\alpha}{A - Z + \Sigma + qV + r_{\mathcal{P}} (c_1 - x)} \\ \frac{\partial L}{\partial \Sigma} &= 0 : \lambda_\Sigma = (1-\alpha)\left(\frac{1}{r_{\mathcal{P}}} - \lambda_{part}\right) - \lambda_{inv} \\ \frac{\partial L}{\partial c_1} &= 0 : \lambda_{c_1} = (1-\alpha)(1 - \lambda_{part} r_{\mathcal{P}}) - \lambda_{inv} (r_{\mathcal{P}} - 1) \\ \frac{\partial L}{\partial Z} &= 0 : \lambda_z = (1-\alpha)(r_{\mathcal{P}} \lambda_{part} - \pi) \end{aligned}$$

Plug $\frac{\partial L}{\partial c_1}$ into $\frac{\partial L}{\partial \Sigma}$ leads to

$$\lambda_{c_1} = r_{\mathcal{P}} \lambda_\Sigma + \lambda_{inv} \tag{29}$$

Lemma 12 *The constraint $\Sigma \geq 0$ cannot be binding*

Proof. Suppose it is. Then $\lambda_\Sigma > 0$. But (29) then implies that $\lambda_{c_1} > 0$ and $c_1 = 0$. But in this case the investment constraint is always violated. ■

Lemma 13 *The investment constraint does not bind*

Proof. Suppose it does. Then $\lambda_{inv} > 0$ and from (29) we have $c_1 = 0$. So the binding investment constraint means $\Sigma = r_{\mathcal{P}} x - qV$. But with $c_1 = 0$ and $\Sigma = r_{\mathcal{P}} x - qV$, the participation constraint is always violated. ■

Since $\lambda_\Sigma = 0$ and $\lambda_{inv} = 0$, we know that $\lambda_{c_1} = 0$. This implies that $\lambda_{part} r_{\mathcal{P}} = 1$, and therefore $\lambda_z = (1-\alpha)(1-\pi) > 0$. Therefore $Z = 0$. Using $\frac{\partial L}{\partial \alpha} = 0$ and $\lambda_{part} r_{\mathcal{P}} = 1$ we get

$$\frac{1}{r_{\mathcal{P}}} = \frac{\frac{\Sigma}{r_{\mathcal{P}}} + c_1 - x + \pi A + qV + \lambda_\alpha}{A + \Sigma + qV + r_{\mathcal{P}}(c_1 - x)}$$

which leads to

$$A\left(\frac{1}{r_{\mathcal{P}}} - \pi\right) = qV\left(1 - \frac{1}{r_{\mathcal{P}}}\right) + \lambda_\alpha$$

and finally

$$\lambda_\alpha = (1-\pi)\, q\, (A - (1-q)\, V)$$

Therefore $\lambda_\alpha > 0$ and $\alpha = 0$.

The cost of the program is

$$\begin{aligned} \Psi &= x - c_0 - \frac{1}{r_{\mathcal{P}}}\left(A + qV - V_{out}^{\theta}(\tilde{r})\right) \\ \Psi &= x - c_0 - \frac{qV - \max\left(qV - \tilde{r}(x - c_0), c_0\right)}{r_{\mathcal{P}}} \end{aligned}$$

If the outside option is to invest, then

$$\Psi = \frac{r_{\mathcal{P}} - \tilde{r}}{r_{\mathcal{P}}}(x - c_0)$$

If the outside option is to do nothing then

$$\Psi = \frac{1}{r_{\mathcal{P}}}\left((x - c_0)\, r_{\mathcal{P}} - qV + c_0\right)$$

Using $(r_{\mathcal{P}} - 1)\, c_{\mathcal{P}} = r_{\mathcal{P}} x - qV$, we can rewrite it as

$$\Psi = \frac{r_{\mathcal{P}} - 1}{r_{\mathcal{P}}}(c_{\mathcal{P}} - c_0)$$

D.2 Comparison of pure programs

Consider the case where the outside option of the good bank is to invest. For pure capital injections, we get

