Time Varying Risk Aversion: Evidence from Russian Lottery Bonds

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JEL Classification: G1, D1, D8.
Keywords: Risk aversion, risk seeking, lottery pricing, investor sentiment.

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This paper provides direct evidence on the risk aversion of participants in a securities market. It uses the prices of lottery bonds issued by the Imperial Russian Government in 1864 and 1866 to estimate the time-variation in investor risk aversion. Time variation in investor risk aversion is then compared to the dynamics of the Russian bond market over the period 1889 to 1904, and increases in risk aversion are positively associated with increases in the price of a risk-free asset. Implications of a Consumption CAPM model for a relationship between changes in interest rates and changes and levels of risk aversion are tested. Evidence supporting the model is found. The paper provides evidence on the role of risk aversion in securities market dynamics.

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1 Introduction

Risk aversion is a fundamental concept in modern financial economics. Investors demand compensation for holding a portion of their wealth in assets with uncertain payoffs. The degree of aversion to risk determines the amount of compensation required, and is related to the returns on risky assets. Yet, empirically risk aversion is very difficult to measure – it requires prices of gambles with known probabilities. Many authors have suggested clever, indirect empirical approaches to estimate risk aversion (see, for example, Grossman and Shiller (1982)). More recently, Brav, Constantinides and Geczy (2002) use household-level quarterly consumption data from the Consumer Expenditure Survey (CEX) produced by the Bureau of Labor Statistics (BLS) and calculate the relative risk aversion (RRA) coefficient as the ratio of the sample mean of the equity premium and the covariance of consumption growth with the market index, although noisy consumption growth creates estimation problems. Guiso and Paiella (2001) are interested in the relationship between risk aversion and wealth. They use household survey data to construct a direct measure of absolute risk aversion based on the maximum price a consumer is willing to pay to enter a lottery. They argue that offering large lotteries is a better way to characterize the risk aversion of expected utility maximizers.

Schilbred (1973) used the data from the Italian bond market to estimate the market price of risk in a mean-variance equilibrium model. The bonds in the study had uncertainty over redemption. A lottery was arranged before each payment date, in order to determine which bonds were redeemed. Recently, researchers have discovered the unique possibilities of lottery bonds to allow the study of risk preferences. For example, Green and Rydqvist (1997) use Swedish bonds whose coupons are determined by lottery to estimate the pricing of idiosyncratic risk. These bonds, issued by the Swedish Treasury, have a fixed total coupon payment on any given date, but the allocation of the payment across bonds within the issue is determined by lottery.

Green and Rydqvist (1999) use Swedish lottery bonds to study the effects of differential tax rates on income and capital gains on ex-day price behavior. Florentsen and Rydqvist (2002) use the pricing of Danish lottery bonds which make coupon payments by lottery to study tax-based explanations of abnormal ex-day returns. They find evidence consistent with the costly arbitrage argument. Florentsen and Rydqvist (2002) also find that the variance of the coupon lottery influences the ex-day returns on the bonds. Ex-day returns increase with the variance of the coupon lottery, suggesting that lottery risk influences securities pricing and requires compensation.

Lottery bonds have a long history in financial markets, and have been issued in a number of different forms and structures.[1] In this paper, I use the market prices of Russian lottery bonds issued in the 1860's as the basis for estimating risk aversion. These bonds differ significantly from those studied by previous authors in that they have embedded lottery rights, and they traded for long periods of time in the secondary market at observable prices. By observing the market prices of these bonds it is possible to compute prices paid to participate in lotteries. Thus, we have a unique opportunity to study risk aversion in a non-experimental setting.

Using this data it is possible to directly estimate an Arrow-Pratt measure of risk aversion and its fluctuation through time for a representative participant in the lottery. In contrast with survey based studies that report aversion towards lotteries, I find evidence of risk seeking behavior. A risk aversion index based on lottery bond prices is also estimated. The time variation in the index is a proxy for changes in market risk preferences. The time variation in investor risk aversion is then compared to the dynamics of the Russian bond market over the period 1889 to 1904. I find evidence of a positive relationship between changes in risk aversion and changes in prices of risk-free bonds. This result is in accord with economic intuition that higher risk aversion is associated with higher demand for a safe asset and hence higher equilibrium bond prices. I also test implications of a simple one factor Consumption CAPM model with heterogeneous income risk for a relationship between changes in the risk free rates and changes in the level of risk aversion. I find evidence supporting the model. In addition, Granger (1969) causality tests indicate that changes in investor risk preferences cause changes in the interest rate. The paper thus provides some evidence on the

[1] The Danish government issued the first lottery bond in 1948. In Sweden lottery bonds have been issued since 1918 (Green and Rydqvist 1997).

role of risk aversion in security market dynamics. The intertemporal fluctuation of risk aversion is consistent with behavioral models and empirical evidence suggesting that investor sentiment fluctuates.

This paper is structured as follows. The next section describes Russian lottery bonds. Section 3 develops the method used for estimating the Arrow-Pratt measure of absolute risk aversion from lottery prices. Section 4 reports the results and Section 5 concludes.

2 Historical Background and Description of Lottery Bonds

2.1 The Reforms of Alexander II

In the years following the Crimean War (1853–1856), Russian Tsar Alexander II engaged in political and economic reforms. Tsar Alexander II liberalized foreign trade and foreign investment regulations, and in 1861 abolished the system of peasant slavery. The government lifted many limitations on private economic activities. These reforms stimulated the development of private enterprise, and fueled the growth of the Russian capital market. In 1856 six public companies with total capital of 15.5 million rubles were registered; in 1857, 14 public companies (capital of 300 million rubles); in 1858, 36 public companies were registered (total capital of 51.3 million rubles).[2] By 1865, the Russian Finance Ministry was collecting and publishing detailed price and capitalization information about its public securities markets.

A key point is that securities issued by new public companies competed with state bonds and government-run banks and often offered or promised higher rates of return. Net withdrawals from banks totalled 229 million rubles in 1858 and 355 million rubles in 1859. The assets of the state banks were mostly informal loans to the government. Due to the large withdrawals, the banks faced insolvency. Additional issuances of paper currency and foreign loans did not provide funds sufficient to cover withdrawals. Banks even gave depositors interest-bearing fixed-term bonds instead of cash. All these measures, however, could not save the banking system. In 1860 the old banks were abolished. The State Bank of the Russian Empire was created and assumed all assets and liabilities of the former government-run banks. The creation of the State Bank did not stop the wave of withdrawals. The demand for capital was high and the government even lifted the

[2] Vagner A. *Russkie bumazhnye den'gi.* Kiev, 1871, as quoted in Ukhov (2002).

ban on private borrowing from abroad–until the mid 1850's foreign capital could enter Russia only through state foreign loans. But the prestige of the Russian government suffered serious damage after the loss of the Crimean War and the State found it difficult to obtain loans abroad. Foreign investors, however, willingly purchased stocks and bonds issued by private entities, thereby investing in Russian economy. Thus, the "curtain" established by Tsar Nicholas I out of fear of foreign democratic ideas was lifted under the pressure from the expanding economy. Competitive capital markets, freed by the liberal economic policy, forced the government to look for innovative solutions to finance the budget deficit. Newly designed state lottery bonds provided a solution.

2.2 Description of Russian Lottery Bonds

The Russian Imperial Government issued lottery bonds for the first time in November of 1864. The official title of the issue was "Internal 5% Lottery Bonds." In order to attract small investors, the bonds were issued with relatively small face value of 100 rubles per bond. The total value of the issue was 100 million paper rubles, - one million individual securities were placed. All bonds were issued as bearer securities.

The bonds did not have a fixed maturity date. Instead, they were called and retired by the government in series. Securities to be recalled were determined at random. Each bond had a serial number and an individual number. Twice a year a recall drawing for serial numbers took place. When a bond was called, the owner received the face value of the bond (100 rubles) and a recall premium. The premium was set at 20 rubles per bond for the first series of recalled bonds and gradually increased to 50 rubles per bond for the bonds that were called in the later years. All bonds were to be recalled within 60 years from the issue date.

These were 5% bonds, with semiannual coupons paid on July 1st and January 2nd of every year (each coupon payment equalled 2.50 rubles). Coupons were paid for 60 years or until recall. All terms of the issue, including the provisions for calling the bonds and the amounts of monetary prizes, were printed on the back of the bond certificate in two languages: Russian and German.

