Learning from Prices and the Dispersion in Beliefs

Snehal Banerjee *
Graduate School of Business
Stanford University
`snehal.banerjee@stanford.edu`

January 11, 2007

First Draft: August 2006

JOB MARKET PAPER

Abstract

I study whether investors use an asset's price to update their beliefs about its payoffs. I show that in a dynamic economy, the relationship between the dispersion in investor beliefs and return-volume characteristics can be used to empirically answer this question. When investors have rational expectations (RE) and condition on the price, I show that belief dispersion is positively related to expected returns, return volatility, and correlation between volume and absolute returns, but negatively related to return autocorrelation. When investors do not use the price, as in a difference of opinions (DO) model, these relationships are reversed. I test these predictions using the cross-section of returns, volume and analyst forecast dispersion over different horizons. The results are consistent with the RE model at quarterly and longer horizons, but reject both models at the monthly horizon. I show that a hybrid model with some DO investors and some RE investors can match the short horizon evidence. The empirical results are consistent with the idea that while some investors do not use the price in the short run, they increase their conditioning on the price over time.

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*I would like to thank Anat Admati, Carlos Corona, Peter DeMarzo, Michael Grubb, Scott Joslin, Yaniv Konchitchki, Ilan Kremer, Stefan Nagel, Nelli Oster, Paul Pfleiderer, Natalya Teplitsky and seminar participants at the Stanford GSB for useful discussions and comments. All errors are mine. The latest version can be downloaded from `http://www.stanford.edu/~snbanerj/`

1 Introduction

Do investors update their beliefs about an asset's payoffs using the information in its price? On the one hand, the approach in much of the finance literature assumes rational expectations (RE), which implies that investors agree on the interpretation of signals and therefore condition on prices to infer the private information of others. Other branches of the literature, however, suggest a number of reasons for why investors may not condition on prices when updating their beliefs. In models of differences of opinions (DO), investors "agree to disagree" about the distribution of payoffs and signals, and therefore, do not use prices to update their beliefs.[1] Investors may also not use the price if they simply do not know how to use prices correctly, or if they exhibit behavioral biases.[2]

Although these models differ sharply in their assumptions about whether investors learn from prices, they are often difficult to distinguish empirically. Many of the models that have been tested are static, and thus limited in the scope of their predictions. The underlying assumptions cannot be directly tested because investor beliefs, information and learning are not observable. Moreover, many of the predictions cannot be easily tested either, because estimating or calibrating models with unobservable belief heterogeneity is difficult. Finally, by picking the right unobservable parameters, one can generate empirically indistinguishable predictions from different types of models. For example, consider the static Hellwig (1980) RE model and the Lintner (1969) counterpart, in which investors condition only on their private signals. By setting the investors' risk aversion and the precisions of their signals appropriately, one can make these models observationally equivalent in terms of their predictions about returns, volume and investor belief dispersion. In general, trying to detect whether investors use prices is virtually impossible using empirical tests based on these static models.[3] In this paper, I show that moving to a dynamic framework is an important step in overcoming these challenges, since this allows one to generate a broader set of predictions that *can* be used to test whether investors learn from prices.

A second objective of this paper is to develop a framework to study how belief dispersion is related to returns and volume. The empirical literature in this area is mainly descriptive, and not driven by specific model predictions. Much of the recent theoretical literature has narrowly focused on the negative dispersion-return relationship documented by Chen, Hong and Stein (2002), Diether, Malloy and Scherbina (2002) and others. While some papers provide theoretical explanations for this negative relationship, others develop models that predict a positive relationship and find support

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[1] In most DO models, the beliefs of other investors are assumed to be common knowledge and so there is no need to condition on prices. In some recent models, this common knowledge assumption is relaxed, and investors may condition on prices to update their higher order beliefs (e.g. Banerjee, Kaniel and Kremer (2006)).

[2] The learning literature (see Blume, Bray and Easley (1982) for an early survey) has found that convergence to rational expectations through learning requires investors to have an extensive structural knowledge of the economy and of other investors' beliefs. This level of sophistication may not reasonable for many investors. The behavioral finance literature suggests a number of cognitive biases, including over-confidence about private information and under-reaction to public information, which may lead investors to not condition on prices.

[3] A notable exception is Miller (1977) who shows that short sales constraints in a DO economy lead to a negative relationship between returns and belief dispersion.

for this opposite effect instead (e.g. Qu, Starks and Yan (2003)). The disparity in the models' assumptions and predictions make them difficult to compare, and given the conflicting empirical evidence, the link between dispersion and expected returns remains unclear. In this paper, I provide a theoretical framework to study the relationship between dispersion and a broader set of return-volume characteristics, and identify some important channels through which they are related.

I compare a dynamic RE model, in which investors condition on prices in addition to their private information, to a corresponding DO model, in which they only condition on their private information. I show that the relationship between the equilibrium belief dispersion and return-volume characteristics can be used to empirically distinguish the two models. Specifically, in the RE model, a higher belief dispersion is associated with higher expected returns, higher return volatility, and higher correlation between volume and absolute returns, but lower autocorrelation in returns. In the DO model, a higher level of disagreement is associated with lower expected returns, volatility and volume-absolute return correlation, but higher autocorrelation in returns. For both models, dispersion is positively related to expected volume and variance in volume. In the analysis below, the precise reason for why investors do not condition on prices is not important. What is important for the predictions is the dynamic nature of the economy, and the fact that investors condition on price in one model, but not in the other.

I test the models' predictions on the cross-section of equity returns and volume over multiple horizons, and use scaled versions of analyst forecast dispersion as proxies for the belief dispersion of investors. I use a Fama-MacBeth regression framework and control for other sources of heterogeneity across stocks. I find that the results for the quarterly and longer horizons are consistent with the predictions of the RE model. At the monthly horizon, however, the evidence is mixed. In particular, the autocorrelation in short horizon returns is positively related to dispersion (consistent with DO), but expected returns and volatility also show a positive relationship to dispersion (consistent with RE). I also find that controlling for the variable that is used to scale analyst forecast dispersion may partially reconcile the conflicting evidence about the dispersion - expected return relationship in the existing literature.[4]

The empirical evidence is consistent with the notion that while investors appear to have rational expectations in the long run and condition on the information in prices, some investors do not condition on prices in the short run. One interpretation of these results is that investors update their beliefs about the informativeness of other investors' signals over time. Initially, some investors exhibit differences of opinions, and rely mostly on their private information when updating their beliefs. If the disagreement persists, however, each investor updates the likelihood that other investors have payoff relevant information. As a result, investors begin to learn from prices as they would in a RE model over time. As an extension, I describe a hybrid model with both RE and DO agents, and numerically show that a relatively small fraction of DO investors is needed to match the monthly horizon results.

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[4] In particular, Diether, Malloy and Scherbina (2002) scale by absolute mean forecast and find a negative relationship between returns and dispersion, while Qu, Starks and Yan (2003) scale by lagged price and find a positive relationship.

The framework I develop is based on dynamic, Hellwig (1980)-type models. Investors receive information about the risky asset in the form of symmetric, conditionally independent, noisy private signals about next period's dividends. Since information is short-lived, investors do not have hedging demands, and their optimal allocation depends only on their beliefs about the dividend and the price in the next period. In the DO model, investors rely only on their private signals when updating their beliefs about the asset's payoffs, while in the RE model, investors also incorporate the price. Market clearing determines the equilibrium price, and aggregate supply shocks prevent the price from being fully-revealing.

Return and volume characteristics in both models are driven by the belief dispersion and the perceived risk from holding the asset. The dispersion in beliefs is measured as the variance in the investors' equilibrium expectations about the payoffs. The risk depends on the investors' posterior beliefs about the variance in payoffs. In both models, the expected return on the asset is a linear function of the risk. Moreover, this risk is higher when the price is more sensitive to the aggregate supply shocks. This implies that higher risk is associated with high return volatility and stronger mean reversion in prices, which leads to lower autocorrelation. High risk is also associated with higher correlation between absolute returns and volume, because supply shocks are the only component common to returns and volume. Finally, expected volume and the variance in volume are increasing in the posterior dispersion in beliefs for both models.

Given these dependencies, the empirical predictions used to distinguish the two models follow from the difference in the relationship between risk and dispersion in the models. In both models, the noise in the private signals across all investors determines not only the investors' uncertainty about payoffs, but also the degree to which they disagree in equilibrium. As a result, I characterize the relationship between risk and dispersion by studying how each changes as a function of this signal noise. For both models, the dispersion in beliefs is low when signals are very precise or very noisy (and so investors put little weight on the signals), but high when signal noise is in the middle. In contrast, the risk that investors face is a hump-shaped function of signal noise in the RE model but a U-shaped function in the DO model. These patterns lead to a positive relationship between risk and belief dispersion for the RE model, but a negative relationship in the DO model.

The non-monotonic behavior of risk that allows one to distinguish the two models critically depends on the dynamic nature of the economy. In a static setup, the risk is always increasing in the signal noise for both models because it only depends on the uncertainty in next period's dividends. However, in a dynamic economy, investors have the ability to resell the asset, and so, are exposed to the variance in next period's price. The behavior of this additional risk as a function of signal noise depends on whether investors learn from the price. In the RE model, when investors condition on the price to update their beliefs, there is a feedback effect through their demands that makes prices more sensitive to dividend and supply shocks. Furthermore, as investors rely more on the price, the variance of prices becomes higher. The weight on prices, and the resulting level of risk, is low when private signals are very informative or very noisy (which makes prices very uninformative), but higher for intermediate signal noise. As a result, risk in the RE model has a hump-shape, and so is positively related to dispersion. In the DO model, investors do not condition on prices and there is no feedback effect. The patterns in risk are driven by the uncertainty about next period's dividend and the variance in next period's price due to future dividend shocks. The first component of risk increases in signal noise and is high when signals are very noisy, while the second component decreases in signal noise and is high when signals are precise. As a result of these opposing effects, the total risk in the DO model first decreases and then increases in signal noise, and so is negatively related to dispersion.

The rest of the paper is organized as follows. In the next section, I briefly highlight some of the related literature. I describe the basic framework and derive the equilibria in the finite horizon RE and DO models in Sections 3.1 through 3.3. These finite horizon models are useful in clarifying intuition, as they provide a link between the static and dynamic setups. While the finite horizon equilibria always exist and are unique in each model, they are recursively defined and not very tractable. In Section 3.4, I formally derive the risk-dispersion relationships for the RE and DO models, using the stationary equilibria of the corresponding, infinite horizon, overlapping generations versions. I also show that while these stationary equilibria need not always exist, they do so when the supply shock variance is small. In Section 3.5, I derive the empirical predictions of each model in terms of the observable return-volume characteristics. Section 4 presents the empirical analysis of the paper, including a description of the regression specifications and a discussion of the results at different horizons. In Section 5, I describe the hybrid model and present a numerical example that is consistent with the monthly horizon results. In Section 6, I conclude.

2 Related Literature

The current paper is most closely related to Lang, Litzenberger and Madrigal (1992). The authors develop empirical predictions about the relationship between volume, price changes and changes in average forecasts around earnings announcements based on static models to distinguish between competitive (Walrasian) and rational expectations equilibria, with and without aggregate noise. As in the current paper, the authors find support for the noisy rational expectations equilibrium. However, their tests are based on static models and rely on the difference in levels of regression coefficients, which makes them more difficult to detect and more sensitive to mis-specification. In contrast, the predictions in this paper are based on dynamic models that rely on the signs of the relationships between disagreement and a number of return-volume characteristics. These differences in sign are easier to detect, and arguably more robust to the underlying specification.

The paper also contributes to the theoretical and the empirical literature on the dispersion in beliefs and return-volume characteristics. Diether, Malloy and Scherbina (2002) motivate their hypotheses with the optimistic pricing model in Miller (1977). A similar argument is made by Park (2005), who extends the Harrison and Kreps (1978) model to show that speculative pricing can lead to the negative dispersion-return relationship. While these papers also use DO models, they rely crucially on the presence of short sales constraints, which prevent full revelation of information. Johnson (2004) argues that the negative relationship can be explained using an option pricing result that states that for a levered firm, expected returns should decrease with idiosyncratic risk, which is potentially related to dispersion in forecasts. Ang and Ciccone (2001) claim that forecast dispersion is a proxy for firm transparency, and as a result, opaque firms, which have high forecast dispersion, usually have lower returns. Zhang (2006) claims that the negative relationship is due to behavioral biases and dispersion in forecasts is a proxy for information uncertainty. In contrast to the complexity of some of these models, I argue that the negative relationship between risk and dispersion is an immediate consequence of investors using only their private signals in a dynamic economy.

While the relationships between dispersion and other return-volume characteristics have been empirically studied in the existing literature, only a few have been theoretically modeled. Ajinkya, Atiase and Gift (1991) find that volume is positively related to analyst forecast dispersion. Ajinkya and Gift (1985) and Lobo and Tung (2002) find that firms with higher forecast dispersion have higher volatility in returns. Shalen (1993) develops a two period rational expectations model in which dispersion is positively related to the correlation in volume and absolute price changes and to the volatility in prices. Verardo (2002) finds a positive relationship between forecast dispersion and autocorrelation in returns and argues that this is consistent with the over-reaction and self-attribution model of Daniel, Hirshleifer and Subrahmanyam (1998), the under-reaction to public news model of Hong and Stein (1999) and the parameter uncertainty model of Lewellen and Shanken (2002). The current paper shows that these relationships may arise more generally, and may only depend on whether investors condition on prices or not.

Finally, the paper relates broadly to the rational expectations and difference of opinions literatures. In the RE literature, the models in the paper are most closely related to the long-lived investor model in Wang (1994), and the overlapping generations models in Spiegel (1998), Watanabe (2002) and Biais, Bossaerts, and Spatt (2005). However, the questions they address are different. Wang (1994) studies the relationship between returns, volume and ex-ante information asymmetry (signal noise). Spiegel (1998) studies the role of rational expectations equilibria in generating excess volatility in returns. Watanabe (2002) studies the impact of noisy rational expectations in generating correlation across assets, and looks at the trading behavior of hierarchically informed agents. Biais et al. (2005) develop a general equilibrium model and then empirically show that a price contingent portfolio out performs passive indexing returns.

Models in which agents exhibit differences in opinion have been useful in explaining a number of features of price and volume dynamics. These include models of speculation, bubbles and crashes (e.g. Harrison and Kreps (1978), Hong and Stein (2003), Scheinkman and Xiong (2003) and Cao and Ou-Yang (2005)), volume and volume-return characteristics (e.g. Kandel and Pearson (1995), Harris and Raviv (1993)), positive autocorrelation in volume (e.g. Harris and Raviv (1993), Banerjee and Kremer (2006)) and more recently, positive autocorrelation in returns (e.g. Banerjee, Kaniel and Kremer (2006)). I add to this literature by providing a dynamic DO model and deriving implications on the relationship between dispersion and return-volume characteristics.

3 The Models

3.1 Finite Horizon Setup

I begin with a finite horizon (T period) setup in which there is a continuum of agents. There are two assets in the economy. The risk-free rate pays an interest rate $r \geq 0$.[5] The risky asset pays dividends $D_t = D + \tilde{\delta}_t$ in period t, where the dividend shock $\tilde{\delta}_t$ is i.i.d. normal: $\tilde{\delta}_t \sim N(0, \sigma_d^2)$. Every period, agent i receives a private signal $Y_{i,t} = \tilde{\delta}_{t+1} + \tilde{s}_{i,t}$ about the dividend shock in the next period, where the error in the signal is i.i.d. normal with noise σ_s^2 (i.e. $\tilde{s}_{i,t} \sim N(0, \sigma_s^2)$). Denote agent i's information set at time t by $\mathcal{F}_{i,t}$. The net supply of the risky asset is positive and normalized to 1. At time t, there is an i.i.d. normal supply shock $\tilde{z}_t$ with mean zero and variance σ_z^2 (i.e. $\tilde{z}_t \sim N(0, \sigma_z^2)$). Supply shocks are a standard assumption in RE models to prevent perfect revelation of information from prices. I restrict attention to i.i.d. shocks for tractability, although numerical solutions show that the main results carry over in a setting with supply shocks that follow a random walk. Moreover, the conclusions do not change if the noise in prices is introduced through non-diversifiable endowment shocks to investors instead of aggregate supply shocks.

