Is Momentum a Self-fulfilling Prophecy?

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Abstract

I show in a simple framework that (1) momentum trading can exist in equilibrium and (2) momentum trading is profitable, but not riskless. The model has several nice properties that fit the empirics well. First, it captures in a parsimonious manner both short-term overreaction and long-term reversals. Second, it predicts that momentum and long-term reversals should be observed in any market where there is noise. Thus, the model gives theoretical support to the empirical evidence that these anomalies are not artifacts of data snooping and to the extant empirical evidence that these anomalies are pervasive. It is interesting to note that momentum traders observe noise shocks and trade on it as information. This trading incorporates a predictive role to the noise. Also, momentum trading is a self-fulfilling action. If agents believe a past price change to be informative of future price changes and act on this belief, it will be true and trading on this belief will be profitable.

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1 Introduction

Speculation and its affect on markets have been a hot debate in the economic literature for centuries.[1] Friedman held a common view of speculators in that they buy low and sell high. Friedman (1953) states: "People who argue that speculation can be destabilizing seldom realize that this is largely equivalent to saying that speculators lose money, since speculation can be destabilizing in general only if speculators sell when the currency is low and buy when it is high." As Hart and Kreps (1987) point out, this is not precise. Speculators buy when they expect prices to increase and sell when they expect prices to decrease. Much to Friedman's horror, momentum is precisely that kind of speculation that he refers to as "market destabilizing" and "money losing." Momentum stocks are stocks that are bought specifically because they have experienced extreme advances and they are sold specifically because they have experienced extreme decreases. Jegadeesh and Titman (1993, 2001) document the wide spread existence and profitability of momentum. They make a strong argument that momentum is a real and robust affect.[2] Momentum is speculation, and according to Friedman, it is speculation of the worst kind. The main objective of this study is to explore if momentum can exist in equilibrium and if so, its implications for markets.

Momentum trading can be viewed as a type of inventory speculation where the asset traded is the storage of wealth (the inventory). That is momentum traders buy (sell) an asset today in *anticipation* of a price increase (decrease) in the next period. I use an inventory model, Muth (1961), to solve an equilibrium setup where an agent's decision to speculate in stock trading is modeled explicitly. I calculate the expected mean and variance generated by a momentum trading strategy. I show in a simple framework that (1) momentum trading can exist in equilibrium, that is, past prices can be useful as predictors of future prices, and (2) momentum trading is profitable, but not riskless.

Not only is there short-term momentum, but long-term reversals have been empirically documented as well. DeBondt and Thaler (1985, 1987) document that stocks have systematic price reversals at the three to five year horizon. They label this anomaly "overreaction." Cutler, Poterba and Summers (1990, 1991) show short-term autocorrelations and long-term negative autocorrelations in excess returns across a range of different asset classes. More recently, Lee and Swaminathan (2000) find that momentum in the US market is reversed at long horizons of three to five years, while Griffin, Ji and Martin (2003) find that momentum profits reverse over a one to five year horizon for many international markets. My model captures this dual effect in a parsimonious manner. Since, in equilibrium, the coefficient on the most recently observed price change is greater than one, my model exhibits overreaction. As there exists a time, T, such that the coefficients become negative, my model captures the notion of long-term reversals. Thus, another prediction of my model is that the existence of short-term momentum

[1] This important debate goes back to Adam Smith (1789).

[2] This argument is supported by a vast literature. Momentum is observed in the US (Jegadeesh and Titman (1993)), internationally (Rouwenhorst (1998)), in industry portfolios (Moskowitz and Grinblatt (1999)), and in book-to-market portfolios (Lewellen (2002)). As momentum is observed in these relatively well-diversified portfolios, momentum is not from idiosyncratic risk factors. See Jegadeesh and Titman (2001), Grundy and Martin (2001), Chen and Hong (2002), and Griffin, Ji, and Martin (2003) for a survey and discussion of the momentum literature.

implies long-term reversals.

A final prediction of my model is that momentum is a self-fulfilling belief. If a subset of agents believe in and use technical trading strategies (past prices are used for predicting future prices), then it becomes so in equilibrium. Wall Street is full of technical traders, even very sophisticated ones. Taylor and Allen (1992) question chief foreign exchange dealers based in London in November of 1988. They find there is a reliance on technical, as opposed to fundamental, analysis at shorter horizons. In their survey, technical analysis is defined as trading "...on the basis of largely visual inspection of past prices, without regard to any underlying economic or 'fundamental' analysis." Interestingly, they find a very high proportion of chief dealers suggest that technical advice may be self-fulfilling. My model captures this perception as it is exactly the short-term technical traders, traders whose strategies are based on past price changes, who realize a *self-fulfilling* equilibrium profit.

My model captures these observed, and previously separate, market properties while making three basic assumptions. First, in the aggregate, demand is non-upward sloping (i.e. demand is flat or downward sloping). Support for this is discussed in Appendix A. Second, in the aggregate, supply is non-downward sloping. I provide new empirical evidence in Appendix B to support this assumption. Finally, there exists a subset of traders who believe that technical trading, momentum specifically, is a valid strategy and act on such beliefs. Assuming linearity of supply and demand in order to make the model tractable, I derive a closed form solution for a market equilibrium with the above properties. These properties are the direct result of allowing momentum trading in the model.