The new feature that distinguished these bonds was the lottery that gave the owner a chance to win cash prizes. Twice a year, on January 2 and July 1, the Board of the State Bank conducted a random drawing to determine which bonds won the prizes. The lottery drawing was followed immediately by the recall drawing.

The random drawing was based on the bond's serial number and it's individual number. Small pieces of cardboard with an individual number and serial number of each outstanding bond were placed in large rotating drums. The members of the Board blindly picked these pieces of cardboard from the drums thus determining which bond won the prize. The same procedure (but for the serial numbers only) took place for determining which bonds were to be recalled.

At each lottery drawing 300 prizes totalling 600,000 rubles were awarded. The specific prizes are shown in **Table 1**. The winner received the payment of the prize three months after the prize was determined. The bond was stamped to record the payment. The bond could participate in all drawings until it was recalled, so, in theory, the owner had a chance of winning on several dates.

Table 1

Prizes Awarded at Each Lottery Drawing

Prize Value, Z_k	Number of Prizes
200,000 rubles	1 prize
75,000 rubles	1 prize
40,000 rubles	1 prize
25,000 rubles	1 prize
10,000 rubles	3 prizes
8,000 rubles	5 prizes
5,000 rubles	8 prizes
1,000 rubles	20 prizes
500 rubles	260 prizes

At the time of original placement, the 1864 issue of lottery bonds was moderately successful. The issue was placed at the price of 98.6 rubles per 100 ruble face-value bond. Later the interest in lottery bonds was fueled by the news of large winnings. The government decided to capitalize on the popularity of the lottery bonds and in 1866 it placed the issue of the "Second Internal 5% Lottery Bonds." The summary of features of the 1866 bonds is presented in the Appendix. The total value of this issue was also 100 million rubles. All the features of the second issue were identical to the features of the 1864 lottery bond, except the drawings were held on March 1st and September 1st, instead of July and January. Coupons on the 1866 issue were paid in March and September. The issue of 1866 was more popular and was placed at 107 rubles per 100 ruble face-value bond.

2.3 Public Perception

According to some observers, lottery bonds were by far the most popular form of government debt. The claim was: "the chance of winning a prize increased as the total number of outstanding bonds declined through recalls." It is reported that the market price of these bonds steadily increased and at times reached 1,000 rubles for one 100 ruble bond. Since the bonds were traded at a premium, the owner faced the risk of recall. The recall premium payment of 20 to 50 rubles could not compensate the bondholder for the loss of the market value. Another innovation appeared - banks offered insurance for the event of recall. This was not the only innovation inspired by the lottery bonds. Banks also purchased lottery bonds and sold *lottery participation rights*. These were securities that gave the right to participate in one lottery drawing and to obtain a specified percentage of the prize in the event of winning. By buying these securities those who could not afford to buy the bond could participate in one lottery drawing.

Lottery bonds had clearly become a part of Russian popular culture. In the 1880s a civil servant in Russia earned between 1,200 and 2,000 rubles a year and supported a family with a middle class life-style on that amount. Clearly, a lottery bond that offered a possibility to win 75,000 or even 200,000 rubles was an exciting security, and perhaps a source of hope. The time of drawings was the time of anticipation and tragedy. A great writer Anton Chehov created several scenes of emotion that involved lottery bonds. In the short story *"75,000"* published in January 1884, he describes happiness in a family when a young wife finds out that her bond has won 75,000 rubles and a deep sorrow when she discovers that the bond has been stolen and she can no longer claim the prize. In another short story, *"The Winning Ticket"*, published on March 9, 1887, a man discovers that the serial number of the bond that won 75,000 published in the newspaper matches the serial number of the bond that he owned. If only the individual numbers matched too, he would be the winner. He does not dare to look at the individual number printed in the newspaper. He dreams about being a winner. Finally he checks. The individual numbers do not match. No prize. In *"The Wedding: A Scene in One Act"*, written in October 1889, lottery bonds are a part of the bridal gift (dowry). The most financial information is contained in Chehov's story *"Everyday Troubles"*, first published on March 28, 1887. The story gives the price of the 1866 lottery bond at 246 rubles and the price of insurance against the recall at 1.10 rubles. The story also describes that the bond could be bought on margin with a down payment of 10 rubles and by borrowing the rest at 7% annual interest, and with the monthly payments of 5 rubles.

In spite of their success, or maybe precisely because of it, lottery bonds attracted criticism. In an interesting parallel to the media outcry during the recent U.S. Dot Com mania, the critics said that the bonds fed interest in public "get rich quick" schemes, instead of building strong ethics of prudence and saving. Curiously, contemporary critics complained that the bonds were an expensive way to raise capital, because large amount of money had to be paid in prizes, as well as in recall payments. They, of course ignored the premium paid for the lottery payoff.[3]

3 Calculating Risk Aversion from Lottery Prices

The method for estimating the Arrow-Pratt measure of absolute risk aversion from lottery prices is developed in this section. Let $U(W)$ be a twice continuously differentiable utility of wealth function. The Arrow-Pratt absolute risk aversion is defined as $R_A(W) \equiv -U''(W)/U'(W)$. Relative risk aversion is defined as $R_R(W) \equiv W R_A(W)$.

Consider a lottery with K prizes $\{Z_k\}_{k=1}^K$ paid with probabilities $\{\alpha_k\}_{k=1}^K$. The lottery pays zero with probability $\left(1 - \sum_{k=1}^K \alpha_k\right)$. Let λ be the price that a risk averse investor pays to participate in the lottery. Then λ (lottery participation fee) is such that investor is indifferent,

$$\begin{aligned} U(W) &= \mathbb{E}U\left(W + \widetilde{Z} - \lambda\right) \\ U(W) &= \left(1 - \sum_{k=1}^K \alpha_k\right) U(W - \lambda) + \sum_{k=1}^K \alpha_k U(W + Z_k - \lambda). \end{aligned}$$

[3] Tarankov (1992) reports that cost of capital was estimated back then at 6.34%. Consistent with Florentsen and Rydqvist (2002), we find that the lottery feature may indeed have raised, rather than lowered the cost of capital. The estimate for the cost of capital computed based on the bond pricing developed in this paper equals 6.5713%. Absent the lottery feature the cost of capital by our calculation falls to 5.3645%. Thus, lottery prizes in effect contributed a significant amount to the cost of debt. Cost of capital is computed as the discount rate that sets the bond price at issue to par value of 100.00. All calculations assume a flat yield curve (all annualized interest rates are assumed to be the same). Calculation were carried out as if there was no cancellation of bonds by the Soviet government, an event not anticipated in the late 19th century.

Use a Taylor series approximation to obtain

$$U(W) = \left(1-\sum_{k=1}^{K}\alpha_k\right)\left[U(W)-U'(W)\lambda+\frac{1}{2}U''(W)\lambda^2\right] + \sum_{k=1}^{K}\alpha_k\left[U(W)+U'(W)(Z_k-\lambda)+\frac{1}{2}U''(W)(Z_k-\lambda)^2\right] \tag{3.1}$$

Simplify (3.1), cancel $U(W)$, divide both sides by $U'(W)$, and use the definition of risk aversion $R_A(W)$ to obtain,

$$R_A(W) = -\frac{U''(W)}{U'(W)} \approx \widehat{a} = \frac{\sum_{k=1}^{K}\alpha_k Z_k - \lambda}{\frac{1}{2}\lambda^2 + \frac{1}{2}\sum_{k=1}^{K}\alpha_k Z_k^2 - \lambda\sum_{k=1}^{K}\alpha_k Z_k}. \tag{3.2}$$

To estimate the risk aversion coefficient, $\widehat{a}$, using (3.2) we need to know the value of lottery prizes, Z_k. For lottery bonds the prize values were printed on the bond certificates, so they are known from the contract specification. I show next how to calculate the probabilities α_k of winning the prizes. As an additional useful result we show how to compute the expected value of the lottery winnings. Finally, we will need to know the participation fee λ. We will show how λ is related to the market prices of the lottery bonds. In other words, we will show how to use market prices of lottery bonds to compute the market price λ of the lottery ticket that pays prizes $Z_1, \ldots, Z_K$ with probabilities $\alpha_1, \ldots \alpha_K$.