Agents maximize utility over final period wealth. They are endowed with CARA utility, and for notational simplicity, risk aversion is set to 1. Denote agent i's demand for the risky asset at time t by $X_{i,t}$, and the dollar return from holding the risky asset between period t and $t+1$ as $\tilde{R}_{t+1}$. This implies that for $t < T-1$,

$$\tilde{R}_{t+1} = P_{t+1} + D_{t+1} - (1+r)P_t$$

and $R_T = D_T - (1+r)P_{T-1}$. Note that resale of the asset is not possible in the last trading period. Using this notation, the agent's wealth process is of the form:

$$\tilde{W}_{i,t+1} = (W_{i,t} - X_{i,t}P_t)(1+r) + X_{i,t}(P_{t+1} + D_{t+1}) = W_{i,t}(1+r) + X_{i,t}\tilde{R}_{t+1}.$$

Conjecture a linear equilibrium of the form:

$$P_t = a_t\tilde{\delta}_{t+1} + b_t\tilde{z}_t + K_t, \tag{1}$$

where a_t, b_t and K_t are price coefficients. This implies that returns are normally distributed. Since the dividend shocks and supply shocks are i.i.d. and information is short lived, there are no hedging demands and the multi-period optimization problem reduces to a simple form. The final wealth of

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[5] Since the model has a finite horizon, it is well defined for $r = 0$. This will not be the case in the infinite horizon model of Section 3.4.

agent i can be re-written as:

$$\tilde{W}_{i,T} = W_{i,t}(1+r)^{T-t} + X_{i,t}\tilde{R}_{t+1}(1+r)^{T-t-1} + \sum_{s=t+1}^{T-1} X_{i,s}\tilde{R}_{s+1}(1+r)^{T-s-1}$$

For $s > t$, each optimal demand $X_{i,s}$ is independent of period t-information. Given the assumption of CARA utility, the optimal demand at time t can be written as:

$$X_{i,t} = \arg\max_{X_{i,t}} E[-\exp\{-\tilde{W}_{i,T}\}|\mathcal{F}_{i,t}] = \arg\max_{X_{i,t}} E[-\exp\{-X_{i,t}\tilde{R}_{t+1}\}|\mathcal{F}_{i,t}] \times \text{constant}$$

Hence agent i's demand at time t depends only on her beliefs about the return $\tilde{R}_{t+1}$.[6] These beliefs in turn depend on the posterior beliefs about next period's dividends. Denote agent i's beliefs at time t about the dividend shock $\tilde{\delta}_{t+1}$ as $\tilde{\delta}_{t+1}|\mathcal{F}_{i,t} \sim N(\mu_{i,t}, \hat{\sigma}^2_{d,t})$. These will be specified in the next two sub-sections. Then her beliefs about the return are given by:

$$\tilde{R}_{t+1}|\mathcal{F}_{i,t} \sim N(D + E[P_{t+1}] - (1+r)P_t + \mu_{i,t}, \hat{\sigma}^2_{R,t})$$

where the posterior variance in payoffs $\hat{\sigma}^2_{R,t}$ is given by:

$$\hat{\sigma}^2_{R,t} = \text{var}[P_{t+1}] + \hat{\sigma}^2_{d,t}$$

Note that the investors' beliefs at time t about the price in the next period is unaffected by her signal and so are equal to her unconditional beliefs. Since the private signals are identically distributed across agents, the posterior variance in dividends, $\hat{\sigma}^2_{d,t}$, and the posterior variance in returns, $\hat{\sigma}^2_{R,t}$, are the same across all agents.

Market clearing sets the aggregate demand equal to the aggregate supply in each period. Denoting the aggregate over the continuum of agents by the integral, the following holds:

$$\int_i X_{i,t} di = 1 + \tilde{z}_t \tag{2}$$

This completes the description of the finite horizon setup. In the next two sub-sections, I characterize the agents posterior beliefs about dividends in each type of model, and then use the market clearing condition (2) to characterize the equilibrium.

3.2 Difference of Opinions

Consider the difference of opinions model first, where the agents' updating problem is easier. Each agent updates her beliefs about next period's dividend shock using only her private signal. Denote

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[6] In particular, this implies that the model does not change if we replace long-lived agents with overlapping generations who live for one period.

the regression coefficient of next period's dividend shock on her private signal by Π_S^{DO}:

$$\Pi_S^{DO} = \frac{\text{cov}(Y_{i,t}, \tilde{\delta}_{t+1})}{\text{var}[Y_{i,t}]} = \frac{\sigma_d^2}{\sigma_d^2 + \sigma_s^2}$$

In this case, the regression coefficient does not depend on the period. It increases in the prior variance of the dividend shock and decreases in the private signal noise. As a result of Bayesian updating, we know that agent i's posterior beliefs are given by $\tilde{\delta}_{t+1}|\mathcal{F}_{i,t} \sim N(\mu_{i,t}, \hat{\sigma}_{d,t}^2)$, where:

$$\mu_{i,t} = \Pi_S^{DO} Y_{i,t} \qquad \text{and} \qquad \hat{\sigma}_{d,t}^2 = \sigma_d^2(1 - \Pi_S^{DO}) \tag{3}$$

The posterior variance in dividends, $\hat{\sigma}_{d,t}^2$, is the fraction of the prior variance unexplained by the signal. In this sense, the regression coefficient, Π_S^{DO}, measures the fraction of the prior variance in dividends that can be explained by the signal. Suppose also that each investor believes that the unconditional distribution of the other signals is given by:

$$Y_{i,t} \sim N(\bar{Y}, \sigma_s^2) \text{ i.i.d.}, \qquad \bar{Y} = \int_i Y_{i,t} di \sim N(0, \sigma_d^2) \tag{4}$$

where $\bar{Y}$ is independent of $\tilde{\delta}_{t+1}$. This implies that investors have difference of opinions since each investor believes that the signals of other investors are uninformative. This also ensures that their beliefs about the distribution of the price in the next period P_{t+1} are consistent with the true distribution of the price. Substituting these beliefs in the agent's demand for the risky asset over the continuum of investors and using a law of large numbers to replace $\int_i Y_{i,t} di$ by $\tilde{\delta}_{t+1}$, leads to the following lemma.

Lemma 1. *Suppose beliefs about the next period's dividends are given by (3), prices are of the form (1), and investors' unconditional beliefs about other signals are given by (4). Then the unique DO equilibrium is characterized by the following price coefficients:*

(a) For the last trading period:

$$a_{T-1} = \frac{\Pi_S^{DO}}{1+r}, \qquad b_{T-1} = -\frac{\hat{\sigma}_{d,t}^2}{1+r}, \qquad \textit{and} \qquad K_{T-1} = \frac{1}{1+r}[D - \hat{\sigma}_{d,t}^2]$$

(b) For earlier trading periods $t < T - 1$:

$$a_t = \frac{\Pi_S^{DO}}{1+r}, \qquad b_t = -\frac{\hat{\sigma}_{R,t}^2}{1+r}, \qquad \textit{and} \qquad K_t = \frac{1}{1+r}[D + K_{t+1} - \hat{\sigma}_{R,t}^2]$$

where $\hat{\sigma}_{R,t}^2 = a_{t+1}^2\sigma_d^2 + b_{t+1}^2\sigma_z^2 + \hat{\sigma}_{d,t}^2$.

The equilibrium always exists, is unique, and is defined recursively. The sensitivity to dividend shocks, a_t, is constant over time. The sensitivity to supply shocks, b_t, is driven by the posterior variance in payoffs. In the last trading period, the posterior variance in payoffs is simply the posterior variance in dividends, $\hat{\sigma}^2_{d,t}$, and so increases in the signal noise, σ^2_s.

In earlier trading periods, the posterior variance in payoffs is given by $\hat{\sigma}^2_{R,t}$, which is non-monotonic in the signal noise σ^2_s. In order to gain some intuition for this, note that the posterior variance in returns $\hat{\sigma}^2_{R,t}$ can be decomposed into three components: (i) uncertainty about dividends in the next period $\hat{\sigma}^2_{d,t}$, (ii) uncertainty due to the next period's price sensitivity to subsequent dividend shocks $a^2_{t+1}\sigma^2_d$, and (iii) uncertainty due to next period's price sensitivity to next period's supply shock $b^2_{t+1}\sigma^2_z$. The two dividend components, (i) and (ii), behave differently as functions of signal noise. Remember that the regression coefficient decreases with signal noise. The first dividend related component $\hat{\sigma}^2_{d,t}$ is the residual uncertainty in next period's dividend and so linearly decreases in the regression coefficient Π^{DO}_S - as a result, it increases in σ^2_s. The price sensitivity to dividend shocks, a_{t+1}, is linear in the regression coefficient since more informative signals imply that prices are more responsive to future dividend shocks. This implies that the second dividend component of posterior variance, $a^2_{t+1}\sigma^2_d$, increases quadratically in Π^{DO}_S, and as a result, decreases in σ^2_s. Moreover, the sum of the dividend components $\hat{\sigma}^2_{d,t} + a^2_{t+1}\sigma^2_d$ first decreases and then increases in the signal noise. For low signal noise (high Π^{DO}_S), the quadratic term dominates, and so the sum decreases in the signal noise. For high signal noise (low Π^{DO}_S), the linear term dominates and so the sum increases in the signal noise.

The price coefficient b_{t+1} decreases linearly in the next period's posterior variance. The last period's coefficient, b_{T-1} is linear in $-\hat{\sigma}^2_{d,t}$ and so decreases in signal noise. Therefore, the posterior variance in period $T-1$ initially decreases and then increases in the signal noise (the expression is a quadratic expression in Π^{DO}_S, which decreases in σ^2_s), which implies that the price coefficient b_{T-2} increases and then decreases in the σ^2_s. As a result, the posterior variance in period $T-2$ also decreases and then increases in σ^2_s. Repeating this argument, one can show that in all earlier periods ($t < T-1$), the price coefficient b_t increases and then decreases in σ^2_d, and as a result, the posterior variance in returns $\hat{\sigma}^2_{R,t}$ decreases and then increases in σ^2_d. The value of σ^2_s that minimizes the posterior variance $\hat{\sigma}^2_{R,t}$ changes over time and is difficult to characterize explicitly.[7] In general, the posterior variance in return $\hat{\sigma}^2_{R,t}$ first decreases and then increases in the signal noise σ^2_d.

Next, let us turn to the measure of belief dispersion. In the current setup, agents share a common prior about the distribution of returns. This implies that the only source of disagreement is the difference in the private signals. If the ex-ante dispersion in private signals were empirically observable, one could derive the comparative statics with respect to signal noise σ^2_s to compare the model's implications with the data. However, empirical proxies for belief dispersion are usually ex-post and not ex-ante, and one needs a comparable theoretical measure. I use the dispersion in the agents posterior expectations about dividends as the measure for belief dispersion. Specifically, the posterior

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[7] In the limit equilibrium defined in Section 3.4, one can explicitly identify the level of σ^2_s that minimizes the price sensitivity.

dispersion in beliefs $\hat{\sigma}^2_{\mu,t}$ is given by:

$$\hat{\sigma}^2_{\mu,t} = \text{var}[\mu_{i,t} - \bar{\mu}_{i,t}] \qquad \text{where} \qquad \bar{\mu}_{i,t} = \int_i \mu_{i,t} di.$$

This measure accounts for any variation in the average beliefs about the dividends. In this way, one can isolate the effects of belief dispersion from information uncertainty.

For the difference of opinions model, the dispersion in beliefs is simply given by:

$$\hat{\sigma}^2_{\mu,t} = (\Pi^{DO}_S)^2 \sigma^2_s = \left(\frac{\sigma^2_d}{\sigma^2_d + \sigma^2_s} \right)^2 \sigma^2_s$$

In this case, belief dispersion is independent of the period, increases in σ^2_d and is non-monotonic in signal noise σ^2_s. When there is very little noise in the signals (low σ^2_s), the signals are very close to each other, and consequently, so are the posterior beliefs. When there is a lot of noise in the signals (high σ^2_s), agents do not much weight on them, and so there is little posterior dispersion in beliefs. The dispersion in beliefs is highest when $\sigma^2_s = \sigma^2_d$.

Figure 1 shows a numerical example of these results. It plots the price coefficients a_t and b_t and the posterior dispersion in beliefs $\hat{\sigma}^2_{\mu,t}$ as functions of the signal noise for different periods.[8] Since a_t and $\hat{\sigma}^2_{\mu,t}$ do not change over time, the plots for all the periods coincide. Also, while the last trading period coefficient b_{T-1} monotonically decreases in σ^2_s, in general b_t increases and then decreases in σ^2_s.

3.3 Rational Expectations

In the rational expectations model, agents condition on both their private signals and the price in updating their beliefs about the next period's dividend shocks. Denote the regression coefficients at time t of next period's return on the private signal and the price as $\Pi^{RE}_{S,t}$ and $\Pi^{RE}_{P,t}$ respectively. These coefficients are given by:

$$(\Pi^{RE}_{S,t}, \Pi^{RE}_{P,t}) = (\sigma^2_d, a_t \sigma^2_d) \begin{pmatrix} \sigma^2_d + \sigma^2_s & a_t \sigma^2_d \\ a_t \sigma^2_d & a^2_t \sigma^2_d + b^2_t \sigma^2_z \end{pmatrix}^{-1}$$

Denote the ratio of variances $\theta_t = \frac{a^2_t \sigma^2_d}{b^2_t \sigma^2_z}$. Then the coefficients can be re-written as:

$$\Pi^{RE}_{S,t} = \frac{\sigma^2_d}{\sigma^2_d + \sigma^2_s + \sigma^2_s \theta_t} \qquad \text{and} \qquad \Pi^{RE}_{P,t} = \frac{1}{a_t} \frac{\sigma^2_s \theta_t}{\sigma^2_d + \sigma^2_s + \sigma^2_s \theta_t}$$

As a result, agent i's posterior beliefs about the dividend shock are given by $\tilde{\delta}_{t+1} | \mathcal{F}_{i,t} \sim N(\mu_{i,t}, \hat{\sigma}^2_{d,t})$, where:

$$\mu_{i,t} = \Pi^{RE}_{S,t} Y_{i,t} + \Pi^{RE}_{P,t} (P_t - K_t) \qquad \text{and} \qquad \hat{\sigma}^2_{d,t} = \sigma^2_d (1 - \Pi^{RE}_{S,t} - a_t \Pi^{RE}_{P,t}) \tag{5}$$

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[8] It also plots the limit equilibrium, which shall be discussed in Section 3.4.

The posterior variance in dividends is still the fraction of the prior variance that is unexplained by the two signals. However, in this case it is time varying, since agents use the price to update their beliefs, and this has a time varying distribution. Substituting these beliefs in the agent's demand for the risky asset, aggregating over the continuum of investors, and using a law of large numbers to replace $\int_i Y_{i,t} di$ with $\tilde{\delta}_{t+1}$, leads to the following lemma, which I prove in the appendix.

Lemma 2. *Suppose beliefs about the next period's dividends are given by (5) and prices are of the form given in (1). Then the unique RE equilibrium is characterized by the following price coefficients:*

(a) For the last trading period:

$$a_{T-1} = \frac{\Pi^{RE}_{S,T-1}}{1 + r - \Pi^{RE}_{P,T-1}}, \quad b_{T-1} = -\frac{\hat{\sigma}^2_{d,t}}{1 + r - \Pi^{RE}_{P,T-1}}, \quad \text{and} \quad K_{T-1} = \frac{1}{1+r}[D - \hat{\sigma}^2_{d,t}]$$

(b) For earlier trading periods $t < T - 1$:

$$a_t = \frac{\Pi^{RE}_{S,t}}{1 + r - \Pi^{RE}_{P,t}}, \qquad b_t = -\frac{\hat{\sigma}^2_{R,t}}{1 + r - \Pi^{RE}_{P,t}}, \qquad \text{and} \qquad K_t = \frac{1}{1+r}[D + K_{t+1} - \hat{\sigma}^2_{R,t}]$$

where $\hat{\sigma}^2_{R,t} = a^2_{t+1}\sigma^2_d + b^2_{t+1}\sigma^2_z + \hat{\sigma}^2_{d,t}$.

The equilibrium exists and is unique. The expressions in the RE model are complicated by the fact that agents condition on the price, which is an endogenous signal. Also, comparative static results are difficult to prove due to the recursive nature of the equilibrium. In the rest of the section, I shall describe the features of the equilibrium without proofs, but will provide some intuition for these results. I shall then provide more formal arguments for the limit equilibrium in section 3.4, which is more tractable.

The price coefficient a_t can be re-written as the fraction of variance explained by the signals, discounted by the risk free rate:

$$a_t = \frac{1}{1+r}(\Pi^{RE}_{S,t} + a_t \Pi^{RE}_{P,t}) = \frac{1}{1+r} \frac{\sigma^2_d + \sigma^2_s \theta_t}{\sigma^2_d + \sigma^2_s + \sigma^2_s \theta_t}$$

This implies that a_t is positive, less than $1/(1+r)$, increases in the prior variance in dividends, σ^2_d, and decreases in the supply shock noise, σ^2_z, and the signal noise, σ^2_s. The posterior variance in dividends, $\hat{\sigma}^2_{d,t}$, linearly decreases in the fraction of the explained variance. As a result, the dividend related components $a^2_{t+1}\sigma^2_d + \hat{\sigma}^2_{d,t}$ first increase and then decrease in the signal noise, as in the DO case.

The price sensitivity to supply shocks as a function of σ^2_s behaves differently in the RE model. In particular, the price coefficient b_t initially decreases and then increases with σ^2_s. This is due to the fact that the agents use the price to update their beliefs about dividends, and hence expose themselves to the supply shocks not only through the aggregate supply, but through their own beliefs. Moreover, the weight agents put on the price in updating their beliefs first increases and then decreases as a function of signal noise. When signal noise is low, price sensitivity to dividend shocks a_t is high, and so prices are informative about the dividend shock in the next period. In this range, the price acts as a substitute source of information - an increase in σ_s^2 leads to an decrease in the weight on the private signal $\Pi_{S,t}^{RE}$, but an increase in the weight on the price $\Pi_{P,t}^{RE}$. However, when signal noise is very large, a_t is small, and prices are not very informative. In this range, the weight on the price $\Pi_{P,t}^{RE}$ decreases with σ_s^2.