Lemma 14 *The optimal capital injection program satisfies*

Proposition 12 • *If* $\tilde{r}(x-c_0) > \frac{r_{\mathcal{P}}}{r_{\mathcal{P}}-1}N$*, then* $\alpha > 0$ *and*

$$\begin{aligned} m &= x - c_0 - \frac{N}{r_{\mathcal{P}}-1} \\ \alpha &= \frac{\tilde{r}(x-c_0) - \frac{r_{\mathcal{P}}}{r_{\mathcal{P}}-1}N}{A + qV - \frac{r_{\mathcal{P}}}{r_{\mathcal{P}}-1}N} \\ \Psi^e &= (1-\alpha)m - \alpha(\pi A + c_0 + N). \end{aligned}$$

• *Otherwise* $\alpha = 0$ *and*

$$\Psi^e = m = \frac{(r_{\mathcal{P}} - \tilde{r})(x - c_0)}{r_{\mathcal{P}}}.$$

Proof. Assume first that the investment constraint binds. Then

$$\hat{c}_1 = \frac{r_{\mathcal{P}}x - qV}{r_{\mathcal{P}}-1} = x - \frac{N}{r_{\mathcal{P}}-1}$$

As long as $r_{\mathcal{P}}\hat{c}_1 - \tilde{r}c_0 - (r_{\mathcal{P}} - \tilde{r})x > 0$, we get α from the participation constraint. Using

$$r_{\mathcal{P}}\hat{c}_1 - \tilde{r}c_0 - (r_{\mathcal{P}} - \tilde{r})x = \tilde{r}(x - c_0) - \frac{r_{\mathcal{P}}}{r_{\mathcal{P}}-1}(qV - x)$$

we get

$$\alpha = \frac{\tilde{r}(x-c_0) - \frac{r_{\mathcal{P}}}{r_{\mathcal{P}}-1}N}{A + qV - \frac{r_{\mathcal{P}}}{r_{\mathcal{P}}-1}N}$$

Otherwise, the constraint $\alpha = 0$ binds and we get c_1 from the investment constraint

$$c_1 = \frac{\tilde{r}c_0 + (r_{\mathcal{P}} - \tilde{r})x}{r_{\mathcal{P}}}.$$

■

For asset buy-backs, we get

Lemma 15 *The optimal asset buyback program satisfies*

Proposition 13 • *If* $\tilde{r}(x-c_0) > \frac{r_{\mathcal{P}}}{r_{\mathcal{P}}-1}N$*, then* $Z > 0$ *and*

$$\begin{aligned} m &= x - c_0 - \frac{N}{r_{\mathcal{P}}-1} \\ Z &= \tilde{r}(x-c_0) - \frac{r_{\mathcal{P}}}{r_{\mathcal{P}}-1}N \\ \Psi^z &= m - \pi Z \end{aligned}$$

- *Otherwise* $Z = 0$ *and*

$$\Psi^z = m = \frac{(r_{\mathcal{P}} - \tilde{r})(x - c_0)}{r_{\mathcal{P}}}$$

Proof. Assume first that the investment constraint binds. Then

$$\hat{c}_1 = \frac{r_{\mathcal{P}} x - qV}{r_{\mathcal{P}} - 1}$$

As long as $r_{\mathcal{P}} \hat{c}_1 - \tilde{r} c_0 - (r_{\mathcal{P}} - \tilde{r}) x > 0$, we get Z from the participation constraint. Using

$$r_{\mathcal{P}} \hat{c}_1 - \tilde{r} c_0 - (r_{\mathcal{P}} - \tilde{r}) x = \tilde{r}(x - c_0) - \frac{r_{\mathcal{P}}}{r_{\mathcal{P}} - 1}(qV - x)$$

we get

$$Z = \tilde{r}(x - c_0) - \frac{r_{\mathcal{P}}}{r_{\mathcal{P}} - 1} N$$

Otherwise, the constraint $Z = 0$ binds and we get c_1 from the investment constraint

$$c_1 = \frac{\tilde{r} c_0 + (r_{\mathcal{P}} - \tilde{r}) x}{r_{\mathcal{P}}}.$$