3.1 Expected Lottery Value

Let us compute the expected value (without discounting) of the lottery drawing number j, $\mathbb{E}[\text{Lottery}(j)]$. At each prize drawing j, 300 prizes were awarded. First, the drawing for the largest prize, prize 1 was conducted. Pieces of paper with the numbers identifying all outstanding bonds were placed into a drum and one piece of paper was drawn at random from the drum to determine the winner. At the time of the drawing j there were O_j bonds outstanding and each bond was equally likely to win the first prize, with probability of being selected and winning the first prize $\alpha_1 = \frac{1}{O_j}$. After a bond won a prize, the piece of paper with the number of the winning bond was returned into the drum. So the number of pieces of paper in the drum stayed constant and equalled O_j during drawings for all 300 prizes. A bond, however, could win only once. If a bond number was drawn more than once, the bond would win only the first time. In all subsequent cases the bond would be returned into the drum and a repeated drawing for the prize would take place.

For a bond to win the second prize it would have to not win the first and to win the second, so that

$$\alpha_2 = (1 - \alpha_1) \frac{1}{O_j - 1} = \frac{1}{O_j}.$$

Therefore, for a bond to win prize number k, it would have to not be drawn in the first $k - 1$ drawings and to be drawn on the $k-$th drawing. Therefore:

$$\mathbb{E}\left[\text{Lottery}(j)\right] = \sum_{k=1}^{300} \alpha_k Z_k, \qquad \alpha_k = \frac{1}{O_j} \text{ for all } k.$$

Let $D(t, j)$ be the discount factor used to obtain the present value at time t of a payment that is made at time j. The notation $D(t, j + 3m)$ means that lottery winnings determined at the lottery drawing j were actually paid three months after the date of the drawing. Then the expected value of all lottery winnings between time t and time i, discounted to obtain the present value at time t, $L(t, i)$ equals

$$L(t, i) = \sum_{j=i_1}^{i} D(t, j + 3m) \cdot \mathbb{E}\left[\text{Lottery}(j)\right],$$

where i_1 is the first drawing at or after time t.

3.2 Participation Fee and Bond Prices

Investors pay to participate in the lottery. For example, an investor could purchase a bond before a drawing date and sell the bond afterwards. Such a transaction is equivalent to purchasing a right to participate in the lottery drawing. We are interested in establishing a link between the price of such right to participate, λ, and the market prices of the bonds before and after a drawing date.

Let i^- and i^+ denote time before and after the lottery drawing i, respectively. Assume that there is exactly one lottery drawing i between times i^- and i^+. Let $P(i^-)$ and $P(i^+)$ be the observed market prices of the bonds at i^- and i^+. Use λ_i to denote the value of the right to participate in the lottery i.

More notation is needed. Write $C(t, i)$ for the present value at t of all coupons received between t and i, and $F(t, i)$ for the present value at t of the recall payment paid if recalled at i.

The bond price just before a drawing date, $P(i^-)$, is the price of (a) the claim to all coupon payments between i^- and i, including the coupon payable at i, (b) the right to participate in the lottery drawing i, (c) the right to receive the recall payment if the bond is recalled at i, and (d) the right to hold on to the bond if the bond is not recalled. Hence the bond price equals the sum of the prices of these four components, as follows:

$$P\left(i^{-}\right) = C\left(i^{-}, i\right) + \lambda_i + \mu_i \cdot F(i^{-}, i) + (1 - \mu_i) \cdot P\left(i^{+}\right),$$

where $\mu_i = \mathbb{P}\{\text{Recall on drawing } i \mid \text{Outstanding at } i^{-}\}$. Hence the right to participate in the i-th drawing is related to the bond market prices via

$$\lambda_i = P\left(i^{-}\right) - C\left(i^{-}, i\right) - \mu_i \cdot F(i^{-}, i) - (1 - \mu_i) \cdot P\left(i^{+}\right). \tag{3.3}$$

Probability of recall, μ_i, can be computed from the number of bonds outstanding at the time of the i-th drawing, O_i,

$$\mu_i = n_i / O_i,$$

where n_i is the total number of bonds recalled at i. Both n_i and O_i can be determined for any date from the contract specification. It remains to provide formulae for $C(t, i)$ and $F(t, i)$.

3.3 Coupon Payments and Recall Payment

Next, derive the expressions for $C(t, i)$ and $F(t, i)$, where i is the drawing where the bond is recalled (for sure) and t is any time before i. Let i_1 be the number of the first drawing that the bond will participate in (the first drawing at or after time t). Let $D(t, j)$ be the discount factor.

Coupons in the amount of 2.50 rubles were paid semiannually on the days of the drawings, beginning from the first drawing (1 July 1865 for the 1864 lottery bonds and 1 September 1866 for the 1866 lottery bonds). There is one special case. For the bonds recalled on the first drawing two coupons were paid - the one due at the time of the drawing and the coupon due six months afterwards. The second coupon was paid not at its due date but three months after the drawing at which the bond had been recalled. The indicator function $\mathbf{1}\{i = 1\}$ is used to account for this special case,

$$\mathbf{1}\{i = 1\} = \begin{cases} 1 & \text{if } i = 1 \\ 0 & \text{if } i \neq 1 \end{cases}$$

Using the indicator function, the expression for $C(t, i)$ becomes:

$$C(t, i) = \left(\sum_{j=i_1}^{i} D(t, j) \cdot 2.50 \right) + \mathbf{1}\{i = 1\} \cdot D(t, i + 3m) \cdot 2.50.$$

The amount paid to the owner when a bond was recalled varied from 120 to 150 rubles depending on the date of recall. The payment was made three months after the bond had been drawn for recall. Using the table printed on the bond certificate:

$$F(t,i) = D(t, i+3m) \cdot \begin{cases} 120 & \text{if } 1 \leq i \leq 20 \\ 125 & \text{if } 21 \leq i \leq 50 \\ 130 & \text{if } 51 \leq i \leq 70 \\ 135 & \text{if } 71 \leq i \leq 90 \\ 140 & \text{if } 91 \leq i \leq 100 \\ 145 & \text{if } 101 \leq i \leq 112 \\ 150 & \text{if } 113 \leq i \leq 120 \end{cases}$$

3.4 Pricing Lottery Bonds

The methodology of pricing 1864 and 1866 lottery bonds is identical since both issues had identical terms. Here the formula for pricing 1866 lottery bonds is derived, with the formula for the 1864 issue being identical except for the specific *calendar* dates of issue, coupon payments, and drawings. The calendar dates for which the formulae are valid are also different for 1864 and 1866 bonds, - it is meaningless to speak of pricing 1866 bonds on January 1, 1865 when they were not issued yet.

Let P_t^* denote the price of the lottery bond at time t. The star (*) indicates a theoretical bond price, not the observed market value. There were 120 drawings in total, on March 1 and September 1 of each year, beginning September 1, 1866. Drawings can be indexed by $i \in \{1, 2, ..., 120\}$ with drawing number 1 being the drawing of September 1, 1866. For the first 60 drawings, on each of the drawing dates, two separate and independent drawings took place: a *prize drawing* was conducted first to determine which of the outstanding bonds won the lottery prizes; a *cancellation* (or *recall*) *drawing* took place immediately after the prize drawing. Beginning with drawing number 61 (the drawing of September 1, 1896) the September drawings included only cancelation (recall) drawings - prize drawings were not conducted in September any longer. Without loss of generality, we can treat all drawings as if they included a prize drawing followed by a cancellation drawing, except for drawings $\{61, 63, 65, ..., 117, 119\}$ we set the value of the prizes to zero.

The objective is to derive an expression for the price of lottery bond, P_t^*, on a given date, t. Time t is such that there is at least one drawing that did not yet occur. Time t can be such that no drawings took place yet - we are pricing the bond on the date before or on September 1, 1866. If time t is the date of a drawing, we assume that the price P_t^* is calculated *before* any drawings take place. Also, since coupon payments and drawings take place on the same date, assume that the coupon was not yet paid - P_t^* includes the coupon payment on the date t.