In the RE equilibrium, changes in the supply shock component dominate those in the dividend related components. As a result, the posterior variance in returns $\hat{\sigma}_{R,t}^2$ first increases and then decreases as a function of signal noise. Agents put a higher weight on the price for intermediate levels of σ_s^2, and so the exposure to supply shock uncertainty is higher in this range than in the extremes.

The posterior dispersion in beliefs in the RE equilibrium is given by:

$$\hat{\sigma}_{\mu,t}^2 = (\Pi_{S,t}^{RE})^2 \sigma_s^2 = \left(\frac{\sigma_d^2}{\sigma_d^2 + \sigma_s^2 + \sigma_s^2 \theta_t} \right)^2 \sigma_s^2$$

As in the DO model, belief dispersion first increases and then decreases as a function of signal noise. The dispersion is highest when σ_s^2 satisfies the relevant first order condition. An example of these results can be seen in Figure 2. It plots the price coefficients a_t and b_t and the posterior dispersion in beliefs $\hat{\sigma}_{\mu,t}^2$. All three variables do change over time, unlike the DO model - however, they do exhibit the patterns described above.

In the last two sub-sections, I have described finite horizon versions of the DO and RE models. There exists a unique equilibrium in each case, and I have described how these equilibria differ across the models. In particular, for the DO model, I find the posterior variance in returns $\hat{\sigma}_{R,t}^2$ initially decreases and then increases in signal noise. In contrast, for the RE model, the posterior variance in returns initially increases and then decreases in signal noise. The posterior dispersion in beliefs in both models increases and then decreases in signal noise. However, analyzing these equilibria is difficult since they are recursively defined and non-stationary. In the next section, I characterize the limit of these equilibria, and show that these limits are equilibria of the corresponding infinite horizon, overlapping generation models. Moreover, I formally analyze some of the results outlined in the previous sections.

3.4 The Limit Equilibria

With some abuse of notation, denote $a_t(a_{t+1}, b_{t+1}, K_{t+1})$, $b_t(a_{t+1}, b_{t+1}, K_{t+1})$ and $K_t(a_{t+1}, b_{t+1}, K_{t+1})$ as the equilibrium price coefficients in a finite horizon (T period) model of the type described in Section 3.1. I make explicit the recursive dependence of the current price coefficients on the next period's price coefficients, but suppress the dependence on the parameters of the model.

Then the "limit equilibrium" of the model is defined as price coefficients a, b, and K, where:

$$a = \lim_{T-t\to\infty} a_t, \qquad b = \lim_{T-t\to\infty} b_t, \qquad \text{and} \qquad K = \lim_{T-t\to\infty} K_t.$$

The limit equilibrium is the limit of the finite horizon equilibrium in the current period as the total number of periods increases to infinity. This is intuitive as the source of non-stationarity is the final period. As one moves further away from the last period, one could expect consecutive periods to resemble each other more. The limit equilibrium need not always exist, as the sequence $\{a_t,\, b_t,\, K_t\}$ need not always converge. However, if it does exist, it is characterized by the following fixed point conditions:

$$a_t(a,b,K) = a, \qquad b_t(a,b,K) = b, \qquad \text{and} \qquad K_t(a,b,K) = K.$$

I show that the stationary equilibria of the analogous overlapping generations models satisfy the above conditions. I also characterize the conditions under which these limits exist, and then derive the comparative statics results discussed in the earlier sections using these limit equilibria.

The setup of the infinite horizon, overlapping generations (OLG) model is very similar to the one in Section 3.1. The dividend process D_t, the signals $Y_{i,t}$, and the aggregate supply shocks, $\tilde{z}_t$ have the same distributions as before. The risk free rate r is assumed to be greater than zero.[9] There is still a continuum of agents in each period, but their preferences are altered. Specifically, the agents of generation t have CARA utility over their wealth at period $t+1$. Risk aversion is still set to 1 for notational simplicity. As before, the dollar return from the risky asset is denoted by $\tilde{R}_{t+1} = P_{t+1} + D_{t+1} - (1+r)P_t$.

Consider stationary equilibria of the form:

$$P_t = a\tilde{\delta}_{t+1} + b\tilde{z}_t + K \tag{6}$$

where the price coefficients a, b and K are constant over time. Returns are normally distributed and agent i in generation t has a demand of the form:

$$X_{i,t} = \arg\max_{X_{i,t}} E[-\exp\{-\tilde{W}_{i,t+1}\}|\mathcal{F}_{i,t}] = \frac{E[\tilde{R}_{t+1}|\mathcal{F}_{i,t}]}{\mathrm{var}[\tilde{R}_{t+1}|\mathcal{F}_{i,t}]}$$

If agent i's posterior beliefs about the dividends in the next period are given by $\tilde{\delta}_{t+1}|\mathcal{F}_{i,t} \sim N(\mu_{i,t}, \hat{\sigma}_d^2)$, then her beliefs about the return are of the form:

$$\tilde{R}_{t+1}|\mathcal{F}_{i,t} \sim N(D + E[P_{t+1}] - (1+r)P_t + \mu_{i,t}, \hat{\sigma}_R^2),$$

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[9] Here the assumption has bite since the average price of the asset is the discounted value of an infinite stream of dividends with positive mean, adjusted for the amount of risk. With $r = 0$, the price would explode.

where the posterior variance in returns $\hat{\sigma}_R^2$ is given by:

$$\hat{\sigma}_R^2 = \text{var}[P_{t+1}] + \hat{\sigma}_d^2$$

Again, since the information is short lived, investors' private signals do not affect their beliefs about next period's price. Since the equilibrium is stationary, the posterior variances are constant over time. Market clearing sets aggregate supply equal to aggregate demand, and so the following holds:

$$\int_i X_{i,t} di = 1 + \tilde{z}_t \tag{7}$$

Next, one needs to specify the beliefs of the agents in either model. I begin with the DO case and then describe the RE model.

3.4.1 Difference of Opinions

In the DO model, agents only use their private signal to update their beliefs. The regression coefficient is again given by:

$$\Pi_S^{DO} = \frac{\sigma_d^2}{\sigma_d^2 + \sigma_s^2}$$

and beliefs about next period's dividend shocks are given by $\tilde{\delta}_{t+1} | \mathcal{F}_{i,t} \sim N(\mu_{i,t}, \hat{\sigma}_d^2)$, where:

$$\mu_{i,t} = \Pi_S^{DO} Y_{i,t}, \qquad \text{and} \qquad \hat{\sigma}_{d,t}^2 = \sigma_d^2 (1 - \Pi_S^{DO}). \tag{8}$$

As before, investors exhibit differences in opinion and so believe that the other signals are distributed as:

$$Y_{i,t} \sim N(\bar{Y}, \sigma_s^2) \text{ i.i.d.}, \qquad \bar{Y} = \int_i Y_{i,t} di \sim N(0, \sigma_d^2) \tag{9}$$

where $\bar{Y}$ is independent of $\tilde{\delta}_{t+1}$. Substituting these beliefs into the optimal demand expression and using the market clearing condition (7) gives us the following lemma.

Lemma 3. *Suppose beliefs about dividends in the next period are given by (8), prices are of the form given in (6), and beliefs about the other signals are given by (9). Then a stationary equilibrium of the DO OLG model exists if*

$$(1 + r)^2 - 4\sigma_z^2 (a^2 \sigma_d^2 + \hat{\sigma}_d^2) \geq 0. \tag{10}$$

If the above condition holds, then the equilibrium price coefficients are given by:

$$a = \frac{\Pi_S^{DO}}{1 + r}, \qquad b = -\frac{\hat{\sigma}_R^2}{1 + r}, \qquad \text{and} \qquad K = \frac{1}{r}[D - \hat{\sigma}_R^2]$$

These expressions also characterize the limit equilibrium of the finite horizon DO model.

Plugging in the expression for $\hat{\sigma}_R^2$ in the expression for b gives the following quadratic equation in b:

$$b^2\sigma_z^2 + b(1+r) + a^2\sigma_d^2 + \hat{\sigma}_d^2 = 0 \tag{11}$$

Condition (10) ensures that the discriminant of the solution to the above equation is non-negative. One interpretation of the condition is based on the recursive definition of b_t in the finite horizon model. The above condition ensures that the noise in prices σ_z^2 is bounded above, given other parameters of the model. If the noise σ_z^2 were larger than this, the supply shock component of the posterior variance would be too large, and this in turn would make the price sensitivity to supply shocks b_t too large in the previous period. Rolling back, this would lead to a sequence of b_t which explodes, and so a limit would not exist. Intuitively, when supply shocks are large, this reduces liquidity and increases the risk premium in the current period. However, this implies that investors in earlier periods face more uncertainty from future prices, which decreases liquidity and increases the risk premium in earlier periods. If the noise σ_z^2 is too large, then the risk premium would explode as one rolls back in time.

The quadratic equation (11) has two possible roots. Numerically, one can show that the finite horizon DO equilibrium price coefficient b_t converges to the root:

$$b = \frac{-(1+r) + \sqrt{(1+r)^2 - 4\sigma_z^2(a^2\sigma_d^2 + \hat{\sigma}_d^2)}}{2\sigma_z^2}.$$

Hence, I focus on this one in the rest of the analysis.[10] As mentioned in Section 3.2, this implies that the price coefficient b first increases and then decreases as a function of signal noise σ_s^2. This in turn implies that the posterior variance in returns initially decreases and then increases in σ_s^2. Combining this with the fact that the posterior dispersion in beliefs $\hat{\sigma}_\mu^2 = (\Pi_S^{DO})^2\sigma_s^2$ remains unchanged from the finite horizon case, gives the following result.

Proposition 1. *Denote the maximizers of the posterior variance in returns* $\hat{\sigma}_R^2$ *and the posterior dispersion in beliefs* $\hat{\sigma}_\mu^2$ *by* s_R^* *and* s_μ^* *respectively. Then,* $s_R^* = \sigma_d^2(2/(1+r)^2 - 1)$ *and* $s_\mu^* = \sigma_d^2$*. Moreover, in the DO model, the posterior variance in returns* $\hat{\sigma}_R^2$ *and the posterior dispersion in beliefs* $\hat{\sigma}_\mu^2$ *are negatively related to each other except when* $\sigma_s^2 \in (s_R^*, s_\mu^*)$*. This region,* (s_R^*, s_μ^*)*, shrinks to zero as* $r \rightarrow 0$*.*

The above follows from the fact that $\hat{\sigma}_R^2$ first decreases and then increases with σ_s^2, while $\hat{\sigma}_\mu^2$ first increases and then decreases with σ_s^2. Moreover, $\hat{\sigma}_R^2$ is lowest when $\sigma_s^2 = \sigma_d^2(2/(1+r)^2 - 1)$. Similarly, $\hat{\sigma}_\mu^2$ is highest when $\sigma_s^2 = \sigma_d^2$. The negative relationship between the posterior variance in returns and the dispersion in beliefs holds for all but a small portion of the parameter space. The same

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[10] In addition to not being the limit of the finite horizon equilibrium price coefficient, the other root of (11) can be ruled out based on the empirical predictions it leads to. Specifically, the other root corresponds to a more volatile equilibrium in which there is strong negative autocorrelation in returns and very high correlation in absolute returns and price changes for a wide range of parameters, neither of which is empirically observed.

result is true in the finite horizon model, but it is more difficult to explicitly find the level at which $\hat{\sigma}^2_{R,t}$ is maximized. Finally, note that the expressions that characterize the equilibria are the same as those in the finite horizon model, except that the time dependent coefficients have been replaced by their stationary counterparts. This establishes the characterization of the limit equilibrium.

3.4.2 Rational Expectations

Next consider the RE model, in which agents update their beliefs about dividends using their private signals and the price. The regression coefficients on the private signal and the price are given by the following expressions respectively:

$$\Pi_S^{RE} = \frac{\sigma_d^2}{\sigma_d^2 + \sigma_s^2 + \sigma_s^2\theta} \qquad \text{and} \qquad \Pi_P^{RE} = \frac{1}{a}\frac{\sigma_s^2\theta}{\sigma_d^2 + \sigma_s^2 + \sigma_s^2\theta}$$

where $\theta = \frac{a^2\sigma_d^2}{b^2\sigma_z^2}$. As a result, beliefs about dividends are given by $\tilde{\delta}_{t+1}|\mathcal{F}_{i,t} \sim N(\mu_{i,t}, \hat{\sigma}_d^2)$, where:

$$\mu_{i,t} = \Pi_S^{RE} Y_{i,t} + \Pi_P^{RE}(P_t - K) \qquad \text{and} \qquad \hat{\sigma}_d^2 = \sigma_d^2(1 - \Pi_S^{RE} - a\Pi_P^{RE}) \tag{12}$$

Substituting these beliefs into the optimal demand and using the market clearing condition gives us the following result.

Lemma 4. *Suppose beliefs about dividends in the next period are given by (12) and prices are of the form given in (6), and denote the ratio of the price coefficients by $x = a/b$. Then a stationary equilibrium of the RE OLG model exists if there is a real solution x of a 6-th order polynomial of the form:*

$$A_0 + A_1x + A_2x^2 + A_3x^3 + A_4x^4 + A_6x^6 = 0 \qquad \textit{where} \qquad A_i > 0 \quad \textit{for all } i \tag{13}$$

If there exists a solution x to the above equation, then $x \in (-1/\sigma_s^2, 0)$ and the equilibrium price coefficients are given by:

$$a = \frac{\Pi_S^{RE}}{1 + r - \Pi_P^{RE}} \geq 0, \qquad b = -\frac{\hat{\sigma}_R^2}{1 + r - \Pi_P^{RE}} \leq 0, \qquad \textit{and} \qquad K = \frac{1}{r}[D - \hat{\sigma}_R^2],$$

where $\theta = x^2\sigma_d^2/\sigma_z^2$. These expressions also characterize the limit equilibrium of the finite horizon RE model.

The coefficients A_i in equation (13) are specified in the appendix. Note that since all the coefficients are positive, the solutions x must be negative. Since a is positive, this implies that b is negative. The condition for which the equilibrium exists is not as transparent as in the DO case, but numerical solutions suggest that there are 2 real roots and 4 imaginary roots - the limit of the finite horizon models corresponds to the more negative real root.[11] Moreover, the parameters over which the

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[11] The root that I focus on is the limit of the finite horizon model. The other real root corresponds to the more volatile

equilibrium exist seem reasonable. Figure 3 compares the parameter ranges over which the stationary equilibrium exists for the two models. For both models, increasing r, decreasing σ_z^2 and decreasing σ_s^2 increase the likelihood of the existence of stationary equilibria. However, an interesting distinction between the models is that the existence of equilibria seems to be quite sensitive to σ_d^2 for the DO model, but not as much for the RE model. While the reason for this is not immediate, it is due to the fact that the relative noise in the signals / prices with respect to the prior variance is what drives the RE equilibrium, and not the absolute levels of the variance per se.

As in the finite horizon model, the posterior variance in returns first increases and then decreases in signal noise. This is due to the fact that for intermediate levels of signal noise, agents put a higher weight on the price, which leads to a higher sensitivity to supply shock noise. A numerical example of this can be seen in Figure 4. This leads to an overall increase in the posterior variance in returns, even though the posterior variance from the dividend components is lower. On the other hand, the posterior dispersion in beliefs increases and then decreases with signal noise, as in all the models. This implies the following result, which is proved in the appendix.

Proposition 2. *There are unique maximizers of the posterior variance in returns* $\hat{\sigma}_R^2$ *and the posterior dispersion in beliefs* $\hat{\sigma}_\mu^2$*, denoted by* s_R^* *and* s_μ^* *respectively. In the RE model,* $\hat{\sigma}_R^2$ *and* $\hat{\sigma}_\mu^2$ *are positively related to each other except when* $\sigma_s^2 \in (s_\mu^*, s_R^*)$.

The region $\sigma_s^2 \in (s_\mu^*, s_R^*)$ is difficult to characterize since these maximizers cannot be solved for explicitly. Numerical examples suggest that the region shrinks as r decreases.

3.5 Observable Characteristics

In this subsection, I characterize the moments of the return-volume distribution and use the posterior variance-dispersion relationships to derive the empirical implications of each model. Instead of using price changes, as is common in OLG models, I use the dollar returns $\tilde{R}_{t+1}$ as the measure of returns. While the expressions are slightly more complicated, this leads to non-zero expected returns and interesting autocorrelation properties. Define volume as the unconditional expectation of absolute trades (i.e. $\tilde{V}_{t+1} = \int |X_{i,t+1} - X_{i,t}| di$). The following expressions are derived in the appendix.

Lemma 5. *Suppose returns are given by* $\tilde{R}_{t+1} = P_{t+1} + D_{t+1} - (1+r)P_t$ *and volume is given by* $\tilde{V}_{t+1} = \int |X_{i,t+1} - X_{i,t}| di$*. Then, for both models, the following expressions characterize the return volume distribution:*

(i) $E[\tilde{R}_{t+1}] = \hat{\sigma}_R^2$

(ii) $var[\tilde{R}_{t+1}] = a^2\sigma_d^2 + b^2\sigma_z^2 + (1-(1+r)a)^2\sigma_d^2 + (1+r)^2 b^2 \sigma_z^2$

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equilibrium, which is analogous to the volatile equilibrium in the DO model. As in the DO model, it exhibits strong negative autocorrelation in returns, and high correlation in volume and absolute returns.