■

We can now show that pure equity is cheaper than pure asset buyback. In the case where $\tilde{r}(x - c_0) < \frac{r_{\mathcal{P}}}{r_{\mathcal{P}} - 1} N$, all the programs are equivalent. If $\tilde{r}(x - c_0) > \frac{r_{\mathcal{P}}}{r_{\mathcal{P}} - 1} N$, we have

$$m = x - c_0 - \frac{N}{r_{\mathcal{P}} - 1}$$

The equity program is characterized by

$$\begin{aligned} \alpha &= \frac{\tilde{r}(x - c_0) - \frac{r_{\mathcal{P}}}{r_{\mathcal{P}} - 1} N}{A + qV - \frac{r_{\mathcal{P}}}{r_{\mathcal{P}} - 1} N} \\ \Psi^e &= m - \alpha(\pi A + c_0 + N + m) \end{aligned}$$

The asset buyback is characterized by

$$\begin{aligned} Z &= \tilde{r}(x - c_0) - \frac{r_{\mathcal{P}}}{r_{\mathcal{P}} - 1} N \\ \Psi^z &= m - \pi Z \end{aligned}$$

So $\Psi^e < \Psi^z$ if and only if

$$\begin{aligned} \alpha(\pi A + c_0 + N + m) &> \pi Z \\ &\Leftrightarrow (1 - \pi) qV > \frac{1 - \pi r_{\mathcal{P}}}{r_{\mathcal{P}} - 1} N \\ &\Leftrightarrow \frac{N}{V} < \frac{(1 - \pi) q (1 - 1/r_{\mathcal{P}})}{1/r_{\mathcal{P}} - \pi} \\ &\Leftrightarrow \frac{N}{V} < (1 - q)(1 - \pi) \end{aligned}$$

But the condition $\tilde{r}(x-c_0) > \frac{r_{\mathcal{P}}}{r_{\mathcal{P}}-1}N$ is equivalent to

$$(1-\pi)(1-q) > \frac{N}{\tilde{r}(x-c_0)}$$

Since $V > \tilde{r}(x-c_0)$, $\frac{N}{V} < \frac{N}{\tilde{r}(x-c_0)} < (1-\pi)(1-q)$ so the last condition is always satisfied, and equity is always cheaper.

Finally, we can verify that the pure equity program is more expensive than the pure debt program. The pure debt program costs $\Psi = \frac{r_{\mathcal{P}}-\tilde{r}}{r_{\mathcal{P}}}(x-c_0)$. Therefore

$$\Psi^e = m - \alpha(\pi A + c_0 + N + m) < \Psi^s = \frac{(r_{\mathcal{P}}-\tilde{r})(x-c_0)}{r_{\mathcal{P}}}$$

if and only if

$$\begin{aligned}
x - c_0 - \frac{N}{r_{\mathcal{P}}-1} - \frac{\tilde{r}(x-c_0) - \frac{r_{\mathcal{P}}}{r_{\mathcal{P}}-1}N}{A + qV - \frac{r_{\mathcal{P}}}{r_{\mathcal{P}}-1}N}\left(\pi A + qV - \frac{N}{r_{\mathcal{P}}-1}\right) \quad &< \quad \frac{(r_{\mathcal{P}}-\tilde{r})(x-c_0)}{r_{\mathcal{P}}} \\
&\Leftrightarrow \quad A + qV < r_{\mathcal{P}}\pi A + r_{\mathcal{P}} qV \\
&\Leftrightarrow \quad A\left(\frac{1}{r_{\mathcal{P}}} - \pi\right) < \left(1 - \frac{1}{r_{\mathcal{P}}}\right) qV \\
&\Leftrightarrow \quad A < (1-q)V
\end{aligned}$$

which is always true.