Let $\mu_t(i) = \mathbb{P}\{\text{Recall on drawing } i \,|\text{Outstanding at } t\}$ be the probability of a bond being recalled during the recall drawing number i, given that as of time t the bond is outstanding, so it was not recalled during any of the previous recall drawings. Thus, for all drawings scheduled on a date before t, $\mu_t(i) = 0$. It is shown in Appendix B that

$$\mu_t(i) = \begin{cases} 0 & \text{if drawing } i \text{ takes place before time } t \\ \frac{n_{i_1}}{O_{i_1}} & \text{for the first drawing after } t\text{: } i = i_1 \\ \frac{n_i}{O_i} \cdot \prod_{j=i_1}^{i-1} \left[1 - \frac{n_j}{O_j}\right] & \text{for all other } i \end{cases}$$

Let $P_t^*(i)$ denote the price at time t of the bond that is recalled on drawing i for sure. Then, conditioning on the drawing number, i when the bond is recalled, we can write the expression for the price of lottery bond, P_t^*, on the given date, t as

$$P_t^* = \sum_{i=1}^{120} \mu_t(i) \cdot P_t^*(i).$$

Note that in the last expression we can use the notation $P_t^*(i)$ for the price at time t of the bond that is recalled on drawing i even when the date of drawing i comes before the date t, because in these cases $\mu_t(i) = 0$ and we do not need to calculate the value of $P_t^*(i)$.

The price $P_t^*(i)$ consists of $C(t,i)$, the value of all coupons received between time t and time i, discounted to time t; $L(t,i)$,the expected value of all lottery winnings between time t and time i, discounted to obtain the present value at time t; and $F(t,i)$ is the principal payment discounted to obtain the present value at time t. Therefore:

$$P_t^*(i) = C(t,i) + L(t,i) + F(t,i)$$

and hence

$$P_t^* = \sum_{i=1}^{120} \mathbb{P}\{\text{Recall on drawing } i \mid \text{Outstanding at } t\} \cdot [C(t,i) + L(t,i) + F(t,i)] \tag{3.4}$$

The formulae for $C(t,i)$ and $F(t,i)$ in the decomposition (3.4) were established earlier. We now have explicit expressions for all the components in the decomposition of the bond price P_t^*.

4 Empirical Study

4.1 Data

The data for this study consists of prices of the 1864 and 1866 lottery bonds. The prices were manually retrieved from various issues of the newspaper *Novosti i Birzhevaya Gazeta.* For each lottery date from January 1889 until January 1905, a 16-year period, two prices are collected.[4] The first one is the price on a trading date before the lottery drawing date. The second one is the ex-lottery price, which is a market price on a trading date after the lottery. There are 3 missing observations due to incomplete records, leaving 42 data points.[5]

A few additional assumptions are required. First, for purposes of avoiding the estimation of a full bond pricing model, we assume a flat yield curve. Second, we assume the probability of default by the government is zero. We know, of course, that these bonds eventually defaulted. However, we would argue that expectations of such an event were minimal for much of the period of interest. Also, we use the *change* in bond prices around the lottery drawing date to estimate risk aversion. If bond prices before and after the lottery date both contain the same default information, then we do not need to account for a default probability.

For each drawing date t, use (3.3) to compute the value of the lottery participation right, λ_t, from the bond prices. I then use (3.2) to obtain the estimate of the Arrow Pratt measure of risk aversion at time t, $\widehat{a}_t$. This procedure produces a time series of the estimated risk aversion coefficient.

The first result is that out of 42 estimates only one is above zero, indicating risk aversion. All the other estimates of the Arrow-Pratt measure of risk aversion are negative, meaning *risk seeking*. This is in sharp contrast with the most studies based on surveys, where the authors find

[4] A note on the calendars and dates. The Russian Empire used the Julian calendar. It differs from the Gregorian calendar, used in the West at the time and throughout most of the world now. The Julian calendar specifies that every year that is a multiple of 4 is a leap year. The Gregorian calendar excludes leap years in century years not divisible by 400. So, for example 1900 was not a leap year, although 2000 was. At the time the Julian calendar was 11 days behind the Gregorian calendar. Russia converted from the Julian calendar to the Gregorian calendar in 1918 – February 1, 1918 was followed by February 14, 1918.

[5] For the 1864 bond there are two missing observations during the period; one that corresponds to the July 1, 1890 lottery and the other that corresponds to the January 2, 1895 lottery drawing. For the 1866 bond there is one missing observation that corresponds to the March 1, 1898 drawing.

self-reported risk aversion towards lottery tickets. The Table 2 shows the summary statistics for the estimated risk aversion coefficient.

Table 2

Risk Aversion Estimate Summary Statistics

Mean	-0.00010
Median	-0.00008
StDev	0.00006
Minimum	-0.00027
Maximum	2.50×10^{-6}

For comparison, Guiso and Paiella (2001) find that in their survey, 96 percent of participants are risk averse, and 4 percent are either risk neutral (3.6% of the sample) or risk loving (0.04% of the sample). The average absolute risk aversion coefficient *among risk neutral and risk seeking individuals* in their sample is, $\widehat{a} = -0.005$.

Since estimated coefficient of risk aversion is negative in our sample (with exception of one value), for the rest of the paper it is convenient to work with the *risk seeking coefficient*, defined as the negative of the estimated risk aversion coefficient,

$$RS_t = -\widehat{a}_t.$$

A positive value of the risk seeking coefficient means risk seeking, and a negative value means risk aversion.

Figure 1 is a time series plot of the risk seeking coefficient RS_t estimated from the change in prices of 1864 bonds and 1866 bonds. Estimation errors are reflected on the plot as differences in values of the coefficients obtained from the two bonds when observations are close in time. Still, both series are close in value. This is as expected, since the estimates are obtained from price changes on two bonds that were very close substitutes to each other. It is interesting to note that the peak of risk seeking in March of 1899 corresponds to a high level of stock prices on the St. Petersburg Stock Exchange.

The estimate of the risk aversion coefficient is based on λ_t, the value to investors of the right to participate in the lottery. This value is computed from the price change on the lottery date. Hence, two prices around each lottery date are required to compute each risk aversion estimate. The number and frequency of observations in the risk aversion series thus depend on the number and frequency of lottery drawing dates. This limitation on the number of observations will affect the power of tests that use this data. The power to detect relationships will be further affected by the estimation errors. It is shown next how levels of prices can be used to construct a proxy for risk aversion, increasing the number of observations and power of the tests.

4.2 Risk Attitude Index

Market prices of lottery bonds on any trading date, however, contain information on attitudes towards risk. To extract such information, we can compare the market price of a lottery bond to the price that a rational risk-neutral investor would pay for it. The rational price can be computed using the bond pricing equation (3.4). We now develop this approach more formally. Equation (3.4) for pricing a lottery bond on date t can be written to highlight the fact that bond price consists of two components, (a) the value of all coupons and expected recall premium; and (b) the value of all future lotteries,

$$
\begin{aligned}
P_t^* &= \sum_{i=1}^{120} \mathbb{P}\{\text{Recall on drawing } i \mid \text{Outstanding at } t\} \cdot [C(t,i) + L(t,i) + F(t,i)] \\
&= \sum_{i=1}^{120} \mathbb{P}\{\text{Recall on drawing } i \mid \text{Outstanding at } t\} \cdot [C(t,i) + F(t,i)] \\
&\quad + \sum_{i=1}^{120} \mathbb{P}\{\text{Recall on drawing } i \mid \text{Outstanding at } t\} \cdot L(t,i).
\end{aligned}
$$

The first sum is the value of the bond, absent the lottery feature. The second sum is the fair value of the future lotteries.

The market price of all future lottery winnings, $\mathbb{M}_t$ [Lotteries], equals the market price of the bond minus the fair value of all coupons and recall payments,

$$
\mathbb{M}_t\,[\text{Lotteries}] = P\,(t) - \sum_{i=1}^{120} \mathbb{P}\{\text{Recall on drawing } i \mid \text{Outstanding at } t\}\,[C(t,i) + F(t,i)]\,.
$$

Given the market price of the bond and the discount rate we can now compute $\mathbb{M}_t$ [Lotteries]. The fair value at time t of future lotteries is given by the sum

$$\mathbb{E}_t\left[\text{Lotteries}\right] = \sum_{i=1}^{120} \mathbb{P}\{\text{Recall on drawing } i \mid \text{Outstanding at } t\} \cdot L(t,i),$$

which can be computed for any date t. Define the Risk Seeking Index at time t as the ratio of the market value of the future lotteries to the fair value of the lotteries,

$$RSI_t = \frac{\mathbb{M}_t\left[\text{Lotteries}\right]}{\mathbb{E}_t\left[\text{Lotteries}\right]} - 1.$$

The index value equals 0 when investors will pay exactly the expected value of future lotteries and corresponds to the case of risk-neutrality. A value of the index below 0 corresponds to risk aversion, since the market value of the lotteries is less than their expected value. Finally, the index above 0 indicates risk seeking behavior. For example, value of $RSI_t = 1$ means that investors will pay twice as much as the expected value of all future lottery winnings.