There is a burgeoning literature to rationalize momentum. The first model proposed was by Berk, Green, and Naik (1999). As a consequence of optimal investment choices, a firm's current assets and growth options change in predictable ways. Slow turnover in the firm's project portfolio leads to persistence in both the firm's asset base and its systematic risk, which makes expected returns positively correlated with lagged expected returns. In their model, firms that perform well tend to be those that have discovered particularly valuable investment opportunities. As they exploit those opportunities, their systematic risk changes. Johnson (2002) develops a model based on Berk et al's growth option intuition. Momentum effects occur when expected dividend growth rates vary over time. Johnson's model does not fit the empirical facts of momentum very well. To achieve large effects, growth rate shocks must decay too slowly. A two-regime switching model is introduced for which persistent shocks occur only in the short-lived regime. This two-regime switching version of Johnson's model fits the data well. Both these models depend on stochastic growth rates to generate momentum. Neither captures the observed long-term reversals.

One approach to rectifying the simultaneous existence of both intermediate-term momentum and long-term reversals in returns has been to appeal to human cognitive biases borrowed from the psychology literature. Barberis, Shleifer, and Vishny (1998), like my model, utilize a single asset. They utilize a Bayesian representative agent that exhibits both representativeness and conservatism in order to generate the dichotomous results of momentum and long-term reversals. In their model, earnings follow a random walk, yet the agent believes the true process is a two-regime switching model, with one mean-reverting regime and one trending regime.

The authors show that their model can produce both empirical anomalies using reasonable parameter values. A second paper by Daniel, Hirshleifer, and Subrahmanyam (1998) utilize two different cognitive biases to generate the dual nature of returns. In their model, overconfidence of private signals causes overreaction, while biased self-attribution causes a slow readjustment of the overreaction as public information becomes available. Both these models generate the dichotomous anomalies with a representative agent who possesses offsetting cognitive biases. A third model by Hong and Stein (1999), unlike the above models, generates the observed dichotomous anomalies using two sets of agents. Each set of agents optimally utilize their universe of information. However, each agent's subset of information is limited. One set of agents, newswatchers, forecast based only on private signals. The other set of agents, momentum traders, utilize only past price changes, similar to my model. One further assumption that information diffuses slowly over the newswatchers is necessary to generate the observed empirical irregularities. Empirical support for these models is presented in Bhojraj and Swaminathan (2004).

My model is most similar to the behavioral literature in that my model imbues agents with behavior based on observations from real world traders. My model is also able to rectify the coexistence of momentum and long-term reversals, as well as other empirically observed phenomenon. I add to the insight gained by this literature in that I explore and demonstrate a new potential source of pricing anomalies. I also demonstrate a potential mechanism of how noise affects market prices. The mere fact that a subset of agents believe that past price changes are informative creates an equilibrium where this is so. If there are noisy shocks to the economy, then this noise will be incorporated into prices via the momentum trader's actions. Thus, anomalies such as momentum and long-term reversals not only coexist, they can have a common source. Both anomalies can be the result of a self-fulfilling prophecy in which a subset of traders trade on past price changes as informative of future price change.

This paper provides a first order explanation for the wide existence of momentum, not just in prices, but in earnings (Chan, Jegadeesh, and Lakonishok (1996)), in diversified portfolios (Lewellen (2002)), and in diverse markets (Rouwenhorst (1998), Griffin, Ji and Martin (2003)). My model, posited in the simplest form possible, demonstrates that the mere existence of noise in the market (in the form of noise trading) is sufficient to sustain momentum and long-term reversals in equilibrium. It seems that Black (1986) was right when he exclaimed: "In the end, my response to the skepticism of others is to make a prediction: someday, these conclusions will be widely accepted. The influence of noise traders will become apparent. . . . If my conclusions are not accepted, I will blame it on noise."

The remainder of the paper is organized into four additional sections. Section 2 discusses the assumptions of the economy and the model setup. Section 3 solves for the equilibrium of the model. Here I show that existence of noise traders is a sufficient condition for momentum to occur in stock market pricing and for markets to fail the weak form of market efficiency. Thus, past price changes are useful for future price movements. The results also suggest that momentum pricing is most likely to occur in stocks where noise trading is more prevalent. Hong, Lim, and Stein (1999) show that momentum is empirically stronger in stocks where the prior information is less well developed, which suggests these stocks experience more noise in pricing. This is consistent with what my model predicts. Section 4 provides suggestions for future research, summarizes, and concludes.

2 Assumptions and Model Setup

In section 2.1, I provide the general overview of the economy. In section 2.2, I specify the assumptions mathematically.

2.1 Assumptions of the economy

1. Assume a single risky security and a single riskless security. Implicitly, the model assumes that there is unlimited borrowing and lending.
2. There are two possible sources of demand for the risky asset. One demand is the classic downward sloping Marshallian demand. The other demand is from gathering information from observing past price changes. Thus the model allows agents to trade on past information if it is profitable.
3. There are no transaction costs or similar types of frictions.

2.2 Model setup

Given the above assumptions, I can state more formally my design.

1. Information. The information set, I_t, is used by the momentum traders and includes past market prices $p_t, p_{t-1}, p_{t-2}, \ldots$ or equivalently (to a constant) all past price changes $\Delta p_t, \Delta p_{t-1}, \Delta p_{t-2}, \ldots$.
2. Forecast of the state variable. The central problem is to study the evolution of the expectation of the state variable, $E[\Delta p_{t+1}|I_t]$. It is shown in section 3 that an equilibrium exists where expectations of price changes can be written in terms of past price changes.
3. Demand Equation. I assume that, in the aggregate, demand is either flat or downward sloping in the price level. This Marshallian demand for the asset is denoted by D_t, which has the following formulation:

   $$D_t = -\gamma p_t + \tilde{\eta}_t$$

   The slope of the demand equation is the coefficient $-\gamma$, while $\tilde{\eta}$ is a random shock to demand that is normally distributed with mean zero and variance σ_η^2. It is assumed that $\gamma \geq 0$. Appendix A discusses the existing support for this assumption.
4. Supply Equation. I assume that, in the aggregate, supply is flat or upward sloping in the price level. This Marshallian supply for the asset is denoted by S_t, which has the following formulation:

   $$S_t = \phi p_t$$

The slope of the supply equation is the coefficient ϕ. Again, it is assumed that $\phi \geq 0$. Note that only one of the three supply/demand equations has to have a stochastic shock in order to guarantee that future prices are random. Shocks could be included in all three equations. Appendix B discusses the existing support for this assumption.