E Proof of Proposition 11

We will guess that at a solution it turns out that $m_B < \frac{x-q^2V}{(1-q)} - c$ so the second expression is the relevant IC_G.The Lagrangian for this problem is $L(m_B, \alpha_B, m_G, \alpha_G, \lambda_{IC_G}, \lambda_{IC_B}, \lambda_{PC_G}, \lambda_{PC_B}, \lambda_{A_G}, \lambda_{A_B})$ and is given by:

$$\begin{aligned}
L \quad = \quad & \pi\{(1-\alpha_G)m_G - \alpha_G(\pi A + c_0 + N)\} + (1-\pi)\{(1-\alpha_B)m_B - \alpha_B(c_0+N)\} \\
& -\lambda_{IC_G}[(1-\alpha_G)(A + qV + c_0 + m_G - x) - (1-\alpha_B)(A + c_0 + m_B)] \\
& -\lambda_{IC_B}[(1-\alpha_B)(qV + c_0 + m_B - x) - (1-\alpha_G)q(V + c_0 + m_G - x)] \\
& -\lambda_{PC_B}[(1-\alpha_B)(c_0 + m_B + qV - x) - q(V - \tilde{r}(x-c_0))] \\
& -\lambda_{PC_G}[(1-\alpha_G)(A + qV + c_0 + m_G - x) - A - qV + \tilde{r}(x-c_0)] \\
& -\lambda_{A_G}\alpha_G \\
& -\lambda_{A_B}\alpha_B
\end{aligned}$$

F_{α_G} :

$$\begin{aligned}
\frac{\partial L}{\partial \alpha_G} \quad &= \quad \pi\{-m_G - (\pi A + c_0 + N)\} + \lambda_{IC_G}(A + qV + c_0 + m_G - x) \\
& \quad\quad +\lambda_{PC_G}(A + qV + c_0 + m_G - x) - \lambda_{IC_B}q(V + c_0 + m_G - x) - \lambda_{A_G} \\
&= \quad 0
\end{aligned}$$

or

$$\begin{aligned} & \lambda_{IC_G}\left(A+qV+c_0+m_G-x\right)+\lambda_{PC_G}\left(A+qV+c_0+m_G-x\right) \\ = \; & \lambda_{IC_B}q\left(V+c_0+m_G-x\right)+\lambda_{A_G}+\pi\left(m_G+\pi A+c_0+N\right) \end{aligned}$$

F_{m_G} :

$$\frac{\partial L}{\partial m_G}=\pi\left(1-\alpha_G\right)-\lambda_{IC_G}(1-\alpha_G)+\lambda_{IC_B}(1-\alpha_G)q-\lambda_{PC_G}(1-\alpha_G)=0$$

or

$$\pi-\lambda_{IC_G}+\lambda_{IC_B}q-\lambda_{PC_G}=0$$

F_{α_B} :

$$\begin{aligned} \frac{\partial L}{\partial \alpha_G} \; = \; & -\left(1-\pi\right)\left(m_B+c_0+N\right)-\lambda_{IC_G}\left[\left(A+c_0+m_B\right)\right] \\ & -\lambda_{IC_B}\left[-\left(qV+c_0+m_B-x\right)\right]-\lambda_{PC_B}\left[-\left(c_0+m_B+qV-x\right)\right] \\ & -\lambda_{A_B} \end{aligned}$$

or

$$\begin{aligned} & \left(1-\pi\right)\left(m_B+c_0+N\right)+\lambda_{IC_G}\left[\left(A+c_0+m_B\right)\right]+\lambda_{A_B} \\ = \; & \lambda_{IC_B}\left(qV+c_0+m_B-x\right)+\lambda_{PC_B}\left(c_0+m_B+qV-x\right) \end{aligned}$$