(iii) $cov[\tilde{R}_{t+1}, \tilde{R}_{t+2}] = a(1-(1+r)a)\sigma_d^2 - b^2(1+r)\sigma_z^2$

(iv) $E[\tilde{V}_{t+1}] = \sqrt{\frac{4}{\pi}(\sigma_z^2 + \sigma_s^2\frac{a^2}{b^2})}$

(v) $var[\tilde{V}_{t+1}] = \frac{2(\pi-2)}{\pi}(\sigma_z^2 + \sigma_s^2\frac{a^2}{b^2})$

(vi) $cov[\tilde{V}_{t+1}, |\tilde{R}_{t+1}|] = \Psi((2+r)b\sigma_z^2)$

where Ψ*, which transforms the covariance of two normals to the covariance between their absolute values, is defined in the appendix.*

There are a few immediate implications of the model setup on the distribution of returns and volume. Volume is independent of dividend shocks and the autocorrelation in volume is constant. These follow from the fact that the agents' information is short-lived and symmetric. Persistent and / or hierarchical information can generate correlation between volume and dividend shocks, and non-trivial autocorrelation patterns in volume (see Wang (1994) for example). Expected volume and volatility in volume are monotonic transformations of each other - these results stem from the fact that volume is a half-normal random variable.

For both models, expected returns have the standard form of risk per unit of the asset ($\hat{\sigma}_R^2$) multiplied by total supply of the asset (which is normalized to 1). Hence the patterns in Propositions 1 and 2 apply directly to expected returns. The volatility and autocovariance in returns can be re-written as

$$\begin{aligned}
\text{var}[\tilde{R}_{t+1}] &= \hat{\sigma}_R^2(1+(1+r)^2) + a\sigma_d^2(1+r)((1+r)^2-1) - (1+r)^2\sigma_d^2, \qquad \text{and} \\
\text{cov}[\tilde{R}_{t+1}, \tilde{R}_{t+2}] &= -\left((1+r)\hat{\sigma}_R^2 + a\sigma_d^2((1+r)^2-1) - (1+r)\sigma_d^2\right)
\end{aligned}$$

respectively and are primarily driven by the posterior variance $\hat{\sigma}_R^2$. This is especially apparent as we take the limit of the above expressions when $r \to 0$. Intuitively, this is because when risk is high, prices are more sensitive to supply shocks. This implies that prices are more volatile, and that they exhibit stronger mean reversion. As a result, higher risk is associated with higher volatility and more negative auto-correlation in returns.

Volume in these models is driven by the variance in the signed trade $\Delta X_{i,t,t+1} = X_{i,t+1} - X_{i,t}$. This variance can be re-written as

$$\text{var}[\Delta X_{i,t,t+1}] = 2\sigma_z^2 + 2\sigma_s^2\frac{a^2}{b^2} = 2\sigma_z^2 + 2\frac{\hat{\sigma}_\mu^2}{(\hat{\sigma}_R^2)^2}$$

There are two sources of volume - supply shocks and informed trade. Informed trade increases in the posterior dispersion in beliefs, $\hat{\sigma}_\mu^2$, but decreases in the perceived risk, $\hat{\sigma}_R^2$. The expected volume and the variance in volume are monotonic transformations of the above expressions and so increase in the dispersion in beliefs and decrease in the posterior variance in returns. Finally note that given the simplifying assumptions, the only common component between returns and volume are supply shocks - this is because dividend shocks do not affect the volume, and the dispersion in beliefs does not affect prices (while the average beliefs do). These results are summarized in the following proposition, which is proved in the appendix. To clarify, a hump-shaped function first increases and then decreases in its argument, while a U-shaped function first decreases and then increases.

Proposition 3. *In both models, the posterior dispersion in beliefs, expected volume and variance in volume first are hump-shaped functions of signal noise* σ_s^2. *In the RE model, expected returns, variance in returns and the covariance in returns are hump-shaped functions of signal noise, but the autocorrelation in returns is U-shaped. In the DO model, expected returns, variance in returns and the covariance in returns are U-shaped functions of signal noise, but the autocorrelation in returns is hump-shaped.*

The above proposition will provide the basis for empirically distinguishing between the two models. Specifically, in the RE model, belief dispersion is positively related to expected returns, variance in returns, covariance between absolute returns and volume, but negatively related to autocorrelation in returns. In the DO model, the relationships between dispersion and these return-volume characteristics are reversed.

4 Empirical Analysis

In going from the theoretical model to the empirical tests, some caveats must be kept in mind. First, I assume that the comparative static results derived in a single asset model can be tested using the cross-section of stocks. In particular, this implies that investors are risk averse to individual stocks. However, I do not need information uncertainty to be the primary determinant of returns, and I do include controls in the empirical analysis to account for other sources of heterogeneity in returns and volume across firms. Also, note that the level of risk aversion is not important to the results - the *signs* of the cross-sectional effects, and not their magnitudes, are used to distinguish the two models. So even if investors are only mildly risk averse to individual stocks, the results should carry through.[12] Finally, while this problem is potentially severe for the dispersion-expected return relationship, the relationships between dispersion and the other return-volume characteristics should be robust to this concern.

The second caveat is that I assume the empirical distribution of the parameters of the model are such that the comparative static results are not dominated by the effects when $\sigma_s^2 \in (s_R^*, s_\mu^*)$ for the DO case and $\sigma_s^2 \in (s_\mu^*, s_R^*)$ for the RE case. Numerical analysis shows that these regions are small,

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[12] This seems to be a reasonable assumption, given the extant literature on under-diversification of investors (e.g. Barber and Odean (2001), Goetzmann and Kumar (2005)). Moreover, while there may be well-diversified investors trading in these stocks, they may not necessarily be the marginal investors. Instead, specialized investors with private information, who are also likely to be concentrated in these stocks, may be the ones who determine the return / volume characteristics.

and shrink to zero when $r \to 0$. If the distribution of σ_s^2 is sufficiently disperse so that not too much weight in the region between the two optima, then the average cross-sectional effects of each model coincide with those in Proposition 3. Since the model is very stylized, and parameterizing it is difficult, I do not attempt to derive and test restrictions on the distribution of σ_s^2.[13]

Finally, a number of papers document biases in analyst forecasts, including over-optimism (e.g. Hong and Kubik (2003)), under-reaction to public information and over-confidence in private information (e.g. Abarbanell and Bernard (1992)), and herding and anti-herding of forecasts (e.g. Hong, Kubik and Solomon (2000), Bernhardt, Campello and Kutsoati (2004)). This potentially raises concerns about the use of analyst forecast dispersion as a proxy for investor belief dispersion. While admittedly very noisy, there are a number of reasons for why analyst dispersion may be a good proxy. First, using this proxy enables me to compare my empirical analysis to the existing literature. Second, the results depend on the cross-sectional variation in dispersion, and so are presumably unaffected by biases, unless analyst biases are systematically different across stocks. Even if the analyst forecast bias varies systematically across stocks, these effects should be mitigated by the controls I use. Finally, other measures like volume, breadth of ownership and short interest, may be affected by factors unrelated to belief dispersion, while at least theoretically, analyst dispersion seems to be more directly related to the dispersion of investor beliefs.

I use returns and volume data and standard proxies for belief dispersion to test the implications of each model. I use monthly return and volume data from CRSP, and as in the previous literature, the measure of volume is log turnover.[14] For the measures of belief dispersion, I use proxies that are based on the dispersion in analyst forecasts of annual earnings per share from IBES.[15] In order to have enough firms in each cross-section, and to make the results more comparable with the existing literature, I present results on data from January 1983 to December 2005. Results do not qualitatively change if one begins in 1978. For firm specific data like outstanding shares and total assets, I use COMPUSTAT.

Given that analyst forecast dispersion is in terms of dollar per share, one must control for some measure of firm size to avoid a mechanical relationship between size and dispersion (larger firms have higher earnings per share and so higher dispersion). The literature suggests a number of variables that can be used to scale analyst dispersion e.g. absolute lagged earnings, lagged price, absolute mean earnings forecast. However, these variables seem to have a strong effect on the measure of

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[13] Looking ahead to the summary statistics in Table 1, the left skewness in the proxies for belief dispersion are reassuring as they suggests that many of the firms have relatively small posterior dispersion, and so have either small σ_s^2 or large σ_s^2. I also repeat the analysis for a subsample of firms with low dispersion and find similar results (see Table 5). This suggests that the firm characteristics for large σ_s^2 and small σ_s^2 firms are similar to those in the middle, and so the firms in the range $\sigma_s^2 \in (s_R^*, s_\mu^*)$ do not dominate the results.

[14] The monthly log turnover is calculated as $\log\left(\frac{\text{volume}_t}{\text{shares outstanding}_t} + 0.0051\right)$ where the constant is an adjusted version of the weekly constant in Richardson, Sefcik and Thompson (1986) that was picked to fit the normal distribution.

[15] I use the summary file for the data unadjusted for stock splits, now available through WRDS, to avoid the rounding error documented in Diether, Malloy and Scherbina (2002) and others.

dispersion and often have opposite implications for the joint distribution of returns / volume and dispersion. To avoid favoring one model over the other, I use a number of proxies for belief dispersion. In addition to using the unadjusted analyst forecast dispersion (denoted by AFD), I also use dispersion scaled by absolute mean forecast (denoted by AFD/abs(MEST)), scaled by lagged price (denoted by AFD/PRICE), and scaled by standard deviation of earnings per share (denoted by AFD/STDEPS). In view of the discrepancies in the existing literature, the comparisons between AFD/abs(MEST) and AFD/PRICE are of particular interest. The scaling variables are always lagged and calculated on the June of the previous year to avoid any look-ahead biases. The standard deviation in earnings per share is calculated using the realized earnings per share in the 4 quarters of the last year. Observations in which the share price is lower than $5 or the number of analysts is lower than 2 are excluded. Observations in which the scaling variable is zero are also dropped.

Before testing the models, one must to specify the horizon over which returns and volume are measured. The models do not provide much guidance as to what the appropriate empirical analog to the investment horizon should be. Hence, I repeat the empirical tests for different horizons. For brevity, I report the 1-month, 3-month and 1-year results, but the results from 6-month horizon are qualitatively similar. The proxies for dispersion are measured in the month before to prevent look ahead biases. I require firms for the 1-month and 3-month specifications to have at least 12 months of return and volume data, and for the 12-month specification to have at least 24 months of return and volume data, so as to make the samples more comparable. I perform monthly Fama-MacBeth regressions and adjust for autocorrelation in the estimates using Newey West standard errors.

4.1 Summary Statistics

Table 1 presents the summary statistics for the data that is used, for each of the three reported horizon specifications. The number of firms per year tends to decrease as the horizon increases - this is to be expected as the longer horizon analysis requires firms to have more monthly observations. However, the variables suggest that the distribution of the firms is still comparable across the specifications. There is a selection bias in the dataset as it is restricted to IBES firms with at least 2 analysts following them - this subset of firms seems to be large and relatively more profitable than the market as a whole. The distribution of the firms' sizes is skewed to the left since the median size is substantially smaller than the mean. I multiply AFD/PRICE by 10 and divide AFD/STDEPS by 10 so that the mean values of the dispersion in beliefs proxies are of the same order of magnitude. All the measures of dispersion appear to be left skewed. As mentioned earlier, this alleviates one potential concern of taking the models to the data.

Two other interesting features emerge across the different horizons. The autocorrelation in returns seems to be slightly negative at the monthly horizon, strongly positive at the quarterly horizon and then slightly negative at the annual horizon again. This is indicative of the short-run reversals, medium run continuation and long run reversals that has been documented in the earlier literature.

Also, note that the correlation between the absolute value of returns and volume is positive but decreasing as the horizon increases, eventually settling at a mean of around 0.09.[16]

4.2 Regression Analysis

Recently, a popular approach to testing cross-sectional implications of forecast dispersion has been to compare returns from sorted portfolios. While this approach has much appeal, it is difficult to control for other correlated explanatory variables. Instead, I use a regression framework in which to study test these models. I run monthly Fama-MacBeth regressions of returns and volume on the proxies of dispersion and other variables to test how various volume and return characteristics change with dispersion. Let $R^j_{t+1,n}$ denote the n period return of firm j starting in period $t+1$, and let $V^j_{t+1,n}$ denote the n period volume of firm j starting in period $t+1$. Finally, denote by $DISP_t$ one of the four proxies for analyst forecast dispersion (i.e. AFD, AFD/abs(MEST), AFD/PRICE and AFD/STDEPS). Then the cross-sectional regressions in the Fama MacBeth procedure are given by:

$$
\begin{aligned}
\text{E(R)} &: \; R^j_{t+1,n} = \alpha_t + \beta_t DISP_t + \text{(controls)} + e^j_{t+1} && (14)\\
\text{Var(R)} &: \; \sum_n (R^j_{t+1,1})^2 = \alpha_t + \beta_t DISP_t + \text{(controls)} + e^j_{t+1} && (15)\\
\text{AC(R)} &: \; R^j_{t+1,n} = \alpha_t + \beta_t (R^j_{t+1-n,n} \times DISP_t) + \gamma_t R^j_{t+1-n,n} + (R^j_{t+1-n,n} \times \text{controls}) + e^j_{t+1} && (16)\\
\text{C(|R|,V)} &: \; V^j_{t+1,n} = \alpha_t + \beta_t (|R^j_{t+1,n}| \times DISP_t) + \gamma_t |R^j_{t+1,n}| + (|R^j_{t+1,n}| \times \text{controls}) + e^j_{t+1} && (17)\\
\text{E(V)} &: \; V^j_{t+1,n} = \alpha_t + \beta_t DISP_t + \text{(controls)} + e^j_{t+1} && (18)\\
\text{Var(V)} &: \; \sum_n (V^j_{t+1,1})^2 = \alpha_t + \beta_t DISP_t + \text{(controls)} + e^j_{t+1} && (19)
\end{aligned}
$$

Each label (E(R), E(V), etc.) denotes the return-volume characteristic that I test using the given specification. E(R) and E(V) are regressions of returns and volume on a proxy for dispersion - this is used to test the relationship between dispersion and expected returns and expected volume. In the Var(R) and Var(V) regressions, I proxy for the variance in returns and volume using the sum of squared monthly returns and monthly volume respectively. In the AC(R) regression, I regress future returns on lagged returns and the interaction of dispersion with lagged returns. The coefficient on the interaction term measures the marginal effect of dispersion on the autocorrelation in returns. Similarly, the C(|R|,V) specification measures the marginal effect of dispersion on the covariance between volume and absolute returns. The null and alternative hypotheses for the two models are as follows:

H0: Under the null hypothesis of dispersion not affecting returns and volume characteristics, the coefficients on the dispersion proxy, β_t, in all the specifications (14) - (19) are zero.

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[16] These observations provide additional support for ruling out the high volatility equilibriums that appear in the OLG versions of the models. These equilibriums are characterized by strong negative autocorrelation in returns and extremely high levels of volume-absolute return correlation.

- **H1:** Under the alternative of the RE model, the coefficients on the dispersion proxy, β_t, in the E(R), Var(R), C(|R|,V), E(V) and Var(V) specifications are positive, but the coefficient in the AC(R) specification is negative.
- **H2:** Under the alternative of the DO model, the coefficients on the dispersion proxy, β_t, in the E(R), Var(R), and C(|R|,V) specifications are negative, while the rest are positive.

The controls included in the regression are: (i) log of market value of equity, (ii) the market to book ratio, (iii) the number of analysts who report their estimates, and (iv) the scaling variables for the dispersion proxies, i.e. 1/abs(MEST), 1/PRICE, and 1/STDEPS. The first three regressions have been documented to explain the cross-section of returns and volume characteristics. The scaling variables are included to control for the scaling effects that the proxies for dispersion might have. I present the results without controls in Table 2 and with the controls in Table 3. Finally, in Table 4, I control for other sources of cross-sectional predictability in returns by using excess returns from the four factor model (i.e. using the residuals from the regression of firm returns on the market premium, and returns from the SMB, HML and UMD portfolios), in addition to using the above controls.

Let us begin with the results in Table 2. The results are mixed for the monthly horizon, but support the RE model in the 3 month and 1 year horizons. It is reassuring to note that the coefficients on volume and variance in volume are positive and many are significant. There is also some evidence of the results from the existing literature. In particular, as in Diether, Malloy and Scherbina (2002), the 1-month E(R) coefficient for AFD/abs(MEST) is negative, although not significant. In contrast, the 1-month E(R) coefficient for AFD/PRICE is positive and significant, as in Qu Starks and Yan (2003). At longer horizons, the coefficients on all the dispersion measures are positive for the E(R) regression. Moreover, across all horizons, all the significant coefficients for E(R) and Var(R) are positive, which is evidence against the DO model. The only evidence against the RE model are the significant, positive coefficients of AC(R) for all proxies of dispersion, and the significant, negative coefficient of C(|R|,V) for AFS/STDEPS. As a result, the 1-month horizon evidence is not consistent with either model. On the other hand, all the significant coefficients for the quarterly and annual results are in the direction predicted by the RE model. Moreover, most of the other coefficients also have the same sign as those predicted by the RE model.