We use 99 monthly observations of market prices of 1866 lottery bond to compute the index time series, RSI_t. Discount rates are the yields on the 5% Russian Government perpetuity of 1822.[6] Figure 2 is a plot of the RSI and the time series of the estimated risk seeking coefficient. Both series move together (the correlation coefficient equals 0.67, significant at 1%). From late 1897 until 1900 the risk seeking coefficient estimated from bond price changes on lottery dates diverges from the risk seeking index based on the bond price levels. The index displays less volatility during this period than the estimated risk seeking coefficient. After 1900, however, the RSI index becomes more volatile than the estimated risk seeking coefficient.

The RSI Index value starts at 7 in 1889 and quickly falls to 5 where it stays until June 1895. It then begins to grow, at what appears to be an exponential rate until it reaches the maximum of 16.5 at the end of January 1902. Then there is a rapid decline in the risk seeking index, followed by growth. Index summary statistics is presented in the following table.

[6] I thank William N. Goetzmann for sharing bond yield data.

Table 3

Risk Seeking Index Summary Statistics

Mean	7.328
Median	6.253
StDev	3.310
Minimum	4.258
Maximum	16.497

These numbers indicate that, on average, investors were willing to pay 8 times the expected value of winnings to participate in the lottery! Often this number was above 10, and not once there was any evidence of risk aversion.

Having constructed the measure of risk attitude, we proceed to developing testable hypothesis about the relationship between interest rates and risk aversion.

4.3 One Factor Consumption CAPM

Consider a standard one-period Consumption CAPM equilibrium model with negative exponential utility of consumption and jointly normally distributed endowment and asset payoffs. There is a continuum of agents, $h \in H$, each endowed with initial wealth W_0^h. There is one risky asset in the economy, the market, and a risk free bond. Both assets are assumed to be in zero net supply. Trading takes place at time 0. All uncertainty is resolved at time 1, when the risky asset pays off a random amount, $\widetilde{z}_m$, and terminal endowment $\widetilde{e}^h$ is also realized. At $t = 1$ riskless asset pays 1 unit of the consumption good. At this time agents consume all their wealth. Payoffs are generated according to one factor model,

$$\begin{aligned} \widetilde{e}^h &= \overline{e}^h + b^h \widetilde{f} + \widetilde{\varepsilon}^h, \\ \widetilde{z}_m &= \overline{z} + \widetilde{f}. \end{aligned}$$

The factor $\widetilde{f}$ and the agent's idiosyncratic endowment shock $\widetilde{\varepsilon}^h$ are independently normally distributed with

$$\begin{aligned}\mathbb{E}\left[\widetilde{\varepsilon}^h\right] &= \mathbb{E}\left[\widetilde{f}\right]=0,\\ \mathbb{E}\left[\widetilde{\varepsilon}^h\widetilde{f}\right] &= 0,\\ \mathbb{E}\left[\left(\widetilde{\varepsilon}^h\right)^2\right] &= \sigma^2_{\varepsilon^h}, \qquad \mathbb{E}\left[\widetilde{f}^2\right]=1.\end{aligned}$$

An agent h has a factor loading b^h; $\overline{z}$ is the expected payoff on the market portfolio, and $\overline{e}^h$ is the expected value of the agent's terminal endowment.

At time 0 agent $h \in H$ solves the consumption-portfolio problem

$$\begin{aligned}&\max_{c_0^h,\theta_0^h,\theta_b^h}\left\{-e^{-ac_0^h}-\delta\mathbb{E}e^{-a\widetilde{c}_1^h}\right\},\\ s.t.\ W_0^h &= c_0^h+p_0\cdot\theta_0^h+\theta_b^h/R,\end{aligned}$$

where a is the risk aversion coefficient and $\delta \in (0,1)$ is the discount factor. There are three decision variables: time 0 consumption c_0^h, time 0 post-trade holdings of the market θ_0^h, and time 0 investment in the bond, θ_b^h. Time 0 equilibrium price of the market portfolio is p_0, and R is one plus the risk free rate of return from $t=0$ to $t=1$.

For each agent $h \in H$, asset payoff and terminal endowment $\widetilde{e}^h$ are jointly normally distributed, so standard CARA-Normal solution method is valid. Define economy-wide averages

$$\begin{aligned}W_0 &\equiv \int_H W_0^h d\mathbb{P}(h), \qquad \overline{e}\equiv\int_H \overline{e}^h d\mathbb{P}(h),\\ b &\equiv \int_H b^h d\mathbb{P}(h), \qquad \widetilde{e}\equiv\int_H \widetilde{e}^h d\mathbb{P}(h).\end{aligned}$$

In equilibrium, the price of the risky asset and the risk-free rate are given by,

$$p_0 = \frac{\overline{z}-aCov\left(\widetilde{z}_m,\widetilde{e}\right)}{R}, \tag{4.1}$$

$$\ln R = -\ln\delta+\left(\overline{e}-W_0\right)a-\frac{a^2}{2}\left[\int_H \sigma^2_{\varepsilon^h}d\mathbb{P}(h)+b^2\right], \tag{4.2}$$

where $Cov\left(\widetilde{z}_m,\widetilde{e}\right)=\int_H b^h d\mathbb{P}(h)=b$. The term in the square brackets is a measure of endowment risk in the economy.

If we assume that there is no terminal endowment payoff, then the relationship between risk aversion and the interest rate is given by

$$\ln R = -\ln\delta - W_0 a.$$

This relationship has a clear economic intuition. One would expect a higher level of risk aversion to be associated with lower risk free interest rate. Higher risk aversion implies higher precautionary savings motive and higher demand for the bond, resulting in a lower risk free rate. The negative relationship between the log-interest rate and risk aversion is equivalent to a positive relationship of the interest rate and risk seeking. This is the first testable hypothesis.

Now consider an economy with endowment shock. Testable implications can be derived from (4.2) by noting that the term in square brackets is a positive constant, and taking the derivative

$$\begin{aligned} \frac{d\ln R}{dt} &= K_1 \cdot \frac{da}{dt} - K_2 \cdot a\frac{da}{dt}, \\ K_1 &\equiv \overline{e} - W_0, \qquad K_2 \equiv \left[\int_H \sigma^2_{\varepsilon^h} d\mathbb{P}(h) + b^2\right] > 0. \end{aligned}$$

To test the hypothesis $K_2 > 0$, note that our measure of risk seeking is a negative of risk aversion, $RS_t = -\widehat{a}_t$, and $\Delta RS_t = -\Delta\widehat{a}_t$, where Δ is the first difference operator.

Write the above as a linear regression in first differences and levels,

$$\begin{aligned} \Delta \ln R_t &= \beta_1 \cdot \Delta RS_t + \beta_2 \cdot RS_{t-1} \cdot \Delta RS_t + v_t, \\ \beta_1 &= -K_1, \qquad \beta_2 = -K_2 < 0. \end{aligned} \tag{4.3}$$

We proceed by investigating the sign of K_1 empirically, and then by constructing a risk seeking index and formally testing the hypothesis $\beta_2 < 0$.

As a third model, briefly consider the Consumption CAPM with power utility of consumption, such as in Brav, Constantinides, and Geczy (2002), with $\gamma > 0$ being the constant coefficient of Relative Risk Aversion. Then, from the Euler equation,

$$\mathbb{E}\left[\delta\left(\frac{\widetilde{c}_1}{c_0}\right)^{-\gamma}(1+Y)\right] = 1,$$

it can be shown that

$$\frac{dY_t}{1+Y_t} = K d\gamma_t.$$

In the last equation Y_t and R_t are related via $R_1 = 1 + Y_t$, and K is is defined as,

$$K \equiv \mathbb{E}\left[\left(\frac{\widetilde{c}_1}{c_0}\right)^{-\gamma}\ln\left(\frac{\widetilde{c}_1}{c_0}\right)\right] / \mathbb{E}\left[\left(\frac{\widetilde{c}_1}{c_0}\right)^{-\gamma}\right].$$

Consumption CAPM with a power utility function assumes that terminal consumption, $\widetilde{c}_1$, has a log-normal distribution. Expected consumption growth rate, $\mathbb{E}\left[(\widetilde{c}_1/c_0)^{-\gamma}\right]$, is positive, and the sign of K is determined by the sign of the numerator. The numerator is the expected value of a log-normal random variable multiplied by a normal random variable. The sign of this expected value depends on the mean and variance of the distribution of terminal consumption, $\widetilde{c}_1$. The numerator is positive for sufficiently high value of the mean. The numerator is negative for low values of the mean. The sign of the numerator depends also on the variance of the terminal consumption. The numerator is negative for high values of the variance. Thus, the value of K will be negative when the expected value of the terminal consumption is low, or when the uncertainty over the terminal consumption, represented by the variance, is high.