5. Momentum or Speculative Demand/Supply Equation. It is assumed that the size of the momentum position will be proportional to the size of the expected price change. The larger the anticipated price change, the larger the speculative position.[3] M_t is the momentum trader's speculation demand (or supply if short) for the asset.

$$M_t = \alpha E[\Delta p_{t+1}|I_t] = \alpha \left(E[p_{t+1}|I_t] - p_t \right)$$

Here the coefficient α can be referred to as the aggressiveness coefficient. The larger α is, the larger is the momentum traders demand response for a given observed price change.

It is important to note that the market price will be set in equilibrium by a balance of supply and demand for the risky asset. This equilibrium market price will reflect and incorporate agents' expectations within the framework of the supply and demand equations.

It is assumed that the supply and demand functions are exogenously given. That the linearity of supply and demand in price and the momentum equation can occur in a rational setting is proved in Stein (1987) for a two-period model. There the linear form in price follows from a quadratic Marshallian utility function (and thus the nomenclature for these agents). In Stein's model, the optimality of the momentum demand equation is also shown using a constant absolute risk aversion utility function. It is not the purpose here to justify these well studied and utilized equations. The main purpose of this paper is to explore, in a tractable framework, what are the observed market implications of the existence of momentum traders.

3 Equilibrium and Momentum

Black (1986) and De Long et al. (1990 a,b) assume that noise traders misperceive the fundamental price but sophisticated investors do not wish to bet against them because prices may continue to diverge from fundamental value. In the De Long et al. (1990 a,b) setups, they assume that noise traders are "dumb" and simply create disturbances to the market pricing and raise the cost of capital. I do not study a rationale for noise in this paper. I assume that noise exists and then evaluate its effects on prices. In the model, momentum traders are traders who trade just on past observed price changes. Of course, such traders may generate additional noise, or may amplify existing noise, in the market. I study the risk that momentum traders might bear.

[3] Academic evidence for this assumption is provided by Taylor and Allen (1992) who question chief foreign exchange dealers based in London in November of 1988. They find there is a reliance on technical, as opposed to fundamentalist, analysis at shorter horizons. They find a very high proportion of chief dealers suggest that technical trading may be self-fulfilling. The vast popular literature on technical trading strategies is evidence that this belief is shared by many non-professional traders as well.

Momentum trading can be viewed as a type of inventory speculation where the asset traded is the storage of wealth (the inventory). Momentum trader's buy an asset today in *anticipation* of a price increase in the next period. I use an inventory model, Muth (1961), to solve an equilibrium setup where an agent's decision to speculate in stock trading is modeled explicitly. I calculate the expected mean and variance generated by a momentum trading strategy. I show in a simple framework that (1) momentum trading can exist in equilibrium, that is, past prices can be useful as predictors of future prices, and (2) momentum trading is profitable, but not riskless.

For tractability, I utilize a model of speculation where momentum (or speculative) demand is formed on past observed price changes. I assume a linear setup for supply, demand, and the speculative demand of the asset. I assume that momentum traders have past price observations, and thus price changes, in their information set. There are two kinds of agents. The first are the standard Marshallian agents which utilize the classic downward-sloping demand and upward-sloping supply curves. The second type of agent are the momentum traders who condition their demand on past price changes. The momentum strategy sorts on past price returns, typically for the past 3 to 12 months. Two simplifications are made. As it is easier to work with an infinite series than a truncated series, I allow the momentum trader to condition on all past prices. The other simplification is that the momentum strategy is either 100% long or 100% short. The model implicitly assumes that the risk-free asset is in infinite supply and that agents can lend and borrow at the risk-free rate.[4] Mathematically, the assumptions on the momentum trader can be written:

$$E[\Delta p_{t+1}|I_t] = c + \sum_{j=0}^{\infty} \beta_j \Delta p_{t-j} \tag{1}$$

The setup is as follows:

$$\begin{aligned} D_t &= -\gamma p_t + \tilde{\eta}_t \\ S_t &= \phi p_t \\ M_t &= \alpha E[\Delta p_{t+1}|I_t] = \alpha \left(E[p_{t+1}|I_t] - p_t \right) \end{aligned} \tag{2}$$

D_t is the demand for the asset, which is assumed downward sloping in the price level. S_t is the supply of the asset and it is assumed that supply is upward sloping in the price level. M_t is the momentum trader's speculation demand (or supply if short) for the asset. It is assumed that the size of the momentum position, M_t, will be proportional to the size of the expected price change. The larger the anticipated price change, the larger the speculative position. The noise term $\tilde{\eta}_t$ is assumed to be i.i.d. $N(0, \sigma_\eta^2)$ and $\tilde{\eta}_t$ is assumed to be independent of the market

[4]By itself, the assumption that all agents can borrow and lend at the risk-free rate is a strong assumption. It basically precludes default from the economy. Thus, all obligations will be paid with 100% certainty. If the usual approximation is made that the government short-term rate is to be used for the risk-free rate, then the assumption implicitly assumes that all agents in the economy have the same default probability as the government. Another way of viewing this assumption is that it implies that the government will step in and honor any obligation that is not met for every agent in the economy. Although this may be true for very large firms or for firms in nationally sensitive areas, it can hardly be true that the government will do the same for the average investor. Again, it is an idealization that introduces some distortion, but hopefully does not affect the first order intuition or directional affect that we seek here. For simplicity, the risk-free rate is set to zero, which is consistent with the no transaction costs assumption.

price. For simplicity, only demand is assumed to be stochastic. The general results that follow do not depend on this simplification. I assume only one risky asset. Finally, γ, ϕ, and α are all non-negative.