Table 3 shows that introducing controls to the regressions increases the significance of coefficients for all horizons. Moreover, the coefficient of C(|R|,V) for AFD/STDEPS is no longer significant. However, the coefficients for AC(R) is still positive for all dispersion proxies at the monthly horizon. Table 4 shows that the cross-sectional patterns are robust to adjusting returns for the four factor model. The significance for the E(R) specification improves at the short horizon, and the coefficient of E(R) on AFD/abs(MEST) becomes positive, although not significant. The coefficients on Var(R) become less significant across all horizons, although there is no change in their signs. The AC(R) coefficients are still positive at the monthly horizon, although their significance is much lower. The AC(R) coefficient for the longer horizons is not only negative, but highly significant. I conclude that adding controls to the regressions improves the significance of the coefficients, and the results are robust to adjusting returns by the four factor model.

The results also highlight the importance of controlling for the scaling variable in expected return-dispersion regressions. Adding controls for the scaling variables makes the expected return coefficients on AFD/abs(MEST) and AFD/PRICE insignificant, although they still have opposite signs. This suggests that at least part of the discrepancy between Diether, Malloy and Scherbina (2002) and Qu, Starks and Yan (2003) stems from the fact that neither paper controls for the effect of the scaling variable. The coefficient on 1/abs(MEST) is strongly negative, which suggests some type of distress risk. This also provides a link to the results in Sadka and Scherbina (2006), who find that higher levels of AFD/abs(MEST) coincide with higher trading costs and lower liquidity. The coefficient on 1/PRICE is positive and significant, and is probably driven by a size or value effect. Finally, the coefficients for other observable characteristics are more consistent across dispersion proxies than those for the expected return regressions, highlighting the usefulness of testing empirical implications of the models on other moments of returns and volume, in addition to expected returns.

Table 5 presents the results from the control regressions reported in Table 3 for a sub-sample of firms with low dispersion. For each month, firms with values of AFD/STDEPS lower than the median value are used. While some of the coefficients are less statistically significant, the signs of these coefficients remain the same and the overall evidence is still supportive of the conclusions drawn earlier. This specification confirms that the empirical evidence is not restricted to firms with high dispersion. It also provides additional evidence in support of our assumptions about the distribution of signal noise across firms. In particular, if the observed relationship between risk and dispersion was due to a high concentration of firms with signal noise between the two optima, s^*_R and s^*_μ, then the relationship would be of the opposite sign for low dispersion firms as they are in either tail of the distribution. Since the effect is similar for low dispersion firms, this implies that most firms lie on either side of the interval between s^*_R and s^*_μ, and our empirical predictions for the two models are correct.

In addition to the specifications above, I have run standard rolling window regressions of estimates of the return-volume moments on the proxies for dispersion (and controls), using annual and bi-annual windows. I find qualitatively similar results - however, the standard errors in these regressions are larger since the number of cross-sectional regressions is significantly reduced (from monthly to yearly, at best). Hence I report the results from the monthly regressions, which should have more power in distinguishing between the two models.

The empirical results in this section reject the DO model in favor of the RE model at quarterly and annual horizons. At the monthly horizon, the evidence rejects both models. The AC(R) coefficients are positive, thus ruling out the RE model, while the E(R), Var(R) and AC(R) coefficients are also positive, thus ruling out the DO model. This seems to suggest that while investors update using the price in the long run, they fail to do so completely in the short run. A possible explanation for the short-run autocorrelation result is the post-earnings announcement drift. This is a well-documented and robust example of investors slowly updating their beliefs, in which the returns of stocks with large earnings surprises continue to drift in the direction of the surprise. If the dispersion in analyst forecasts is high for stocks which have recently had large earnings surprises, then this would explain the positive relationship between dispersion and the autocorrelation in returns over the monthly horizon. In fact, Morse, Stephan and Stice (1991) document that forecast dispersions actually increase after an earnings announcement, and Brown and Han (1992) show that this effect is strongest for stocks with large earnings surprises.

As a reduced-form model of imperfect learning from prices, I describe a hybrid model with both DO and RE investors in the next section. Under certain parameter values, this model can produce dispersion effects that are consistent with the above evidence. Instead of a literal interpretation, the fraction of DO investors in the model can thought of as a measure of how imperfect the learning from prices is. I find that in one numerical example, the fraction of DO investors is less than 30%, suggesting that the degree to which investors fail to update their beliefs using the price is relatively small.

5 Extension - A Hybrid Model

In this section, I briefly describe a hybrid model with both difference of opinions and rational expectations. In particular, consider an overlapping generations economy as in 3.4, in which fraction α of the agents exhibit difference of opinions and the rest exhibit rational expectations. The difference of opinions agents ignore prices in updating their beliefs, while the rational expectations agents do use the price. The dividend process, private signals and aggregate supply process are as before. Once again, conjecture a linear stationary equilibrium of the form:

$$P_t = a\tilde{\delta}_t + b\tilde{z}_t + K \tag{20}$$

Given the information structure, we know that the posterior beliefs of the DO agents are given by $\tilde{\delta}_{t+1} \sim N(\mu_{i,t}^{DO}, \hat{\sigma}_{d,DO}^2)$, where

$$\mu_{i,t}^{DO} = \Pi_S^{DO} Y_{i,t}, \qquad \text{and} \qquad \hat{\sigma}_{d,DO}^2 = \sigma_d^2(1 - \Pi_S^{DO}) \tag{21}$$

and the RE agents are given by $\tilde{\delta}_{t+1} \sim N(\mu_{i,t}^{RE}, \hat{\sigma}_{d,RE}^2)$, where

$$\mu_{i,t}^{RE} = \Pi_S^{RE} Y_{i,t} + \Pi_P^{RE}(P_t - K), \qquad \text{and} \qquad \hat{\sigma}_{d,RE}^2 = \sigma_d^2(1 - \Pi_S^{RE} - \Pi_P^{RE}) \tag{22}$$

The regression coefficients in the above beliefs are as defined before i.e.

$$\Pi_S^{DO} = \frac{\sigma_d^2}{\sigma_d^2 + \sigma_s^2}, \qquad \Pi_S^{RE} = \frac{\sigma_d^2}{\sigma_d^2 + \sigma_s^2 + \sigma_s^2\theta} \qquad \text{and} \qquad \Pi_P^{RE} = \frac{1}{a}\frac{\sigma_s^2\theta}{\sigma_d^2 + \sigma_s^2 + \sigma_s^2\theta}$$

where $\theta = \frac{a^2\sigma_d^2}{b^2\sigma_z^2}$. As before, the DO investors believe that the other signals are distributed as:

$$Y_{i,t} \sim N(\bar{Y}, \sigma_s^2) \text{ i.i.d.}, \qquad \bar{Y} = \int_i Y_{i,t} di \sim N(0, \sigma_d^2) \tag{23}$$

where $\bar{Y}$ is independent of $\tilde{\delta}_{t+1}$.

Let $\hat{\sigma}_{R,DO}^2$ and $\hat{\sigma}_{R,RE}^2$ denote the posterior variance in returns for the DO and RE types respectively. Then market clearing implies the following condition is satisfied:

$$\int_i \alpha X_{i,t}^{DO} + (1-\alpha) X_{i,t}^{RE} di = 1 + \tilde{z}_t$$

where $\quad X_{i,t}^{DO} = \dfrac{D + E[P_{t+1}] + \mu_{i,t}^{DO} - (1+r)P_t}{\text{var}[P_{t+1}] + \hat{\sigma}_{d,DO}^2}, \quad$ and $\quad X_{i,t}^{RE} = \dfrac{D + E[P_{t+1}] + \mu_{i,t}^{RE} - (1+r)P_t}{\text{var}[P_{t+1}] + \hat{\sigma}_{d,RE}^2}.$

Substituting the beliefs in (21) and (22) in the above expression yields the following equilibrium characterization:

Lemma 6. *Suppose beliefs are given by (21), (22), and (23) and there exists an equilibrium of the form (20). Then the price coefficients are given by:*

$$a = \frac{1}{1+r}\left(\chi \Pi_S^{DO} + (1-\chi)(\Pi_S^{RE} + a\Pi_P^{RE})\right), \qquad b = -\frac{\hat{\sigma}_R^2}{1+r-(1-\chi)\Pi_P^{RE}}, \qquad \text{and}$$

$$K = \frac{1}{r}[D - \hat{\sigma}_R^2],$$

where $\quad \hat{\sigma}_{R,DO}^2 = a^2\sigma_d^2 + b^2\sigma_z^2 + \hat{\sigma}_{d,DO}^2, \qquad \hat{\sigma}_{R,RE}^2 = a^2\sigma_d^2 + b^2\sigma_z^2 + \hat{\sigma}_{d,RE}^2,$

$$\hat{\sigma}_R^2 = \left(\frac{\alpha}{\hat{\sigma}_{R,DO}^2} + \frac{1-\alpha}{\hat{\sigma}_{R,RE}^2}\right)^{-1} \qquad \text{and} \qquad \chi = \frac{\alpha}{\hat{\sigma}_{R,DO}^2}\hat{\sigma}_R^2.$$

As the model is complex, I study its characteristics numerically, and present some illustrative examples in Figure 5. In the figure, I plot the price coefficients, posterior variance in returns and the dispersion in beliefs as functions of σ_s^2. Note that the above price coefficients are weighted averages of the DO and RE model coefficients. The weights, denoted by χ and $1-\chi$, depend on the population weighted posterior precisions for each group. The price coefficients are continuous in α, and are equal to the RE and DO coefficients when $\alpha = 0$ and $\alpha = 1$ respectively. For low levels of α, the posterior variance $\hat{\sigma}_R^2$ behaves like in the RE model and first increases and then decreases in signal noise. For high levels of α, $\hat{\sigma}_R^2$ decreases and then increases like in the DO model. For intermediate levels of α, the function transitions smoothly from one extreme to the other.

The posterior measure of dispersion is calculated using the law of total variance. The dispersion in beliefs about the dividend shock in the next period is the sum of the expected value of the conditional posterior dispersions and the variance of the conditional means:

$$\begin{aligned}\hat{\sigma}^2_\mu &= \alpha(\Pi_S^{DO})^2\sigma_s^2 + (1-\alpha)(\Pi_S^{RE})^2\sigma_s^2 + \alpha(\bar{\mu}_{DO,i,t} - \bar{\mu}_{i,t})^2 + (1-\alpha)(\bar{\mu}_{RE,i,t} - \bar{\mu}_{i,t})^2 \\ &= \alpha(\Pi_S^{DO})^2\sigma_s^2 + (1-\alpha)(\Pi_S^{RE})^2\sigma_s^2 + \alpha(1-\alpha)(\bar{\mu}_{DO,i,t} - \bar{\mu}_{RE,i,t})^2\end{aligned}$$

The last term depends on the difference in the posterior conditional expectations in the two types - this difference first increases and then decreases in σ_s^2. As a result, the dispersion in beliefs as a whole initially increases and then decreases in the signal noise σ_s^2. Moreover, increasing α shifts the maximum to the left and closer to the DO maximum at $\sigma_s^2 = \sigma_d^2$.

The expressions for expected returns, volatility and autocorrelation in returns are the same as those in the single type models (as in Lemma 5). Expected returns are again linear in $\hat{\sigma}^2_R$. Hence, for small α, returns are positively related to the dispersion in beliefs, while for large α, the relationship is negative. Similarly, volatility and autocorrelation in returns are also largely driven by $\hat{\sigma}^2_R$ - hence for small α, volatility is positively related and autocorrelation is negatively related to dispersion, and for large α the reverse is true.

Volume in the hybrid economy is the population weighted average of the volume from each type of investor. In particular, volume is given by:

$$\tilde{V}_{t+1} = \int_i \alpha|\Delta X^{DO}_{i,t,t+1}| + (1-\alpha)|\Delta X^{RE}_{i,t,t+1}|di.$$

Since expectations are linear, this implies that the expected volume in the economy is the sum of the expected volume from each type:

$$E[\tilde{V}_{t+1}] = \alpha\sqrt{\frac{2}{\pi}\text{var}[\Delta X^{DO}_{i,t,t+1}]} + (1-\alpha)\sqrt{\frac{2}{\pi}\text{var}[\Delta X^{RE}_{i,t,t+1}]}.$$

The variance and covariance between absolute returns and volume can be calculated using standard covariance expansions:

$$\text{var}[\tilde{V}_{t+1}] = \alpha^2\frac{\pi-2}{\pi}\text{var}[\Delta X^{DO}_{i,t,t+1}] + (1-\alpha)^2\frac{\pi-2}{\pi}\text{var}[\Delta X^{RE}_{i,t,t+1}] + 2\alpha(1-\alpha)\text{cov}[\text{var}[\Delta X^{DO}_{i,t,t+1}], \text{var}[\Delta X^{RE}_{i,t,t+1}]]$$

$$\text{cov}[\tilde{V}_{t+1}, |\tilde{R}_{t+1}|] = \alpha\text{cov}[\Delta X^{DO}_{i,t,t+1}, |\tilde{R}_{t+1}|] + (1-\alpha)\text{cov}[\Delta X^{RE}_{i,t,t+1}, |\tilde{R}_{t+1}|]$$

The above expressions are similar to those in the single type models. As a result, expected volume and variance in volume first increase and then decrease in signal noise. For low α, the covariance between volume and absolute returns increases and then decreases, as in the RE model, while for high α, the pattern is reversed.

While the characteristics of the hybrid model at either extreme (low and high α) are similar to the single type models, it can potentially give rise to interesting comparative static results for intermediate values of α. Figure 5 presents an example of this. I plot changes in dispersion with the associated changes in return-volume characteristics. For $\alpha = 27.5\%$, note that expected returns, volatility, and covariance between volume and absolute returns are positively related to dispersion as in the RE model. The autocorrelation in returns is also positively related to dispersion (although very slightly), as in the DO equilibrium. This is consistent with the empirical evidence found in Section 4.2 for the monthly horizon. If we interpret the fraction of DO investors as the degree to which the average investor fails to incorporate prices, it is somewhat reassuring that the number is not larger. This numerical example supports the notion that there is only partial learning from prices at the monthly horizon.

6 Conclusions

In this paper, I study to what extent investors use the information in prices to update their beliefs about asset payoffs. I develop a dynamic framework to compare a rational expectations model in which investors use the price, and a difference of opinions model in which they do not use the price. I show that the models' implications about the dispersion in beliefs and the joint distribution of returns and volume can be used to distinguish the two. I find that in the RE model, higher dispersion in beliefs is associated with higher expected returns, higher volatility, lower autocorrelation in returns and higher correlation between absolute returns and volume, while in the DO model, the relationships are reversed. In both models, expected volume and variance in volume increase with dispersion.

I test these empirical implications on the cross-section of returns and volume. I find that after controlling appropriately for the scaling variables in the proxies for dispersion, expected returns, volatility, correlation between absolute returns and volume, expected volume and variance in volume are positively related to dispersion. Autocorrelation in returns is negatively related to dispersion in the long horizon specifications, but positively related to dispersion at the monthly horizon. This evidence is consistent with the notion that investors use the price to update their beliefs, as in the RE model, especially at the longer horizons. At the short horizon, the evidence suggests that investors incorporate the information in prices only partially. Using a hybrid model with DO and RE agents to approximate a model with partial updating, I find numerically that a relatively low level of incomplete learning is needed to match the features of the data at the monthly horizon.

A limitation of the current setup is that I use single asset models to derive predictions which I then test on the cross-section of returns. One useful extension would be to develop a multi-asset framework in which to determine whether investors learn from prices. Another interesting area for future research would be to explore the effect of horizon on the learning process of investors. A model in which agents update their beliefs about the informativeness of others' private signals, and endogenously decide how to use the price to update their beliefs would shed light on this.