Using a measure of risk aversion we can test whether the data supports the postulated relationship between risk aversion and bond yields. To do this, we test the statistical significance of the K coefficient in a linear model.

One caveat. The hypothesis is based on an equilibrium analysis that assumes risk aversion of agents, so that the objective function in the utility optimization problem is concave. As reported, we find evidence of risk seeking in our sample. This finding should not be taken to imply that all agents are risk seeking. The time variation in the measure of risk seeking should be taken as a proxy for variation in risk preference in the society. It is the relationship between changes in risk references and changes in interest rates that is of interest.

4.4 Risk Aversion and Bond Yields

We now investigate the effect of risk aversion on yields of risk free Russian bonds. Figure 3 is a plot of the RSI and of the yield on the 1822 perpetuity. Bond yield Y_t is the yield on the 5% Russian Government perpetuity of 1822. To control for serial correlation in yield series, lagged values are included in the regression of log-yield on the estimated risk seeking coefficient:

$$\begin{aligned}
\ln(1+Y_t) &= \underset{(-0.51)}{-0.00142} + \underset{(15.61)}{1.00} \cdot \ln(1+Y_{t-1}) + \underset{(1.37)}{6.52} \cdot RS_t + \underset{(1.19)}{5.57} \cdot RS_{t-1} \\
R^2 &= 89.2\%, \qquad R^2_{adj} = 88.3\%, \qquad DW = 1.86
\end{aligned}$$

The numbers in parenthesis are the t-statistics. The coefficients for the current and the lagged risk seeking estimate are not significant at conventional levels (the p-values are 0.179 and 0.241). Positive value of the coefficient for RS_t is encouraging, however, because it agrees with the postulated hypothesis of a positive relationship between risk seeking and the interest rate.

Next, use the RSI index time series that has more observations to estimate linear model in levels,

$$\begin{aligned}\ln(1+Y_t) &= \underset{(1.04)}{0.000984} + \underset{(42.51)}{0.966} \cdot \ln(1+Y_{t-1}) + \underset{(3.47)}{0.000344} \cdot RSI_t - \underset{(-3.09)}{0.000310} \cdot RSI_{t-1} \\ R^2 &= 95.3\%, \qquad R^2_{adj} = 95.1\%, \qquad DW = 1.77\end{aligned}$$

We find evidence of a positive relationship between risk seeking and risk-free rate. This agrees with economic intuition that higher risk seeking is associated with less demand for a safe asset, lower bond prices, and hence, higher yields. The estimated positive relationship between changes in risk seeking and changed in bond yield is equivalent to a positive relationship between changes in risk aversion and changes in riskless bond prices, - higher degree of risk aversion leading to higher prices for the safe asset.

We first-difference the data and estimate the relation

$$\begin{aligned}\Delta \ln(1+Y_t) &= \underset{(-0.76)}{-0.000066} + \underset{(3.51)}{0.000350} \cdot \Delta RSI_t \\ R^2 &= 11.4\%, \qquad R^2_{adj} = 10.5\%, \qquad DW = 1.72 \\ \Delta \ln(1+Y_t) &= \ln(1+Y_t) - \ln(1+Y_{t-1}) \\ \Delta RSI_t &= RSI_t - RSI_{t-1}\end{aligned}$$

The numbers in parenthesis are t-statistics. The slope coefficient is significant at 1% level. Again, the regression indicates that an increase in risk seeking measure corresponds to an increase in yields on a riskless bond.

The result is also consistent with the notion of competition among government bonds. Increased appetite for risk results in increased demand for the lottery bonds. Yields on other government securities must then increase.

To check robustness, we remove six influential observations and re-estimate the last equation. In the new regression the slope coefficient remains positive and significant at the 1% level.

As another robustness check, we decompose the change in the market value of the future lotteries into the change in the expected lottery value and change in risk preferences. We estimate the following relationship,

$$\begin{aligned}\Delta \mathbb{M}_t\,[\text{Lotteries}] &= \underset{(0.68)}{0.84} + \underset{(2.08)}{6.58} \cdot \Delta \mathbb{E}_t\,[\text{Lotteries}] + \Delta \widehat{u}_t \\ R^2 &= 4.3\%, \qquad R^2_{adj} = 3.3\%, \qquad DW = 1.96\end{aligned}$$

The numbers in parenthesis are t-statistics. The slope coefficient is significant at 5% level. The residuals $\Delta\widehat{u}_t$ correspond to the change in risk preferences. We now estimate

$$\begin{aligned}\Delta Y_t &= \underset{(-0.49)}{-0.000043} + \underset{(3.37)}{0.000024} \cdot \Delta\widehat{u}_t \\ R^2 &= 10.6\%, \qquad R^2_{adj} = 9.7\%, \qquad DW = 1.73\end{aligned}$$

The slope coefficient is significant at 1% level.

Having established a positive relationship between changes in risk seeking and changes in yields, we proceed to testing (4.3) directly. The following linear model is estimated

$$\begin{aligned}\Delta \ln(1+Y_t) &= \underset{(-1.40)}{-0.000115} + \underset{(4.40)}{0.00172} \cdot \Delta RSI_t - \underset{(-3.61)}{0.000134} \cdot RSI_{t-1} \cdot \Delta RSI_t \\ R^2 &= 22.1\%, \qquad R^2_{adj} = 20.4\%, \qquad DW = 1.99\end{aligned}$$

The numbers in parenthesis are t-statistics. The slope coefficients are significant at 1% level. The coefficient on ΔRSI_t is positive, as was found in the previous tests. The coefficient on the product term, $RSI_{t-1} \cdot \Delta RSI_t$, is negative as predicted by the Consumption CAPM model. The model is also estimated with intercept set to zero. The result is

$$\begin{aligned}\Delta \ln(1+Y_t) &= \underset{(4.20)}{0.00162} \cdot \Delta RSI_t - \underset{(-3.41)}{0.000126} \cdot RSI_{t-1} \cdot \Delta RSI_t \\ DW &= 1.94\end{aligned}$$

The numbers in parenthesis are t-statistics. The slope coefficients are significant at 1% level.

We also perform a test of the implication from the CCAPM with power utility of consumption by estimating the model

$$\begin{aligned}\frac{\Delta Y_t}{1+Y_{t-1}} &= \underset{(-0.76)}{-0.000065} + \underset{(3.51)}{0.000350} \cdot \Delta RSI_t \\ R^2 &= 11.4\%, \qquad R^2_{adj} = 10.5\%, \qquad DW = 1.72 \\ \Delta Y_t &= Y_t - Y_{t-1}\end{aligned}$$

The numbers in parenthesis are t-statistics. The slope coefficients are significant at 1% level.

Time variation in the RSI index is a proxy for changes in market risk preferences. When this proxy is used to test theoretical prediction for the relationship between changes in yields and levels and changes in risk preferences, I find strong evidence supporting the model.

4.5 Causality Tests

Classical tests for causality, as introduced by Granger (1969), are used to test whether changes in risk preferences cause changes in the interest rate. To test the null hypothesis that changes in risk preferences do not cause changes in the interest rate, estimate two regressions,

$$\begin{aligned} \text{Unrestricted regression} &: \quad \Delta Y_t = \beta_0 + \beta_1 \Delta Y_{t-1} + \beta_2 \Delta RSI_{t-1} + \epsilon_t \\ \text{Restricted regression} &: \quad \Delta Y_t = \beta_0 + \beta_1 \Delta Y_{t-1} + \epsilon_t \end{aligned}$$

The F statistic is 7.34 and we reject the null (p value is 0.008). We conclude that changes in risk preferences, captured by changes in the index, cause changes in the interest rate.

To test the null hypothesis that changes in the interest rate do not cause changes in the measure of risk preferences, the RSI index, estimate two regressions,

$$\begin{aligned} \text{Unrestricted regression} &: \quad \Delta RSI_t = \beta_0 + \beta_1 \Delta RSI_{t-1} + \beta_2 \Delta Y_{t-1} + \epsilon_t \\ \text{Restricted regression} &: \quad \Delta RSI_t = \beta_0 + \beta_1 \Delta RSI_{t-1} + \epsilon_t \end{aligned}$$

The F statistic for the test $\beta_2 = 0$ equals 0.0039 (p value is 0.95) and we cannot reject the null. We conclude that changes in interest rates do not cause changes in the RSI index, the metric used to reflect investor risk preferences.