F_{m_B} :

$$\frac{\partial L}{\partial m_B}=\left(1-\pi\right)\left(1-\alpha_B\right)+\lambda_{IC_G}(1-\alpha_B)-\lambda_{IC_B}(1-\alpha_B)-\lambda_{PC_B}(1-\alpha_B)=0$$

or

$$\left(1-\pi\right)+\lambda_{IC_G}=\lambda_{IC_B}+\lambda_{PC_B}$$

Combining F_{m_G} and F_{m_B} we get that $\pi+\lambda_{IC_B}q-\lambda_{PC_G}=\lambda_{IC_G}$ and $\lambda_{IC_G}=\lambda_{IC_B}+\lambda_{PC_B}-1+\pi$

$$1-\lambda_{PC_G}-\lambda_{IC_B}\left(1-q\right)=\lambda_{PC_B}$$

$$\lambda_{IC_G}=q\lambda_{IC_B}-\lambda_{PC_G}+\pi$$

From these constraints it follows that either $\lambda_{IC_B}>0$ or $\lambda_{PC_B}>0$ and that either $\lambda_{IC_G}>0$ or $\lambda_{PC_G}>0$.

Conjecture: $\lambda_{IC_B}>0$ or $\lambda_{PC_B}=0$ and $\lambda_{IC_G}=0$ or $\lambda_{PC_G}>0$. Then:

$$\lambda_{PC_G}\left(A+qV+c_0+m_G-x\right)=\lambda_{IC_B}q\left(V+c_0+m_G-x\right)+\lambda_{A_G}+\pi\left(m_G+\pi A+c_0+N\right) \tag{30}$$

$$\frac{\left(1-\pi\right)\left(m_B+c_0+N\right)+\lambda_{A_B}}{\left(qV+c_0+m_B-x\right)}=\lambda_{IC_B} \tag{31}$$

$$\pi+\lambda_{IC_B}q-\lambda_{PC_G}=0 \tag{32}$$

$$\lambda^*_{IC_B}=\left(1-\pi\right) \tag{33}$$

From (32) and (31) we get that $(1-\pi)=\lambda_{IC_B}>0$ and that

$$\lambda^*_{A_B}=0.$$

Then combining these results with (32) we get that

$$\lambda^*_{PC_G} = \pi + (1-\pi)\,q.$$

Then using these results and (30) we get that

$$\begin{aligned}
(\pi + (1-\pi)\,q)\,(A + qV + c_0 + m_G - x) &= (1-\pi)\,q\,(V + c_0 + m_G - x) + \lambda_{A_G} + \pi\,(m_G + \pi A + c \\
(\pi + (1-\pi)\,q)\,A + (\pi + (1-\pi)\,q)\,(qV + c_0 + m_G - x) &= (1-\pi)\,q\,(V + c_0 + m_G - x) + \lambda_{A_G} + \pi\,(m_G + \pi A + c \\
\left(\pi + (1-\pi)\,q - \pi^2\right) A - (1-\pi)\,q\,(1-q)\,V &= \lambda_{A_G}
\end{aligned}$$

From here we get that

$$\lambda_{A_G} = \begin{cases} 0 \text{ if } \pi = 1 \\ > 0 \text{ if } \pi < 1 \end{cases},$$

which implies that

$$\alpha^*_G = 0.$$

Then we can get m_G from the participation constraint of good banks:

$$\begin{aligned}
A + qV + c_0 + m_G - x &= A + qV - \tilde{r}\,(x - c_0) \\
m^*_G &= -(\tilde{r} - 1)\,(x - c_0)\,,
\end{aligned}$$

that is good banks pay to participate.