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8 Appendix

Proof for Lemma 2: The expressions above obtain directly from substituting the beliefs in (5) into the optimal demand expression and then aggregating over the continuum of investors. To show existence, I use induction. Note that at $t = T-1$, the ratio of the price coefficients is given by:

$$\frac{a_{T-1}}{b_{T-1}} = -\frac{\Pi^{RE}_{S,T-1}}{\sigma_d^2(1-\Pi^{RE}_{S,T-1}-\Pi^{RE}_{P,T-1})} = -\frac{1}{\sigma_s^2}$$

This implies that $\theta_{T-1} = \frac{\sigma_d^2}{\sigma_z^2(\sigma_s^2)^2}$, which can be substituted into the regression coefficients to calculate the price coefficients:

$$a_{T-1} = \frac{1}{1+r}\frac{\sigma_d^2(1+\sigma_s^2\sigma_z^2)}{\sigma_d^2+\sigma_s^2\sigma_z^2(\sigma_d^2+\sigma_s^2)} \qquad \text{and} \qquad b_{T-1} = -\frac{1}{1+r}\frac{\sigma_d^2\sigma_s^2(1+\sigma_s^2\sigma_z^2)}{\sigma_d^2+\sigma_s^2\sigma_z^2(\sigma_d^2+\sigma_s^2)}$$

For $t < T-1$, suppose that the coefficients a_{t+1} and b_{t+1} exist. Then some algebra shows that the ratio a_t/b_t satisfies the following cubic equation of the form $A_3x^3 + A_1x + A_0 = 0$:

$$(a_{t+1}^2\sigma_d^2 + b_{t+1}^2\sigma_z^2)\frac{\sigma_d^2\sigma_s^2}{\sigma_z^2}(a_t/b_t)^3 + [(a_{t+1}^2\sigma_d^2 + b_{t+1}^2\sigma_z^2)(\sigma_d^2+\sigma_s^2) + \sigma_s^2\sigma_d^2](a_t/b_t) + \sigma_d^2 = 0$$

The above equation has two imaginary roots and one real root. The real root can be substituted into the expression for θ_t, which can then be plugged into the expressions for the price coefficients. ■

Proof of Lemma 4: The expressions for a and b follow immediately from the market clearing conditions. Let $x = a/b$. Then $\theta = x^2\sigma_d^2/\sigma_z^2$, and the ratio,

$$\begin{aligned}
& x = -\frac{\Pi^{RE}_S}{\hat{\sigma}^2_R} = -\frac{\frac{\sigma_d^2}{\sigma_d^2+\sigma_s^2+\sigma_s^2\theta}}{a^2\sigma_d^2(1+1/\theta)+\hat{\sigma}_d^2} = -\frac{\frac{\sigma_d^2}{\sigma_d^2+\sigma_s^2+\sigma_s^2\theta}}{\frac{1}{(1+r)^2}\left(\frac{\sigma_d^2+\sigma_s^2\theta}{\sigma_d^2+\sigma_s^2+\sigma_s^2\theta}\right)^2(1+1/\theta)\sigma_d^2 + \sigma_d^2\frac{\sigma_s^2}{\sigma_d^2+\sigma_s^2+\sigma_s^2\theta}} \\
\Rightarrow \quad & \frac{(\sigma_d^2+\sigma_s^2\theta)^2}{(1+r)^2}(\theta+1) + x\frac{\sigma_d^2}{\sigma_z^2}(\sigma_s^2x+1)(\sigma_d^2+\sigma_s^2+\sigma_s^2\theta) = 0 \\
\Rightarrow \quad & A_0 + A_1x + A_2x^2 + A_3x^3 + A_4x^4 + A_6x^6 = 0
\end{aligned}$$

where

$$A_0 = \frac{\sigma_z^2}{(1+r)^2}, \qquad A_1 = \sigma_d^2 + \sigma_s^2, \qquad A_2 = \frac{(\sigma_d^2)^2}{(1+r)^2} + \sigma_d^2\sigma_s^2 + \frac{\sigma_d^2\sigma_s^2}{(1+r)^2} + (\sigma_s^2)^2,$$

$$A_3 = \frac{\sigma_d^2\sigma_s^2}{\sigma_z^2}, \qquad A_4 = \frac{2(\sigma_d^2)^2\sigma_s^2}{(1+r)^2\sigma_z^2} + \frac{\sigma_d^2(\sigma_s^2)^2}{\sigma_z^2} + \frac{\sigma_d^2(\sigma_s^2)^2}{(1+r)^2\sigma_z^2}, \qquad \text{and} \qquad A_6 = \frac{(\sigma_d^2)^2(\sigma_s^2)^2}{(1+r)^2(\sigma_z^2)^2}.$$

These coefficients are all positive, and so the solutions x to the above equation must be negative. The second to last equation implies that $x > -1/\sigma_s^2$, as otherwise, the polynomial is greater than zero. At the extremes, the equation characterizing x simplifies:

$$\sigma_s^2 = 0: \quad \sigma_d^2x^2 + (1+r)^2x + \sigma_z^2 = 0, \qquad \sigma_s^2 \to \infty: \quad x \to 0 \tag{24}$$

In particular, the limiting equilibrium corresponds to the more negative solution of x, which implies that $-(1+r)^2/(2\sigma_d^2) > x > -(1+r)^2/\sigma_d^2$ when $\sigma_s^2 = 0$.

The coefficient a can be re-written as

$$a = \frac{\Pi_S^{RE} + a\Pi_P^{RE}}{1+r} = \frac{1}{(1+r)} \frac{\sigma_d^2 + \sigma_s^2\theta}{\sigma_d^2 + \sigma_s^2 + \sigma_s^2\theta},$$

this implies that $b \leq 0$. The following bounds also follow:

$$x \in (-1/\sigma_s^2, 0), \qquad \theta \in (0, \sigma_d^2/(\sigma_z^2(\sigma_s^2)^2)), \qquad a \in [0, 1/(1+r)], \qquad \Pi_S^{RE} \in [0,1], \qquad \text{and}$$

$$\Pi_P^{RE} = (1+r)\frac{\sigma_s^2\theta}{\sigma_d^2 + \sigma_s^2\theta} \in [0, 1+r].$$

■

Proof of Proposition 2: All derivatives are with respect to σ_s^2. The derivative of the regression coefficient Π_S^{RE} can be written as:

$$\frac{\Pi_S^{RE\prime}}{\Pi_S^{RE}} = -\frac{1 + \theta + \sigma_s^2\theta'}{\sigma_d^2 + \sigma_s^2 + \sigma_s^2\theta}$$

This must be negative. Otherwise $1 + \theta + \sigma_s^2\theta' < 0$, which implies $\theta' < -(1+\theta)/\sigma_s^2 < -1/\sigma_s^2$, which in turn implies $\theta < -\log(\sigma_s^2) + \text{constant}$, which violates the fact that $\theta > 0$ for all σ_s^2. Since $\Pi_S^{RE\prime}$ is negative,

$$\theta' > -\left(\frac{1}{\sigma_s^2} + \frac{\sigma_d^2}{(\sigma_s^2)^3\sigma_z^2}\right), \qquad x' < -\frac{\theta x}{2}\left(\frac{1}{\sigma_s^2} + \frac{\sigma_d^2}{(\sigma_s^2)^3\sigma_z^2}\right) < \frac{\sigma_d^2}{2(\sigma_s^2)^3\sigma_z^2}\left(\frac{1}{\sigma_s^2} + \frac{\sigma_d^2}{(\sigma_s^2)^3\sigma_z^2}\right)$$

since $\theta'/\theta = 2x'/x$.

The posterior variance and the dispersion in beliefs can be re-written as products of monotone functions of σ_s^2:

$$\hat{\sigma}_R^2 = -b(1 + r - \Pi_P^{RE}) = -\frac{1}{x}\Pi_S^{RE}, \qquad \hat{\sigma}_\mu^2 = (\Pi_S^{RE})^2\sigma_s^2$$

The derivative of $\hat{\sigma}_\mu^2$ is given by:

$$\frac{\partial}{\partial\sigma_s^2}\hat{\sigma}_\mu^2 = (\Pi_S^{RE})^2\left(1 + 2\sigma_s^2\frac{\Pi_S^{RE\prime}}{\Pi_S^{RE}}\right)$$

This implies that the derivative of $\hat{\sigma}_\mu^2$ is positive when $\sigma_s^2 = 0$ (since $\Pi_S^{RE\prime}$ is bounded) and negative when $\sigma_s^2 \to \infty$. The optimum s_μ^* is characterized by $\sigma_s^2 = -\frac{\Pi_S^{RE}}{2\Pi_S^{RE\prime}}$. For $\sigma_s^2 < s_\mu^*$, the derivative is positive and for $\sigma_s^2 > s_\mu^*$, the derivative is negative. Thus the dispersion in beliefs increases and then decreases in signal noise.

The derivative of $\hat{\sigma}_R^2$ is given by:

$$\frac{\partial}{\partial \sigma_s^2}\hat{\sigma}_R^2 = \frac{x'\Pi_S^{RE} - x\Pi_S^{RE\prime}}{x^2} = \frac{x'}{x^2}\Pi_S^{RE} - \frac{1}{x}\Pi_S^{RE\prime}$$

When $\sigma_s^2 = 0$, $\Pi_S^{RE\prime} = -\frac{1+\theta}{\sigma_d^2}$ and so

$$\frac{\Pi_S^{RE}}{x}\left(\frac{x'}{x} - \frac{\Pi_S^{RE\prime}}{\Pi_S^{RE}}\right) = \frac{1}{x}\left(\frac{\theta'}{2\theta} - \frac{\theta+1}{\sigma_d^2}\right) > \frac{\theta'}{x}\left(\frac{1}{2\theta} - \frac{\sigma_s^2}{\sigma_d^2}\right) > 0$$

Similarly, as $\sigma_s^2 \to \infty$,

$$\frac{x'\Pi_S^{RE} - x\Pi_S^{RE\prime}}{x^2} < \frac{x'\Pi_S^{RE}}{x^2} + \sigma_s^2\Pi_S^{RE\prime} < 0$$

This implies that there is at least one maximum for $\hat{\sigma}_R^2$. Let s^* be the point at which the first order condition is satisfied i.e. for $\sigma_s^2 = s^*$,

$$\frac{x'}{x} - \frac{\Pi_S^{RE\prime}}{\Pi_S^{RE}} = \frac{\theta'(\sigma_d^2 + \sigma_s^2 + 3\sigma_s^2\theta) + 2\theta(1+\theta)}{2\theta(\sigma_d^2 + \sigma_s^2 + \sigma_s^2\theta)} = 0$$

Since $\theta' < 0$, for σ_s^2 slightly less than s^*, the above quantity is positive, while for σ_s^2 slightly greater than s^*, the quantity is negative. This implies that when the derivative crosses zero, it crosses from above, which in turn implies that there is only one maximum of $\hat{\sigma}_R^2$, which is denoted by s_R^*.

To find the relative positions of these maxima, one can use a standard monotone comparative statics argument. Since the posterior variance is positive by definition, the σ_s^2 that maximizes $\hat{\sigma}_R^2$ is the same as the one that maximizes $(\hat{\sigma}_R^2)^2$. Let $Q(\sigma_s^2, \gamma)$ be defined as:

$$Q(\sigma_s^2, \gamma) = (\sigma_s^2 x^2)^{(1-\gamma)} b^2 (1 + r - \Pi_P^{RE})^2$$

Then we know that

$$s_\mu^* = \arg\max_{\sigma_s^2} Q(\sigma_s^2, 0) = \arg\max_{\sigma_s^2} \hat{\sigma}_\mu^2 \qquad \text{and} \qquad s_R^* = \arg\max_{\sigma_s^2} Q(\sigma_s^2, 1) = \arg\max_{\sigma_s^2} \hat{\sigma}_R^2.$$

Finally, taking logs does not change the σ_s^2 that maximizes Q. Since

$$\frac{\partial}{\partial \sigma_s^2}\log(\sigma_s^2 x^2) < \frac{\partial}{\partial \sigma_s^2}\log\left(\frac{1}{\sigma_s^2}\right) < 0,$$

it follows that $\frac{\partial^2}{\partial \sigma_s^2 \partial \gamma}\log(Q) > 0$, which in turn implies $s_\mu^* < s_R^*$. ■

Proof of Lemma 5: For both models, returns are of the form $\tilde{R}_{t+1} = a\tilde{\delta}_{t+2} + b\tilde{z}_{t+1} + K + D + \tilde{\delta}_{t+1} - (1+r)P_t$, and some algebra shows that for both models, the signed trade $\Delta X_{i,t,t+1} = X_{i,t+1} - X_{i,t}$, can be written as:

$$\Delta X_{i,t,t+1} = \tilde{z}_{t+1} - \tilde{z}_t - \frac{a}{b}(\tilde{s}_{i,t+1} - \tilde{s}_{i,t})$$

In particular, notice that changes in demand are independent of the dividend shocks. The expressions in the proposition follow from using the fact that $\tilde{V}_{t+1} = \int |\Delta X_{i,t,t+1}| di$ and using the properties of half-normal distributions. Specifically, if y_1 and y_2 are normally distributed with variance σ_1^2 and σ_2^2 and covariance $\sigma_{1,2}$, then

$$E[|y_i|] = \sqrt{\frac{2}{\pi}\sigma_{y_i}^2}, \qquad \text{var}[|y_i|] = \frac{\pi - 2}{\pi}\sigma_{y_i}^2,$$

$$\text{cov}(|y_1|, |y_2|) = \Psi(\sigma_{1,2}) = \frac{2\sigma_1\sigma_2}{\pi - 2}\left((1-\rho)^{3/2} - 1 + \rho^2\sqrt{1-\rho^2} + |\rho|\text{arctan}\left(|\rho|/\sqrt{1-\rho^2}\right)\right),$$

where $\rho = \frac{\sigma_{1,2}}{\sqrt{\sigma_1^2\sigma_2^2}}$ is the correlation between y_1 and y_2. Finally, note that the covariance between consecutive $\Delta X_{i,t,t+1}$ is given by $-(\sigma_z^2 + \sigma_s^2\frac{a^2}{b^2})$, and so the autocorrelation in volume is constant. ■

Proof for Proposition 3: Remember that in both models, risk can be re-written as

$$\hat{\sigma}_R^2 = a^2\sigma_d^2 + b^2\sigma_z^2 + \sigma_d^2(1 - a(1+r))$$

Using this and the expressions in Lemma 5, we know that the following expressions hold:

(i) $E[\tilde{R}_{t+1}] = \hat{\sigma}_R^2$

(ii) $\text{var}[\tilde{R}_{t+1}] = \hat{\sigma}_R^2(1 + (1+r))^2 + a\sigma_d^2(1+r)((1+r)^2 - 1) - (1+r)^2\sigma_d^2$

(iii) $\text{cov}[\tilde{R}_{t+1}, \tilde{R}_{t+2}] = -((1+r)\hat{\sigma}_R^2 + a\sigma_d^2((1+r)^2 - 1) - (1+r)\sigma_d^2)$

(iv) $E[\tilde{V}_{t+1}] = \sqrt{\frac{4}{\pi}(\sigma_z^2 + \frac{\hat{\sigma}_\mu^2}{(\hat{\sigma}_R^2)^2})}$

(v) $\text{var}[\tilde{V}_{t+1}] = \frac{2(\pi-2)}{\pi}(\sigma_z^2 + \frac{\hat{\sigma}_\mu^2}{(\hat{\sigma}_R^2)^2})$

(vi) $\text{cov}[\tilde{V}_{t+1}, |\tilde{R}_{t+1}|] = \Psi((2+r)b\sigma_z^2)$

Since the expected return results follow immediately from the earlier propositions, I shall focus on showing the other results. Denote $x = a/b$. The volume terms are driven by

$$\varphi = \frac{\hat{\sigma}_\mu^2}{(\hat{\sigma}_R^2)^2} = x^2\sigma_s^2, \qquad \text{and} \qquad \frac{\partial\varphi}{\partial\sigma_s^2} = 2x(\sigma_s^2x' + x)$$

This is positive for $\sigma_s^2 = 0$, and is negative for $\sigma_s^2 \to \infty$, since $2x(\sigma_s^2x' + x) < 2x\sigma_s^2x' + \frac{2}{(\sigma_s^2)^2} < 0$. Finally, it crosses zero from above, which implies there is only one optimum. Hence for both models, expected volume and variance in volume first increases and then decreases in σ_s^2.

Similarly, for both models, the correlation between volume and absolute returns is driven by b^2 since Ψ is symmetric around zero. In the DO model, this implies that it behaves like $(\hat{\sigma}_{R,t}^2)^2$ and so first decreases and then increases in σ_s^2. In the RE model, $b = \frac{a}{x}$, and since b is always negative, maximizing b^2 is the same as maximizing $-b$. The derivatives of b and a are

$$b' = \frac{a'}{x} - \frac{ax'}{x^2}, \qquad a' = \frac{-\sigma_d^2 + (\sigma_s^2)^2\theta'}{(1+r)(\sigma_d^2 + \sigma_s^2 + \sigma_s^2\theta)^2} < 0$$

Note that $a' < 0$ - price sensitivity to dividend shocks decreases as one increases signal noise. When $\sigma_s^2 \to \infty$, $a \to 0$ and so the b' is positive. When $\sigma_s^2 = 0$,

$$b' < \frac{1}{x(1+r)\sigma_d^2} - \frac{ax'}{x^2} < -\frac{1}{(1+r)^3} - \frac{ax'}{x^2} < 0$$

Finally, note that b' crosses zero from below. This implies that for the RE model, b^2 first increases and then decreases in σ_s^2.