5 Conclusion

In this paper I use market prices of lottery bonds to obtain a time series of the estimated Arrow-Pratt measure of absolute risk aversion of market participants. I find that buyers of lottery bonds pay prices consistent with risk seeking behavior over the time span of 16 years. This is in contrast with surveys that find evidence of self-reported risk aversion.

I also find that the degree of risk seeking among market participants varies over time. I use a risk aversion index based on lottery bond prices as a proxy for market risk preferences. The time variation in investor risk aversion index is compared to the dynamics of the Russian bond market over the period 1889 to 1904. I find evidence of a positive relationship between changes in risk aversion and changes in prices of risk-free bonds. This result is in accord with economic intuition that higher risk aversion is associated with higher demand for a safe asset and hence higher equilibrium bond prices. I also test implications of a simple one factor Consumption CAPM model with heterogeneous income risk for a relationship between changes in risk free rate and changes and levels of risk aversion. I find evidence supporting the model. An interesting extension to the paper is to compare the behavior of the Russian stock market to the behavior of risk aversion measures developed in this paper.

A Appendix. 1866 Lottery Bonds: Summary of Contractual Features

- Bond face value: 100 rubles.
- Principal Repayment: The bonds were recalled through random *recall drawings* during 60-year period. Recall Drawings took place on March 1 and September 1 of every year. First recall drawing: September 1, 1866. Last recall drawing: March 1, 1926. The bonds were recalled *in series*. The number of bonds recalled at each recall drawing was stated in the schedule of drawings printed on the back of each bond certificate.
- When a bond was recalled the owner received an amount exceeding the face value. The amount paid depended on the drawing and grew over time from 120 to 150 rubles. The recall payments are summarized in the Table A1. The payment on recalled bonds was made three months after the recall drawing.

Table A1

Recall payment on randomly recalled bonds.

Recall Payment rubles	Total Number of Bonds	Effective Dates
120	45,100	1 Sep 1866 - 1 Mar 1876
125	108,500	1 Sep 1876 - 1 Mar 1891
130	137,800	1 Sep 1891 - 1 Mar 1901
135	222,200	1 Sep 1901 - 1 Mar 1911
140	140,100	1 Sep 1911 - 1 Mar 1916
145	196,500	1 Sep 1916 - 1 Mar 1922
150	149,800	1 Sep 1922 - 1 Mar 1926

- Once a bond is recalled, it no longer earns interest. However, the bonds recalled at the first drawing will receive annual coupon payment at the same time as they receive the recall payment.
- Prize Drawing and recall drawing took place on the same date, according to the schedule. On the day of the drawings prize drawing took place before recall drawing. For the first time

both drawings are conducted on September 1, 1866. Prize drawings take place twice a year during the first 30 years and once per year during the last 30 years.

Table A2

Prizes Awarded at Each Lottery Drawing

Prize Value, Z_k	Number of Prizes
200,000 rubles	1 prize
75,000 rubles	1 prize
40,000 rubles	1 prize
25,000 rubles	1 prize
10,000 rubles	3 prizes
8,000 rubles	5 prizes
5,000 rubles	8 prizes
1,000 rubles	20 prizes
500 rubles	260 prizes

- Prizes are drawn for individual bonds. In a prize drawing a serial number is drawn from one drum and individual bond number is drawn from different drum, both numbers together identifying individual bond. After the winner for a prize is announced the numbers are returned into the drums. However, on any given date a bond can win only one prize.

- The prizes are paid three months after the drawing date. A winning bond remains the property of its owner and can participate in lottery drawings on all dates until it is recalled at one of the recall drawings.

B Appendix

Here we derive the expression for the conditional probability of recall. Probability $\mu_t(i) = \mathbb{P}\{\text{Recall on drawing } i \,|\text{Outstanding at } t\}$ is the probability that a bond will be recalled at recall drawing number i conditional on not being recalled at any of the drawings that took place prior to time t.

At a recall drawing i, n_i bonds are recalled. All numbers n_i are known from the table printed on the bond certificate. There were 1,000,000 bonds originally issued. Therefore, O_i, the number of bonds outstanding at the time of the i-th drawing,

$$O_i = 1,000,000 - \sum_{j=1}^{i-1} n_j.$$

If $i = 1$ the sum is an empty sum and equals zero.

Let i_1 be an integer in the interval $[1, 120]$ that equals the number of the first drawing that the bond will participate in (the first drawing at or after time t). At this drawing n_{i_1} bonds will be recalled from O_{i_1} outstanding bonds. Therefore:

$$\mu_t(i_1) = \frac{n_{i_1}}{O_{i_1}}.$$

For a bond to be recalled at the next drawing after the drawing i_1, the bond must not be recalled at i_1 (event that occurs with probability $1 - n_{i_1}/O_{i_1}$) and the bond must be recalled at this next drawing, $i_1 + 1$. Therefore:

$$\mu_t(i_1 + 1) = \left[1 - \frac{n_{i_1}}{O_{i_1}}\right] \cdot \frac{n_{i_1+1}}{O_{i_1+1}}.$$

Applying this argument by induction, we obtain:

$$\Pr\{\text{Recall on drawing } i \text{ for } i > i_1 \mid \text{Outstanding at } t\} = \frac{n_i}{O_i} \cdot \prod_{j=i_1}^{i-1} \left[1 - \frac{n_j}{O_j}\right]$$

and therefore:

$$\mu_t(i) = \begin{cases} 0 & \text{if drawing } i \text{ takes place before time } t \\ \frac{n_{i_1}}{O_{i_1}} & \text{for the first drawing after } t\text{: } i = i_1 \\ \frac{n_i}{O_i} \cdot \prod_{j=i_1}^{i-1} \left[1 - \frac{n_j}{O_j}\right] & \text{for all other } i \end{cases}$$

It is comforting to know that these probabilities add up to unity, as established below.

Proposition B.1 (Recall probabilities)

$$\sum_{i=1}^{120} \mu_t(i) = 1, \qquad \forall t.$$

Proof. We want to prove: $\sum_{i=1}^{120} \Pr\{\text{Recall on drawing } i \mid \Im_t\} = 1$. By definition, $O_i = 1,000,000 - \sum_{j=1}^{i-1} n_j$ and hence, $O_{i+1} = 1,000,000 - \sum_{j=1}^{i} n_j = 1,000,000 - \sum_{j=1}^{i-1} n_j - n_i = O_i - n_i$. Hence, $O_{i+1} = O_i - n_i$.

At time t we know the integer i_1, - the first drawing at which the bond might be recalled. Write

$$\sum_{i=1}^{120} \mu_t(i) = \sum_{j=1}^{i_1-1} 0 + \frac{n_{i_1}}{O_{i_1}} + \sum_{j=i_1+1}^{120} \frac{n_j}{O_j} \prod_{k=i_1}^{j-1} \left[1 - \frac{n_k}{O_k}\right]$$

Now consider the product

$$\begin{aligned}\prod_{k=i_1}^{j-1} \left[1 - \frac{n_k}{O_k}\right] &= \frac{O_{i_1} - n_{i_1}}{O_{i_1}} \times \frac{O_{i_1+1} - n_{i_1+1}}{O_{i_1+1}} \times \ldots \times \frac{O_{j-2} - n_{j-2}}{O_{j-2}} \times \frac{O_{j-1} - n_{j-1}}{O_{j-1}} \\ &= \frac{O_{i_1+1}}{O_{i_1}} \times \frac{O_{i_1+2}}{O_{i_1+1}} \times \ldots \times \frac{O_{j-1}}{O_{j-2}} \times \frac{O_j}{O_{j-1}} \\ &= \frac{O_j}{O_{i_1}}\end{aligned}$$

where the second equality follows from $O_{i+1} = O_i - n_i$.