Theorem 1 *If the economy evolves according to the economic system of equation (2) and a momentum trader will set demand for the risky asset proportional to the size of the expected price change of the underlying asset according to equation (1),*

$$M_t = \alpha(c + \sum_{j=0}^{\infty} \beta_j \Delta p_{t-j})$$

Then an equilibrium exists for which there is no momentum trading and an equilibrium exists with non-negative momentum trading.

Proof.

In addition to the standard Marshallian supply, S_t, and demand, D_t, there is the contribution from the momentum traders. Their trading activity in each period is to unwind their prior period's holdings, $-M_{t-1}$, while buying their new desired position, M_t. Thus, momentum traders contribute ΔM_t to the demand side of the equilibrium, with a negative value signifying supply to the market. Market clearing gives us the following equilibrium relation,

$$D_t + M_t = S_t + M_{t-1} \tag{3}$$

From equations (1) and (2), this yields:

$$(\alpha\beta_0 - \gamma - \phi)p_t = \alpha(2\beta_0 - \beta_1)p_{t-1} + \alpha \sum_{j=1}^{\infty}(2\beta_j - \beta_{j+1} - \beta_{j-1})p_{t-j-1} - \tilde{\eta}_t \tag{4}$$

Taking a lagged difference, dividing by the leading constant and taking expectations with respect to the current information set, we obtain:

$$\begin{aligned} E[\Delta p_{t+1}|I_t] = & \quad \tfrac{\alpha}{\alpha\beta_0 - \gamma - \phi}\ (2\beta_0 - \beta_1)\Delta p_t \\ & + \quad \tfrac{\alpha}{\alpha\beta_0 - \gamma - \phi}\ \textstyle\sum_{j=1}^{\infty}(2\beta_j - \beta_{j+1} - \beta_{j-1})\Delta p_{t-j} \\ & - \quad \tfrac{1}{\alpha\beta_0 - \gamma - \phi}\ E[\Delta\tilde{\eta}_{t+1}|I_t] \end{aligned} \tag{5}$$

Recall by assumption:

$$E[\Delta p_{t+1}|I_t] = \beta_0 \Delta p_t + \sum_{j=1}^{\infty} \beta_j \Delta p_{t-j} + c \tag{6}$$

In order for an equilibrium to exist, the coefficients of equations (5) and (6) must be identical. This gives the following system of equations:

$$\begin{aligned} \beta_0 &= \tfrac{\alpha}{\alpha\beta_0 - \gamma - \phi}\ (2\beta_0 - \beta_1) \\ \beta_j &= \tfrac{\alpha}{\alpha\beta_0 - \gamma - \phi}\ (2\beta_j - \beta_{j+1} - \beta_{j-1}), \quad \text{for } j = 1, 2, \ldots \\ c &= -\tfrac{1}{\alpha\beta_0 - \gamma - \phi}\ E[\Delta\tilde{\eta}_{t+1}|I_t] \end{aligned} \tag{7}$$

An equilibrium exists in which there is no momentum trading. To see this, let $\alpha = 0$ so that there is no momentum trading. Under the assumption that no one uses past price changes to form expectations of future price changes, the following is an solution to and thus an equilibrium of the above system:

$$\text{(8)} \qquad \begin{array}{ll} \beta_j = 0, & \text{for } j = 0, 1, 2, \ldots \\ c = -\frac{1}{\gamma+\phi}\ E[\tilde{\eta}_t | I_t] & \end{array}$$

Intuitively this is appealing. If there is no momentum trading, that is, no one in the economy is using past prices to form their positions in the risky asset, then there will exist a short-term (one period) contrarian strategy (reversal) as the price move is most likely due to noise. Noise effects are reversed over the next period in the no-momentum equilibrium.

An equilibrium exists in which there is momentum trading. To see this, let $\alpha > 0$ so there is a strictly positive level of momentum trading in the economy. Past price changes are used by a subset of agents in order to form expectations of future price changes. This changes the dynamics of the system and yields the following recursive difference equations for c and the β_j's:

$$\text{(9)} \qquad \begin{array}{ll} \beta_1 = K\beta_0 & \\ \beta_{j+1} = K\beta_j - \beta_{j-1}, & \text{for } j = 1, 2, \ldots \\ c = \frac{1}{\alpha\beta_0 - \gamma - \phi}\ E[\tilde{\eta}_t | I_t] & \end{array}$$

where $K = (\gamma + \phi + 2\alpha - \alpha\beta_0)/\alpha$. Solving this recursive difference equation in terms of β_0 yields the solution:

$$\text{(10)} \qquad \beta_j = \begin{cases} \beta_0 \sum_{n=0}^{\frac{j-1}{2}} (-1)^n \binom{j-n}{n} K^{j-2n} & \text{if } t \text{ is odd} \\ \beta_0 \sum_{n=0}^{\frac{j}{2}} (-1)^n \binom{j-n}{n} K^{j-2n} & \text{if } t \text{ is even.} \end{cases}$$

The term $\binom{j-n}{n}$ is the binomial coefficient of $j - n$ choose n. Finally, it is shown in Theorem (2) that β_0 can be chosen such that $\sum \beta_j = 1$.