Substituting these optimal values in IC_B we get that

$$\begin{aligned}
(1-\alpha_B)\,(qV + c_0 + m_B - x) - q\,(V - (\tilde{r} - 1)\,(x - c_0) - (x - c_0)) &= 0 \\
(1-\alpha_B)\,(qV + c_0 + m_B - x) - q\,(V - \tilde{r}\,(x - c_0)) &= 0
\end{aligned}$$

$$\begin{aligned}
(1-\alpha_B)m_B - \alpha_B\,(qV + c_0 - x) + (qV - (x - c_0)) - (qV - q\tilde{r}\,(x - c_0)) &= 0 \\
(1-\alpha_B)m_B - \alpha_B\,(qV + c_0 - x) + (q\tilde{r} - 1)\,(x - c_0) &= 0 \\
(1-\alpha_B)m_B - \alpha_B\,(qV + c_0 - x) &= (1 - q\tilde{r})\,(x - c_0)
\end{aligned}$$

$$(1-\alpha_B)m_B - \alpha_B\,(N + c_0) = (1 - q\tilde{r})\,(x - c_0)$$

The minimal cost to the government is:

$$\begin{aligned}
&\Psi^* = -\pi\,(\tilde{r} - 1)\,(x - c_0) + (1-\pi)\,(1 - q\tilde{r})\,(x - c_0) \\
=\ &(x - c_0)\,((1-\pi)\,(1 - q\tilde{r}) - \pi\,(\tilde{r} - 1)) \\
=\ &(x - c_0)\,(1 - q\tilde{r} - \pi + \pi q\tilde{r} - \pi\tilde{r} + \pi) \\
=\ &(x - c_0)\,(1 - q\tilde{r} + \pi q\tilde{r} - \pi\tilde{r}) \\
=\ &\frac{r_{\mathcal{P}} - \tilde{r}}{r_{\mathcal{P}}}\,(x - c_0)
\end{aligned}$$

Recall that the minimal cost of the equilibrium without menus is

$$\begin{aligned}
\Psi(\mathcal{P}) &= \frac{r_{\mathcal{P}} - \tilde{r}}{r_{\mathcal{P}}}\,(x - c_0) \\
&= \frac{\frac{1 - \tilde{r}\pi - \tilde{r}(1-\pi)q}{\pi + (1-\pi)q}}{\frac{1}{\pi + (1-\pi)q}}\,(x - c_0) \\
&= (1 - \tilde{r}\pi - \tilde{r}q + \tilde{r}q\pi)\,(x - c_0)
\end{aligned}$$

which is exactly the same! It is straightforward to check that there are no other solutions.

References

AKERLOF, G. A. (1970): "The Market for 'Lemons': Quality Uncertainty and the Market Mechanism," *Quarterly Journal of Economics*, 84, 488Ű500.

AUSUBEL, L. M., AND P. CRAMTON (2009): "No Substitute for the "P" Word in Financial Rescue," *Economist's Voice*, 6(2).

BERNANKE, B. (2009): "The Crisis and the Policy Response," Stamp Lecture, London School of Economics.

CRAMTON, P., AND T. R. PALFREY (1995): "Ratifiable Mechanisms: Learning from Disagreement," *Games and Economic Behavior*, 10, 255–283.

DIAMOND, D., S. KAPLAN, A. KASHYAP, R. RAJAN, AND R. THALER (2008): "Fixing the Paulson Plan," *Wall Street Journal*.

DUFFIE, D. (2009): "the failure mechanics of dealer banks," Working Paper, Stanford GSB.

PHILIPPON, T., AND P. SCHNABL (2009): "Efficient Recapitalization," NBER WP 14929.

SOROS, G. (2009): "The right and wrong way to bail out the banks," *Financial Times*, January 22.

STIGLITZ, J. (2008): "We Aren't Done Yet: Comments on the Financial Crisis and Bailout," *The Economists' Voice*, 5 (5).

Information & Technology

$t = 0$ $t = 1$ $t = 2$

Existing Assets → $a \sim F(.|\,\theta)$

Type θ revealed

New Opportunity $-x$ → $v \sim F(.|\,\theta)$

Benchmark Model