Then

$$\sum_{i=1}^{120} \mu_t(i) = \frac{n_{i_1}}{O_{i_1}} + \sum_{j=i_1+1}^{120} \frac{n_j}{O_j} \frac{O_j}{O_{i_1}} = \frac{n_{i_1}}{O_{i_1}} + \frac{1}{O_{i_1}} \sum_{j=i_1+1}^{120} n_j = \frac{1}{O_{i_1}} \sum_{j=i_1}^{120} n_j$$

Recall that O_{i_1} is the number of bonds outstanding at the time of the drawing i_1. During the drawings i_1 through 120 all outstanding bonds, O_{i_1}, must be recalled:

$$\sum_{j=i_1}^{120} n_j = O_{i_1} \Rightarrow \sum_{i=1}^{120} \mu_t(i) = 1.$$

■

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Figure 1
Estimated Risk Seeking Coefficient
From Price Changes of 1864 and 1866 Lottery Bonds

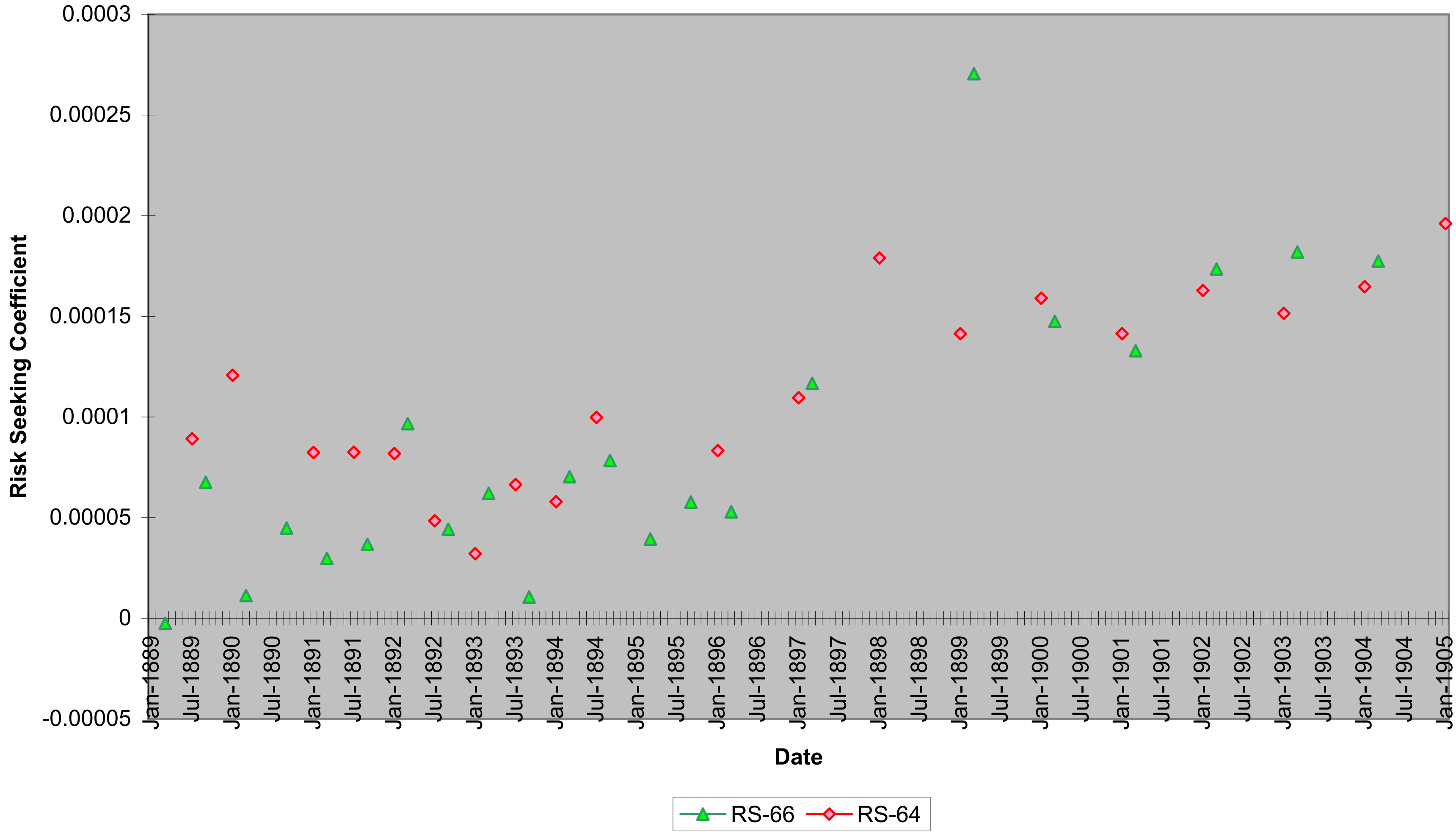

Figure 2
Risk Seeking Index and Estimated Risk Seeking Coefficient

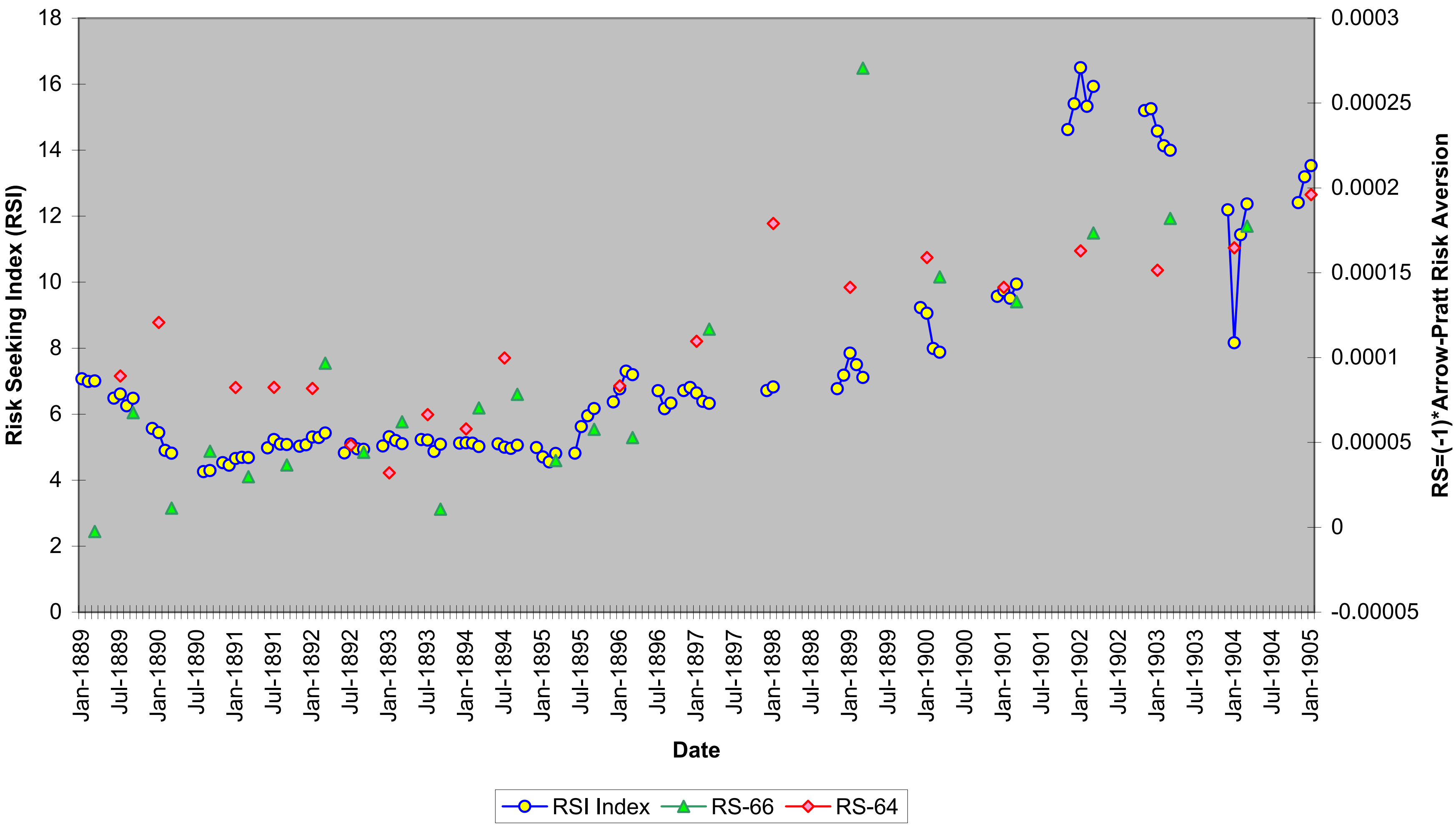

Figure 3
Risk Seeking Index and Government Bond Yield