Equations (8) and (10) complete the proof of the assertion that an equilibrium exists with non-negative momentum trading. Thus momentum trading can exist in a very simple economy with very few assumptions. Past prices can be used to form expectations for future price changes and agents can form profitable trading strategies on past price changes (see Theorem 3).

Q.E.D.

It is important to note that frictions are ignored in this simple model. Any frictions would have obvious mitigating effects. I now turn to the question of how information or the beliefs of active agents are transmitted through prices. The following theorem addresses this.

Corollary 1 *If the economy evolves according to the economic system of equation (2) and a momentum trader will set demand for the risky asset proportional to the size of the expected* *price change of the underlying asset according to equation (1),*

$$M_t = \alpha(c + \sum_{j=0}^{\infty} \beta_j \Delta p_{t-j})$$

Then the noise content of past price changes are embedded in expectations and thus affect the equilibrium.

Proof.

The equation for c in Equation (9) demonstrates how noise affects expectations, which determines momentum demand. The aggressiveness coefficient, α, is a choice variable. It is clear that there exists an α such that $\alpha\beta_0 - \gamma - \phi > 0$. It follows that noise can compound the amount of momentum trading and thus the equilibrium.

Q.E.D.

Remark. This result also makes clear that initially a price change that induces momentum traders to trade may be informationless, as it is just noise. However, by acting on this observed noise, it becomes a reliable predictor of future expected prices. The fact that the existence of noise is all that is necessary to support momentum trading in equilibrium, provides a simple intuition as to why momentum is ubiquitous.

We now address the fact that not only is there short-term momentum, but long-term reversals have been empirically documented as well. This was first documented by DeBondt and Thaler (1985, 1987) who demonstrated that stocks have systematic price reversals at the three to five year horizon. Their findings of stock overreaction are supported by Cutler, Poterba and Summers (1990, 1991), who show short-term autocorrelations and long-term negative autocorrelations in excess returns across a range of different asset classes. More recently, Lee and Swaminathan (2000) find that momentum in the US market is reversed at long horizons of three to five years, while Griffin, Ji and Martin (2003) find that momentum profits reverse over a one to five year horizon for many international markets. the next theorem shows that the model used in this paper not only possesses momentum, but also possesses overreaction.

Theorem 2 *If the economy evolves according to the economic system of equation (2) and a momentum trader will set demand for the risky asset proportional to the size of the expected price change of the underlying asset according to equation (1),*

$$M_t = \alpha(c + \sum_{j=0}^{\infty} \beta_j \Delta p_{t-j})$$

There exists an equilibrium with strictly positive momentum trading that exhibits short-term momentum and long-term reversals in prices.

Proof.

Recall from equation (9) that:

$$\beta_{j+1} = K\beta_j - \beta_{j-1}, \qquad \text{for } j = 1, 2, \ldots \tag{11}$$

Summing equation (11) over $j = 1, 2, \ldots$ we obtain:

$$\sum_{j=1}^{\infty} \beta_{j+1} = K \sum_{j=1}^{\infty} \beta_j - \sum_{j=1}^{\infty} \beta_{j-1} \tag{12}$$

Equation (12) reduces to:

$$\sum_{j=2}^{\infty} \beta_j = K\beta_1 + K \sum_{j=2}^{\infty} \beta_j - \beta_0 - \beta_1 - \sum_{j=2}^{\infty} \beta_j \tag{13}$$

Defining $A = \sum_{j=2}^{\infty} \beta_j$, equation (13) becomes:

$$(K-2)A = \beta_0 - (K-1)\beta_1$$

Recalling that $\beta_1 = K\beta_0$:

$$(K-2)A = (1 + K - K^2)\beta_0 \tag{14}$$

For the infinite sum of weighted past price changes to be valid, we require the weights to sum to one. Thus, we have the relation that $A = 1 - \beta_0 - \beta_1$, which substituted into equation (14) yields:

$$(K-2) = -\beta_0 \tag{15}$$

Returning to equation (12) and defining $B = A + \beta_1$, we get:

$$\sum_{j=1}^{\infty} \beta_j - \beta_1 = K \sum_{j=1}^{\infty} \beta_j - \sum_{j=1}^{\infty} \beta_j - \beta_0$$

or again recalling the definition of β_1:

$$B - K\beta_0 = KB - B - \beta_0$$

$$(K-2)B = -(K-1)\beta_0$$

As we are looking for an equilibrium that has long-term reversals, we desire that $B < 0$, which means that we will require that $\beta_0 > 1$ so that the sum, $\beta_0 + B = 1$ as required. From equation (15) we find that $B = (K-1) < 0$ which gives the important result that $K < 1$ so that the recursive coefficients are convergent. We also do not want wild oscillations so it is required that $K > 0$ as well. We thus find the following bounds on K to hold:

$$0 < K < 1 \tag{16}$$

We need to check that the assumption $\beta_0 > 1$ is consistent with the solution. This follows directly from equation (15) which yields that $\beta_0 = 2 - K$. Applying the bounds above we find that $1 < \beta_0 < 2$ as required. This completes the proof.

Q.E.D.