Let λ denote the expression that drives volatility and autocorrelation in returns:

$$\lambda = (1+r)\hat{\sigma}_R^2 + a\sigma_d^2((1+r)^2 - 1)$$

This implies that the shape of λ is the similar to the shape of $\hat{\sigma}_R^2$ in the same model, and in particular, when $r \to 0$, the two are exactly the same. The derivative of λ is then given by:

$$\lambda' = (1+r)(\hat{\sigma}_R^2)' + a'\sigma_d^2((1+r)^2 - 1), \qquad \text{where} \qquad (\hat{\sigma}_R^2)' = 2aa'\sigma_d^2 + 2bb'\sigma_z^2 - a'\sigma_d^2(1+r)$$

For the DO model, when $\sigma_s^2 = 0$, we know that $(\hat{\sigma}_R^2)' < 0$, and so $\lambda' < 0$. For $\sigma_s^2 \to \infty$, we can rewrite the derivative as

$$\lambda' = a'\sigma_d^2(2a(1+r) - 1) + 2bb'\sigma_z^2 > a'\sigma_d^2 + 2bb'\sigma_z^2 > 0$$

Hence for the DO model, λ initially increases and eventually decreases in σ_s^2. For the RE model, when $\sigma_s^2 \to \infty$, both $(\hat{\sigma}_R^2)'$ and a' are negative, and so λ' is negative too. When $\sigma_s^2 = 0$, the derivative is

$$\begin{aligned} \lambda' &= a'\sigma_d^2(2a(1+r) - 1) + 2(1+r)bb'\sigma_z^2 = -\frac{1}{1+r} + 2(1+r)bb'\sigma_z^2 \\ &> -\frac{1}{1+r} + 2\frac{\sigma_z^2 b}{x}\left(\frac{1}{\sigma_d^2} - \frac{x'}{x}\right) > -\frac{1}{1+r} + 2\frac{\sigma_z^2 a}{x^2}\frac{1}{\sigma_d^2} = \frac{1}{1+r}\left(\frac{2\sigma_z^2}{x^2\sigma_d^2} - 1\right) > 0 \end{aligned}$$

since $\sigma_d^2 x^2 + (1+r)^2 x + \sigma_z^2 = 0$ implies $2 + \frac{2\sigma_z^2}{\sigma_d^2 x^2} > 0$. Hence, for the RE model, λ decreases then increases. ■

9 Figures and Tables

Figure 1: The DO price coefficients a_t and b_t, the posterior dispersion in beliefs $\hat{\sigma}^2_{\mu,t}$, and the posterior variance in returns $\hat{\sigma}^2_{R,t}$ as functions of σ^2_s. The other parameters are set to $r = 0.05$, $\sigma^2_d = 1$, and $\sigma^2_z = 0.1$.

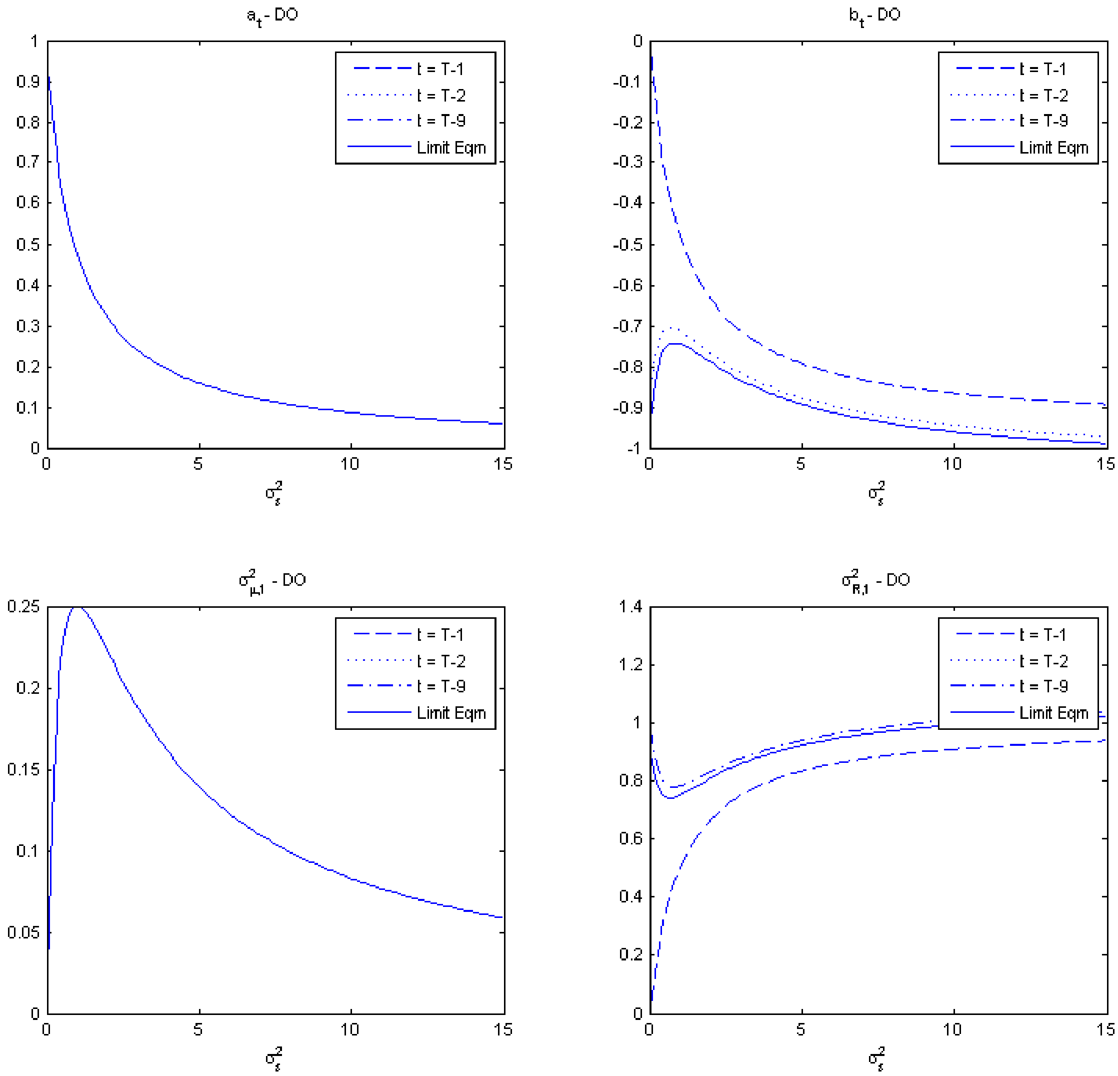

Figure 2: The RE price coefficients a_t and b_t, the posterior dispersion in beliefs $\hat{\sigma}^2_{\mu,t}$, and the posterior variance in returns $\hat{\sigma}^2_{R,t}$ as functions of σ^2_s. The other parameters are set to $r = 0.05$, $\sigma^2_d = 1$, and $\sigma^2_z = 0.1$.

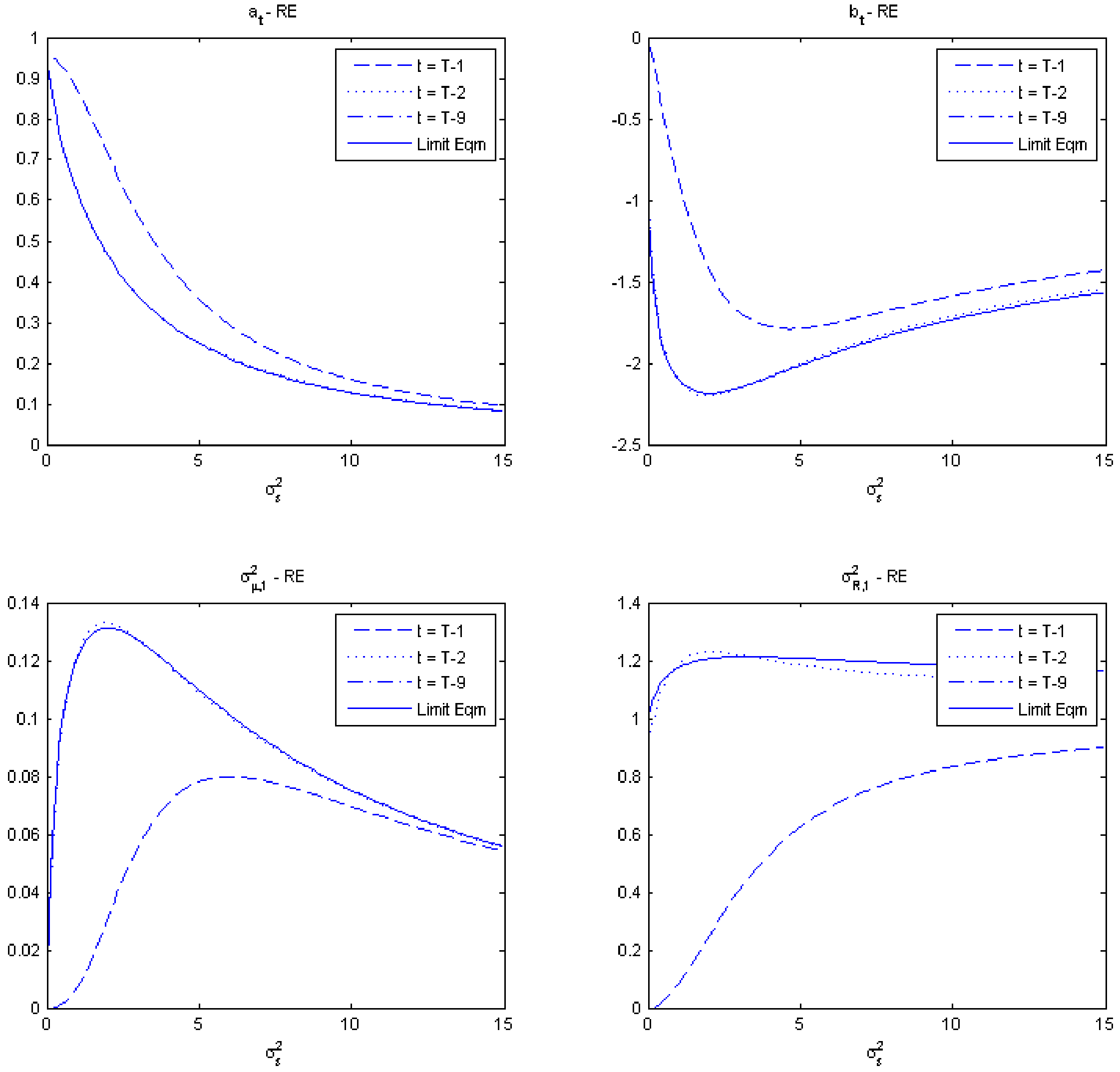

Figure 3: Regions where stationary equilibria exist for the infinite horizon OLG models. The light region represents the range of parameters where a solution exists. Parameter values are as listed in the plots.

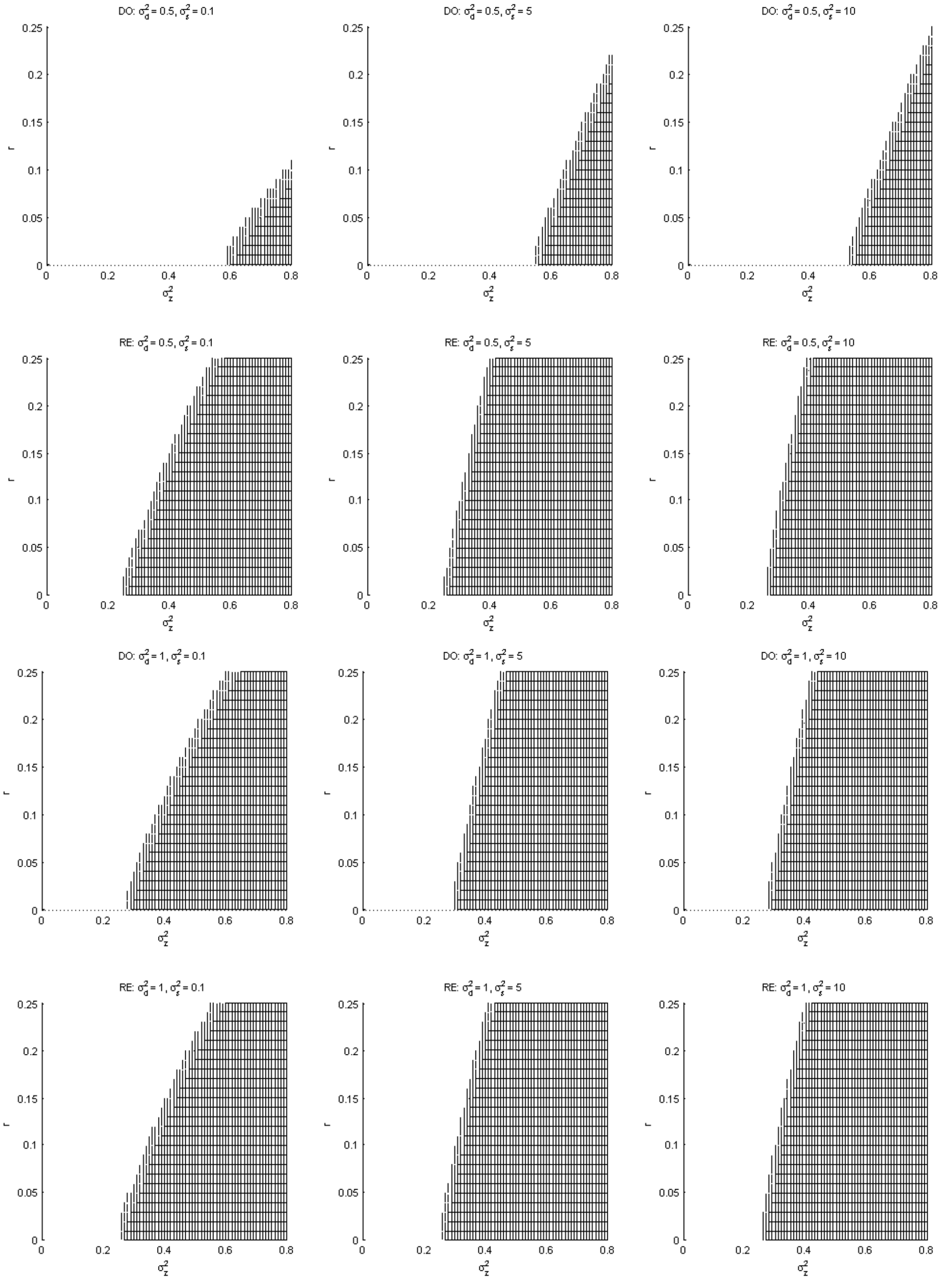

Figure 4: The solution to the OLG RE model, x, and the corresponding price coefficients a and b as functions of r and σ_s^2. The other parameters are set to $\sigma_d^2 = 1$, and $\sigma_z^2 = 0.1$.

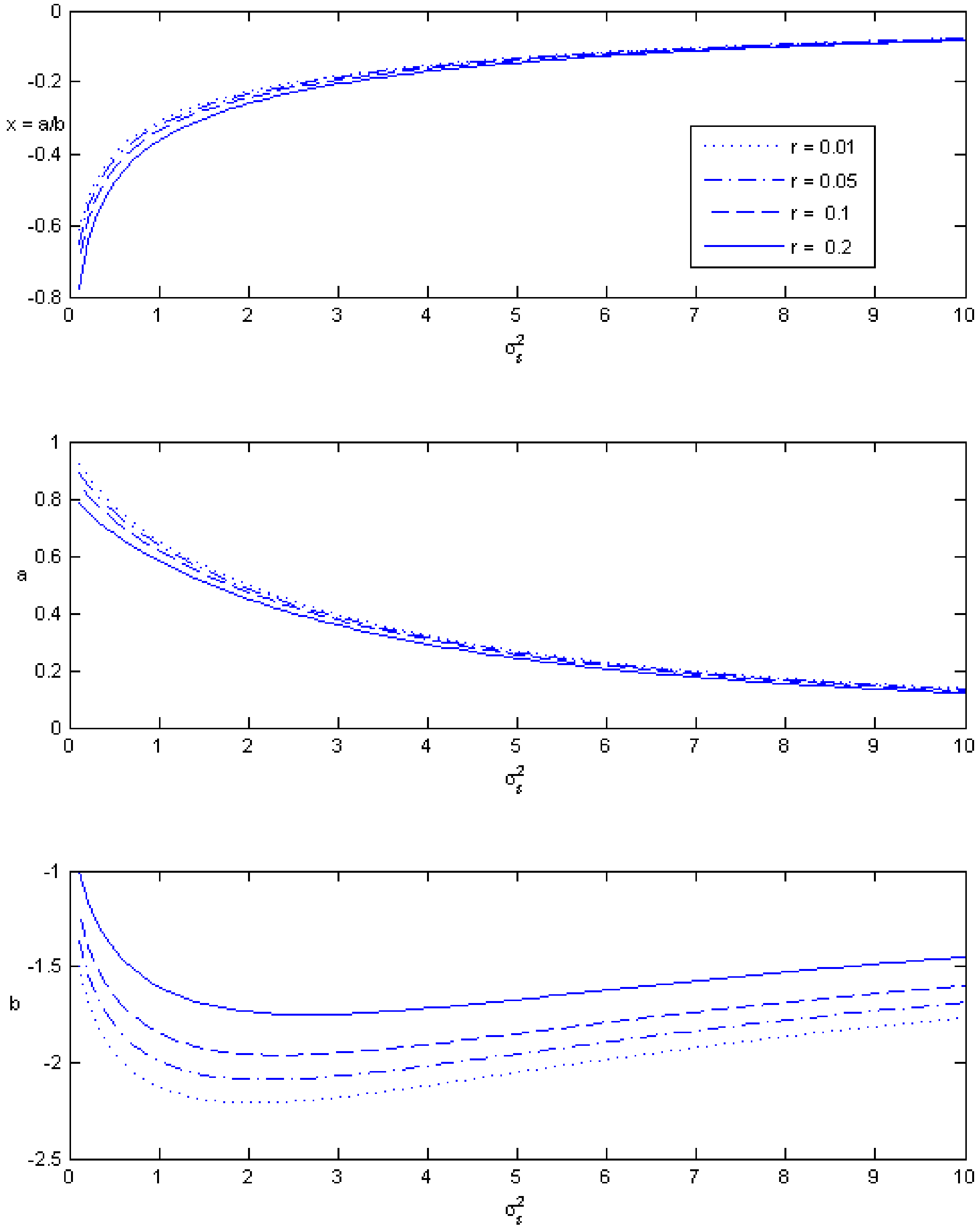

Figure 5: Numerical examples of the hybrid model for three values of α. The price coefficients, a and b, posterior variance in returns, $\hat{\sigma}_R^2$, and dispersion in beliefs, $\hat{\sigma}_\mu^2$, are plotted as functions of σ_s^2. Changes in belief dispersion are plotted with corresponding changes in expected returns, variance in returns, autocorrelation in returns, correlation in volume and absolute returns and expected volume. The other parameters are set to $r = 0.05$, $\sigma_d^2 = 1$, and $\sigma_z^2 = 0.1$.