Remark. The solution with $\beta_0 > 1$ implies that the momentum traders demand is short-term price chasing in that it agrees with the sign of the price change of the previous few days. The coefficient is at least positive for two periods (probably more) as $\beta_1 = K\beta_0$ and $K > 0$. But it is necessary that $\sum \beta_j = 1$. Thus there exists an N such that for $M > N$ the coefficient of the price changes must be negative. This implies that in the long term there is price reversals. Thus, this model exhibits short-term momentum and long-term reversals. The model meets the challenge set forth by Griffin, Ji and Martin (2003) for a model to exhibit both short-term momentum and long-term reversals.

Theorem 3 *If the economy evolves according to the economic system of equation (2) and a momentum trader will set demand for the risky asset proportional to the size of the expected price change of the underlying asset according to equation (1),*

$$M_t = \alpha(c + \sum_{j=0}^{\infty} \beta_j \Delta p_{t-j})$$

Then momentum trading is profitable, but not riskless.

Proof.

I first calculate the expected profitability of a momentum trade. The size of the momentum trade assumed by the speculator is M_t and the realized profit on a unit position is $\tilde{p}_{t+1} - p_t$. Thus the expected profit, π_{t+1}, is:

$$\begin{aligned} E[\pi_{t+1}|I_t] &= E[M_t(\tilde{p}_{t+1} - p_t)|I_t] \\ &= \tfrac{1}{\alpha} M_t E[(\alpha\{\tilde{p}_{t+1} - p_t\}|I_t]) \\ &= \tfrac{1}{\alpha} M_t(\alpha\{E[\tilde{p}_{t+1}|I_t] - p_t\}) \\ &= \tfrac{1}{\alpha} M_t^2 \end{aligned} \tag{17}$$

As α is assumed greater than zero, M_t is non-zero as well. We can conclude that the momentum trade is profitable as the square of the position. Finally, as the momentum trade is financed with the risk-free asset, it must outperform this asset for the portfolio to be attractive. It is clear, that there exists an α such that this is true.

Next we calculate the variance of the momentum trade to demonstrate that it is not riskless. Repeating the above on the second central moment:

$$\begin{aligned} Var[\pi_{t+1}|I_t] &= Var[M_t(\tilde{p}_{t+1} - p_t)|I_t] \\ &= \tfrac{1}{\alpha^2} M_t^2 Var[\alpha(\tilde{p}_{t+1} - p_t)|I_t] \\ &= \tfrac{1}{\alpha^2} M_t^2 \left\{E[\alpha^2(\tilde{p}_{t+1} - p_t)^2|I_t] - E[\alpha(\tilde{p}_{t+1} - p_t)|I_t]^2\right\} \\ &= \tfrac{1}{\alpha^2} M_t^2 \left\{E[\alpha^2(\tilde{p}_{t+1}^2 - 2\tilde{p}_{t+1}p_t + p_t^2)|I_t] - M_t^2\right\} \\ &= \tfrac{1}{\alpha^2} M_t^2 \left\{\alpha^2(E[\tilde{p}_{t+1}^2|I_t] - 2E[\tilde{p}_{t+1}|I_t]p_t + p_t^2) - M_t^2\right\} \\ &= \tfrac{1}{\alpha^2} M_t^2 \left\{\alpha^2(E[\tilde{p}_{t+1}^2|I_t] - E[\tilde{p}_{t+1}|I_t]^2) + \alpha^2(E[\tilde{p}_{t+1}|I_t] - p_t)^2 - M_t^2\right\} \\ &= M_t^2 Var[\tilde{p}_{t+1}|I_t] \\ &= M_t^2 \sigma_p^2 \end{aligned}$$

where σ_p^2 is the variance of the price change. This, of course, is greater than zero as it is a product of two squared terms. This completes this proof.

Q.E.D.

Corollary 2 *If the economy evolves according to the economic system of equation (2) and a momentum trader will set demand for the risky asset proportional to the size of the expected price change of the underlying asset according to equation (1),*

$$M_t = \alpha(c + \sum_{j=0}^{\infty} \beta_j \Delta p_{t-j})$$

Then the equilibrium with strictly positive momentum trading is a self-fulfilling equilibrium.

Proof.

The fact that a subset of agents believe that past price changes are useful for predicting future price changes makes the pursuit of such a strategy profitable, Equation (17). This completes the proof.

Q.E.D.

Remark. Because momentum trading is profitable in equilibrium and momentum trading only exists due to the belief of the agents that pursue the strategy, momentum trading is self-fulfilling. There is a minimal requirement that noise exist in order to ensure that price change is observed to induce trading. Momentum trading has been one of the most difficult pricing anomalies for standard asset pricing models. The documentation of the wide spread existence of momentum is one reason for the anomalies credibility. Here, the model yields a simple intuition as to why both momentum and overreaction should be observed in any market where a subset of agents trade on past price changes as if they were informative.

Corollary 3 *If the economy evolves according to the economic system of equation (2) and a momentum trader will set demand for the risky asset proportional to the size of the expected price change of the underlying asset according to equation (1),*

$$M_t = \alpha(c + \sum_{j=0}^{\infty} \beta_j \Delta p_{t-j})$$

Then information about beliefs contained in past price changes are embedded in price slowly over time. Thus, returns will be autocorrelated.

Proof.

Make a further assumption that expectations equal price in equilibrium. Take the difference of equation (6). This tells us how an agent will update expectations in equilibrium upon the arrival of new information. The following equation gives the price dynamics:

$$(18) \qquad E[p_{t+1}|I_t] - E[p_t|I_{t-1}] = \beta_0 \Delta p_t + \beta_0 (K-1) \Delta p_{t-1} + \sum_{j=2}^{\infty} \left[(K-1)\beta_j - \beta_{j-1}\right] \Delta p_{t-j}$$

As the equilibrium requires that expectation of price be equal to price, the momentum trader's beliefs on information in past price changes will be slowly impounded over time. Returns will be autocorrelated. Price momentum occurs naturally in this simple economy.

Another way to see the autocorrelation follows from the first result proved in Theorem 3. Since momentum trading is on average profitable and the momentum traders overemphasize the most recent price change (i.e. $\beta_0 > 1$ from Theorem 2), the next period price change on average must be of the same sign as the most recent price change.

Q.E.D.

4 Conclusion

By recognizing that momentum trading can be viewed as a type of inventory speculation where the asset traded is the storage of wealth, I solve a simple equilibrium model where an agent's decision to speculate in stock trading is modeled explicitly. I calculate the expected mean and variance generated by a momentum trading strategy. I show that momentum trading can exist and will be limited in equilibrium. Also, the model demonstrates that momentum trading is profitable, but risky.

The empirical literature documents overreaction of prices in the short term and reversals in the long term. To date, rational models have not been able to simultaneously produce both these empirical observations. My simple model captures this dual effect in a parsimonious manner. Since, in equilibrium, the coefficient on the most recently observed price change is greater than one, my model exhibits overreaction. As there exists a time, T, such that the coefficients are negative, my model captures the notion of long-term reversals.

The noise traders provide a crucial role in that price is not fully revealing and the mere existence of noise traders makes the joint existence of momentum and overreaction possible. It is important to note that noise in my model can come from informational changes, such as endowment shocks, or from non-fundamental elements. Momentum traders do not distinguish between these. My main insight and contribution is that noise trading in markets, which can be rational and substantial, is sufficient for momentum and long-term reversals to occur in equilibrium. Momentum traders observe the change in price due to the noise shocks and act on it as if it were information. By doing so, the noise element, in that it is recorded by past prices, acquires a predictive nature via the momentum traders action. The belief that trading on past price changes is profitable makes it so. Thus, momentum trading is a self-fulfilling prophecy.

The effects of noise should not be underestimated. Noise may lead to a better understanding of why consumption patterns, where noise is comparatively low, are different from stock pricing patterns, where noise is quite high. If there were no noise in stocks prices, theory suggests that the solution of one market should generate the shadow price for the other. The presence of noise opens new risk exposures and its further study may lead to a better understanding of how speculative markets function and why the equity premium is as large as it is.

Appendix A: Downward Sloping Demand

There is an extant empirical literature on the downward sloping demand debate. Work by Harris and Gurel (1986) and Blouin, Raedy, and Shackelford (2000) indicate that stocks have a short-term downward sloping demand curve, i.e the price should be momentarily affected by a demand shock due to indexing, but that effect should dissipate once the excess demand is satisfied. That stocks have a long-term downward sloping demand curve with the excess returns permanent is supported by research by Shleifer (1986), Beneish and Whaley (1996), Lynch and Mendenhall (1997), Kaul, Mehrotra, and Morck (2000), and Blume and Edelen (2001). All these studies utilize major stock index additions and deletions to study this effect. Cha and Lee (2001) and Denis, et al. (2003) question the index change study results due to the fact that such price moves could be due to informational effects associated with being included in an index. However, Loderer, Cooney and Van Drunen (1991) and Wurgler and Zhuravskaya (2002) control for information effects and still find downward sloping demand. Probably one of the most interesting results in Wurgler and Zhuravskaya is that they find a surprising lack of substitutability between various assets. For their median stock, they were not able to find "...substitutes that could hedge away even a quarter of the stock's daily return variance."

The claim for downward sloping demand is strengthened as the results are robust across several different methodologies. Mikkelson and Partch (2002) utilize secondary offering and conclude that a large number of shares cannot be sold at the prevailing market price nor at a small cost. Asquith and Mullins (1986) study seasoned equity offerings and find support for the hypothesis that there is a downward sloping demand for a firm's shares. Shleifer and Vishny (1997) and Baker and Savasoglu (2002) find empirical support for risk-arbitrage. As pure arbitrage is required in order to maintain flat demand curves for stocks their results support downward sloping demand. Field and Hanka (2001) study IPO lockup expirations, which are a known event, and find statistically prominent abnormal returns and partial support for downward sloping demand.

There is also theoretical support for downward sloping demand. If the conditions for perfect markets are not satisfied, e.g. there is significant imperfect knowledge of future events, then it is possible for the demand curve to slope down under heterogeneous beliefs. Miller (1977) has a two-period model with N investors. Differing opinions over the uncertainty leads to a downward sloping demand curve for the risky asset. Vives (1987) formalizes Marshall's (1890) idea that when the proportion of income spent on any commodity is small then the income effects are small. This leads to a downward sloping demand curve given the number of assets is large. Merton (1987) in an incomplete markets framework with heterogeneous agents finds that securities can have significant downward sloping demand curves.

Appendix B: Upward Sloping Supply

A common assumption is that the supply of a stock is fixed and typically modeled as one unit total supply. This is a very interesting turn of historical events. From a historical basis, fixed supply of capital or stock was a known limitation to any model. Ricardo (1811) in his classic exposition of the quantity theory of money explicitly made his assumption of fixed supply an instantaneous constraint at any time t. Marshall (1890) in his classic description of the basic principles of economics specifically distinguished between a *short-term* equilibrium in which capital or stock variables are to be considered fixed and a *long-term* equilibrium in which capital is able to adjust to the forces of supply and demand. These early great writers realized the limitations of any model that utilized the simplifying assumption of fixed supply of a stock variable.

In economics, there is a long tradition of capital growth due to the specific assumption of capital accumulation within many models. This is the standard in macro-growth models. In finance, however, the dependence on the one-period model framework, most notably the CAPM, eliminates capital accumulation. The rise to dominance of the CAPM, starting in the mid 1960's, helped to establish the one-period model as the standard in finance. Thus, by association, the assumption of fixed total supply of an asset became a standard in finance as well. Mossin (1969) was the first to carefully delineate this connection. The assumptions, either explicitly stated or implicitly assumed when using the one-period framework, naturally lead to a fixed supply of a financial asset. The first assumption is that there is only one-period in the model. Thus, all production is consumed at the end of the period and firms cease to exist. As there is no intermediate consumption and non-financial assets are assumed not to exist (only risky assets and a risk-free bond are the standard) the total demand for the assets must equal the total wealth endowed in the economy. Finally, market clearing in equilibrium forces the total supply of the asset to equal the total demand of the asset, which has already been assumed to equal the total wealth endowment. Thus, the total supply of the asset is fixed in this one-period setup.

In the modern financial literature, the one-period restriction is usually made for purposes of tractability and to better obtain closed form solutions. However, outside a one-period model, fixed asset supply is far from reality. The supply of assets changes over time and the available supply responds to the current market price. Interestingly, I could find few papers that addressed the slope of the supply curve for financial assets. The one exception, Bagwell (1992), supports this claim, however Bagwell studies the supply curve faced by firms buying their own shares.[5] This is different than the supply curve in my model in which firms are the supply curve. Therefore, I used the CRSP and COMPUSTAT linked database to investigate the time series of the available assets to investors. I collect data for the Russell 1000 index stocks, which constitutes approximately 94% of the total traded stocks in the US market. In order to capture the trading activity of firms in their own stock, I use the firm's current number of treasury stock. Treasury stock is reported on the equity section of the firm's balance sheet.

[5]Three papers provide indirect support for upward sloping supply curves. Bagwell (1991), Bradley, Desai and Kim (1988) and Brown and Ryngaert (1992) empirically measure the heterogeneity among shareholders for a firm's value. The difference is large and violates the assumptions of perfect markets, which is one of the main assumptions that lead to flat supply curves.

By buying and selling from its supply of stock in its treasury a firm can influence the overall supply of stock to investors on a more short-term basis. Dittmar (2000) studies the reasons why firms trade in their own securities. Nohel and Tarhan (1998) conclude that repurchases are not meant to change the firm's capital structure. Rather they are meant to shrink the current assets of the firm. In the long-term there are many ways for a firm to influence the total supply of its stock. There are warrants, convertible bonds, secondary issues, IPO's, and employee stock options, to name a few. To capture this ability of firms to change the total supply of assets to the market I use two measures. The first is the number of common shares outstanding. This captures the current stock that is actually made available to investors. I also consider the number of common shares used to calculate diluted earnings per share. The difference between shares outstanding and shares diluted is a rough measure of the *potential* of new shares to enter trading dependent on market conditions. Figure 1 clearly demonstrates the empirical evidence that the total supply of financial assets is very much not fixed.

[Figure 1 about here.]

Before leaving the topic of how the assumption of fixed total supply of assets became dogma, it is interesting to investigate a few more early classic papers that used the assumption. Other than the CAPM, one of the all time workhorses of finance has been the Lucas (1978) pure exchange economy model. In this framework, production is assumed to be entirely exogenous and to result from a Markov process. Interestingly, Lucas explicitly makes a second assumption that each firm "...has outstanding one perfectly divisible equity share." In a Lucas economy, this assumption is OK. In fact, this assumption is redundant and only serves to normalize the fixed supply of the risky asset to one.[6] Production is *exogenous* and stochastic. Firms will thus not alter production by a change in the investment opportunity set. As such, when the price of a firm's share increases, i.e. when its overall cost of capital decreases, the firm will not try to issue new shares to take advantage of newly profitable investment opportunities. Thus, given *exogenous* and stochastic production, a firm's outstanding equity will be of fixed total supply.

Of course in the real world, production is not exogenous, nor is it random. Firms do observe and react to the current market price of their traded shares. IPO's, secondary issues, stock options, and treasury stock all exist and affect the current supply of stock in the market that is available to investors. My model neither assumes a one-period setup nor assumes exogenous production. My model is dynamic so a unit of supply cannot be justified as in Mossin (1969) for the one-period model. By reacting to price changes, my firm is specifically responding to changes in its investment opportunity set by adjusting the overall supply of its shares in the market. Thus, supply in my model is endogenous and a fixed supply of stock cannot be justified via the Lucas (1978) pure exchange setup.

[6] This is the earliest reference that I have found for the normalization to one of the fixed supply of the risky asset.

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Figure 1

Data for the Russell 1000 stocks were downloaded from the CRSP and COMPUSTAT linked quarterly database on WRDS. This constitutes approximately 94% of the total traded stocks in the US market. For each quarter from January 1985 to October 2003, the mean number of common shares outstanding is plotted against the mean equally weighted price adjusted by the Consumer Price Index. Short term fluctuations are captured by the firm's current number of treasury stock. Treasury stock is reported on the equity section of the firm's balance sheet. Long-term fluctuations are captured by two measures. The first is the number of common shares outstanding. This captures the current stock that is actually made available to investors. I also consider the number of common shares used to calculate diluted earnings per share. The difference between shares outstanding and shares diluted is a rough measure of the potential of new shares to enter trading dependent on market conditions.