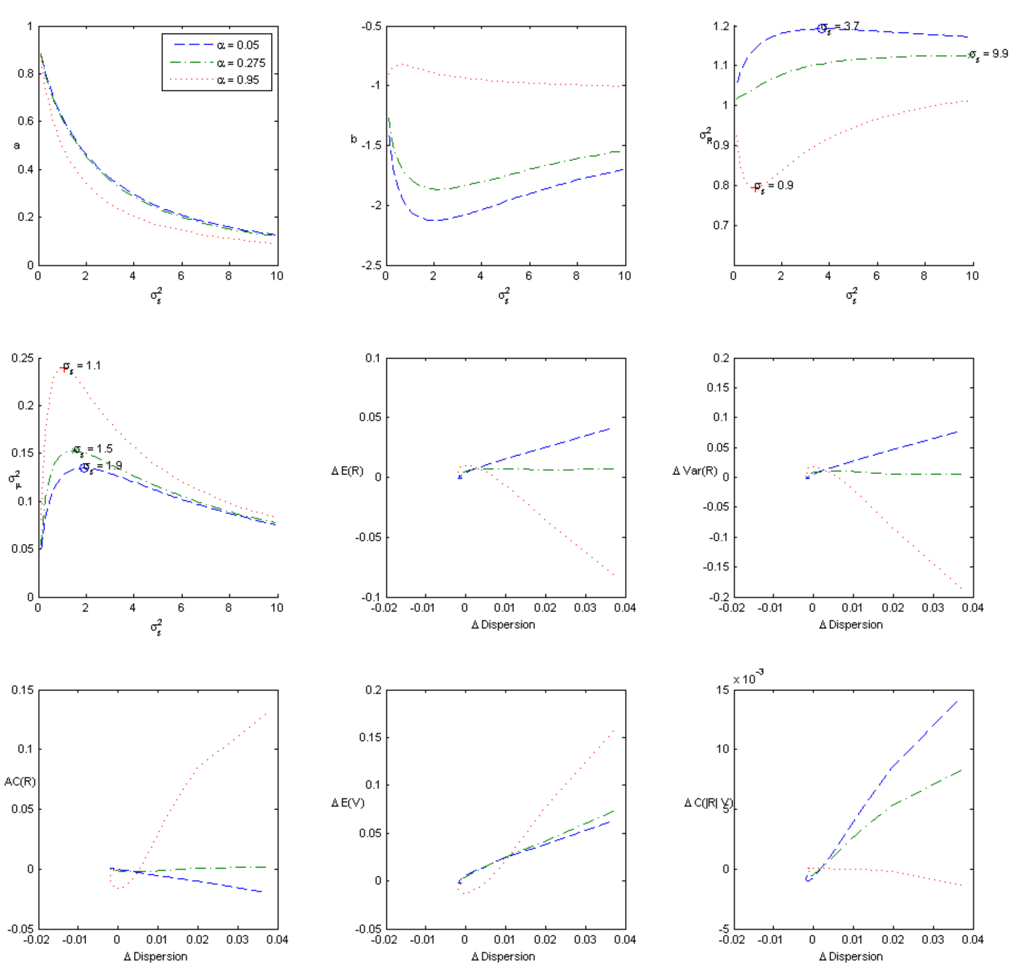

Table 1: Summary Statistics: The table shows summary statistics of the variables used in the analysis for the different horizon-window specifications. The table shows returns (R_{t+1}), autocorrelation in returns (AC(R)), correlation between absolute returns and volume (C(|R|, V)), volume (V_{t+1}), market value of equity (MVE), analyst forecast dispersion (AFD), dispersion scaled by absolute value of the mean analyst estimate (AFD/abs(MEST)), dispersion scaled by price (AFD/PRICE), and dispersion scaled by standard deviation of earnings per share (AFD/STDEPS).

1-Month Horizon	Mean	Median	Std Dev	5%	95%
Number of firms per year	2133.7	2153.5	579.44	1249.5	3123.8
R_{t+1}	0.017093	0.010434	0.137	-0.18033	0.23077
AC(R)	-0.02946	-0.028	0.14112	-0.25653	0.19537
C(|R|, V)	0.26395	0.26666	0.16959	-0.02462	0.531
V_{t+1}	-0.2948	-0.31975	1.0016	-1.9064	1.3884
MVE (in $ millions)	2750	467.96	12532	53.291	9739.2
AFD	0.14601	0.06	2.9529	0.01	0.43
AFD/abs(MEST)	0.15779	0.041667	1.2196	0.006103	0.42857
AFD/PRICE (mult by 10)	0.059153	0.026667	0.1241	0.003008	0.20753
AFD/STDEPS (divd by 10)	0.13556	0.056506	0.37462	0.004082	0.4714

3-Month Horizon	Mean	Median	Std Dev	5%	95%
Number of firms per year	2138.8	2147.5	577.54	1291.9	3130.1
R_{t+1}	0.046316	0.031816	0.23979	-0.29292	0.41945
AC(R)	0.25937	0.27549	0.17512	-0.03517	0.50888
C(|R|, V)	0.19585	0.20006	0.20707	-0.14406	0.52346
V_{t+1}	0.86858	0.83119	0.93433	-0.61421	2.4574
MVE (in $ millions)	2721	462.81	12441	52.973	9608.8
AFD	0.14449	0.06	2.8378	0.01	0.43
AFD/abs(MEST)	0.15779	0.041667	1.2128	0.006135	0.42857
AFD/PRICE (mult by 10)	0.058984	0.026735	0.12313	0.003008	0.20667
AFD/STDEPS (divd by 10)	0.1357	0.056583	0.37436	0.004069	0.47264

12-Month Horizon	Mean	Median	Std Dev	5%	95%
Number of firms per year	2005.4	1972	564.31	1184.7	3085
R_{t+1}	0.19495	0.11845	0.59787	-0.45536	1.0316
AC(R)	-0.05429	-0.06753	0.36656	-0.64166	0.61672
C(|R|, V)	0.091336	0.10304	0.36601	-0.53575	0.69795
V_{t+1}	2.3173	2.2696	0.87721	0.94644	3.8341
MVE (in $ millions)	2650	448.47	12212	51.92	9309.9
AFD	0.14354	0.06	2.9027	0.01	0.42
AFD/abs(MEST)	0.15675	0.041667	1.24	0.006148	0.42778
AFD/PRICE (mult by 10)	0.056418	0.026446	0.11125	0.003083	0.19649
AFD/STDEPS (divd by 10)	0.14951	0.056373	0.42395	0.004124	0.53358

Table 2: Basic Regression Results: The table shows the regression coefficient on the dispersion measure in monthly Fama-MacBeth regressions specified in (14) - (19). Newey-West standard errors are used to correct for serial correlation and coefficients significant at the 5% level are in bold.

1-Month Horizon	E(R)	Var(R)	AC(R)	C(|R| V)	E(V)	Var(V)
AFD	0.000476	0.000239	**0.0309**	**0.592**	**0.137**	**0.274**
pvalue	0.753	0.455	**0**	**0**	**0**	**0**
Adj. R^2	0.00151	0.000383	0.0122	0.125	0.00333	0.00333
AFD/abs(MEST)	-0.00017	**0.00218**	**0.00593**	**0.186**	**0.0665**	**0.133**
pvalue	0.784	**0**	**0.035**	**0**	**0**	**0**
Adj. R^2	0.00164	0.00277	0.0126	0.125	0.00263	0.00263
AFD/PRICE	**0.0134**	**0.0206**	**0.114**	0.496	**0.623**	**1.25**
pvalue	**0.023**	**0**	**0**	0.074	**0**	**0**
Adj. R^2	0.00391	0.00617	0.0136	0.126	0.00797	0.00797
AFD/STDEPS	**0.00189**	**0.00202**	**0.014**	**-0.293**	-0.00101	-0.00202
pvalue	**0.047**	**0**	**0.028**	**0**	0.93	0.93
Adj. R^2	0.000904	0.000811	0.0125	0.126	0.000999	0.000999

3-Month Horizon	E(R)	Var(R)	AC(R)	C(|R| V)	E(V)	Var(V)
AFD	0.00306	-0.00041	-0.0033	**0.285**	**0.125**	**0.241**
pvalue	0.422	0.703	0.766	**0**	**0**	**0**
Adj. R^2	0.00188	0.000373	0.117	0.107	0.0034	0.00328
AFD/abs(MEST)	0.00137	**0.00687**	-0.00082	**0.14**	**0.0756**	**0.154**
pvalue	0.401	**0**	0.79	**0**	**0**	**0**
Adj. R^2	0.00201	0.00491	0.115	0.106	0.0034	0.00348
AFD/PRICE	**0.057**	**0.063**	-0.0115	**0.602**	**0.723**	**1.5**
pvalue	**0.003**	**0**	0.634	**0.011**	**0**	**0**
Adj. R^2	0.0048	0.0108	0.117	0.108	0.00994	0.0102
AFD/STDEPS	0.00419	**0.00722**	0.00077	-0.0917	0.0124	0.0343
pvalue	0.187	**0**	0.921	0.123	0.46	0.304
Adj. R^2	0.00101	0.00184	0.116	0.107	0.00119	0.00121

12-Month Horizon	E(R)	Var(R)	AC(R)	C(|R| V)	E(V)	Var(V)
AFD	0.0389	-0.00763	**-0.0627**	**0.111**	**0.115**	**0.212**
pvalue	0.1	0.257	**0.016**	**0.04**	**0**	**0**
Adj. R^2	0.00148	0.000971	0.029	0.085	0.00345	0.00327
AFD/abs(MEST)	0.0172	**0.0317**	-0.015	**0.147**	**0.113**	**0.23**
pvalue	0.246	**0**	0.22	**0**	**0**	**0**
Adj. R^2	0.00319	0.0126	0.0301	0.0853	0.00751	0.0076
AFD/PRICE	**0.259**	**0.266**	-0.109	**0.692**	**1.07**	**2.24**
pvalue	**0.005**	**0**	0.145	**0**	**0**	**0**
Adj. R^2	0.00637	0.0208	0.0307	0.0858	0.0156	0.0166
AFD/STDEPS	**0.0335**	**0.0372**	-0.0124	**0.131**	**0.108**	**0.237**
pvalue	**0.02**	**0**	0.604	**0.002**	**0**	**0**
Adj. R^2	0.00187	0.00726	0.0297	0.083	0.00398	0.00414

Table 3: Regression Results with Controls: The table shows the regression coefficient on the dispersion measure in monthly Fama-MacBeth regressions specified in (14) - (19). Control variables include scaling variables for the dispersion measures, size, market to book ratio and number of analysts. Newey-West standard errors are used to correct for serial correlation and coefficients significant at the 5% level are in bold.

1-Month Horizon	E(R)	Var(R)	AC(R)	C(|R| V)	E(V)	Var(V)
AFD	**0.00358**	**0.00353**	**0.0452**	**1.11**	**0.29**	**0.581**
pvalue	**0.03**	**0**	**0**	**0**	**0**	**0**
Adj. R^2	0.0373	0.042	0.0236	0.185	0.155	0.155
AFD/abs(MEST)	-0.0005	**0.000755**	**0.012**	**0.25**	**0.0396**	**0.0791**
pvalue	0.397	**0**	**0.009**	**0**	**0**	**0**
Adj. R^2	0.0366	0.0419	0.0233	0.184	0.148	0.148
AFD/PRICE	0.00622	**0.0145**	**0.099**	**1.91**	**0.679**	**1.36**
pvalue	0.165	**0**	**0**	**0**	**0**	**0**
Adj. R^2	0.0379	0.0435	0.024	0.185	0.153	0.153
AFD/STDEPS	0.00149	**0.00292**	**0.0423**	-0.0488	**0.142**	**0.284**
pvalue	0.248	**0**	**0**	0.536	**0**	**0**
Adj. R^2	0.0368	0.0414	0.0233	0.184	0.149	0.149

3-Month Horizon	E(R)	Var(R)	AC(R)	C(|R| V)	E(V)	Var(V)
AFD	**0.0108**	**0.0114**	-0.00263	**0.699**	**0.305**	**0.613**
pvalue	**0.023**	**0**	0.796	**0**	**0**	**0**
Adj. R^2	0.0408	0.0898	0.128	0.179	0.178	0.176
AFD/abs(MEST)	-0.00035	**0.00297**	-0.00437	**0.176**	**0.0428**	**0.0866**
pvalue	0.852	**0**	0.426	**0**	**0**	**0**
Adj. R^2	0.0402	0.089	0.128	0.177	0.17	0.168
AFD/PRICE	0.0256	**0.0436**	-0.0143	**1.48**	**0.726**	**1.46**
pvalue	0.082	**0**	0.552	**0**	**0**	**0**
Adj. R^2	0.0418	0.0913	0.128	0.178	0.176	0.174
AFD/STDEPS	0.000787	**0.00915**	-0.0171	0.0794	**0.151**	**0.303**
pvalue	0.825	**0**	0.194	0.128	**0**	**0**
Adj. R^2	0.04	0.0882	0.128	0.177	0.171	0.169

12-Month Horizon	E(R)	Var(R)	AC(R)	C(|R| V)	E(V)	Var(V)
AFD	0.0456	**0.0536**	**-0.076**	**0.474**	**0.413**	**0.833**
pvalue	0.127	**0**	**0.012**	**0**	**0**	**0**
Adj. R^2	0.0475	0.189	0.0483	0.172	0.204	0.204
AFD/abs(MEST)	0.0158	**0.0136**	-0.0389	**0.195**	**0.0852**	**0.173**
pvalue	0.177	**0**	0.143	**0**	**0**	**0**
Adj. R^2	0.0473	0.184	0.0492	0.17	0.19	0.19
AFD/PRICE	**0.19**	**0.182**	-0.0921	**1.38**	**1.07**	**2.16**
pvalue	**0.048**	**0**	0.161	**0**	**0**	**0**
Adj. R^2	0.05	0.191	0.0487	0.173	0.2	0.2
AFD/STDEPS	**0.0376**	**0.0365**	-0.0445	**0.234**	**0.205**	**0.417**
pvalue	**0.034**	**0**	0.112	**0**	**0**	**0**
Adj. R^2	0.0463	0.185	0.0478	0.169	0.192	0.192

Table 4: Regression Results with Excess Returns: The table shows the regression coefficient on the dispersion measure in monthly Fama-MacBeth regressions specified in (14) - (19). Returns are residuals from a first-stage regression on the four factor model. Control variables include scaling variables for the dispersion measures, size, market to book ratio and number of analysts. Newey-West standard errors are used to correct for serial correlation and coefficients significant at the 5% level are in bold.

1-Month Horizon	E(R)	Var(R)	AC(R)	C(|R| V)	E(V)	Var(V)
AFD	**0.00325**	0.000412	**0.0265**	**1.17**	**0.29**	**0.581**
pvalue	**0.018**	0.102	**0.004**	**0**	**0**	**0**
Adj. R^2	0.0147	0.00911	0.0128	0.171	0.155	0.155
AFD/abs(MEST)	0.000255	**0.00053**	0.0072	**0.295**	**0.0396**	**0.0791**
pvalue	0.613	**0.004**	0.126	**0**	**0**	**0**
Adj. R^2	0.0142	0.01	0.0128	0.169	0.148	0.148
AFD/PRICE	**0.00793**	**0.00625**	**0.0421**	**2.1**	**0.679**	**1.36**
pvalue	**0.019**	**0**	**0.05**	**0**	**0**	**0**
Adj. R^2	0.0151	0.0105	0.0133	0.17	0.153	0.153
AFD/STDEPS	0.00164	**0.000656**	**0.0224**	**-0.362**	**0.142**	**0.284**
pvalue	0.151	**0.049**	**0.014**	**0**	**0**	**0**
Adj. R^2	0.0143	0.00912	0.0127	0.169	0.149	0.149

3-Month Horizon	E(R)	Var(R)	AC(R)	C(|R|V)	E(V)	Var(V)
AFD	**0.00889**	**0.00132**	**-0.0322**	**0.777**	**0.305**	**0.613**
pvalue	**0.007**	**0.015**	**0.002**	**0**	**0**	**0**
Adj. R^2	0.0182	0.0101	0.0965	0.17	0.178	0.176
AFD/abs(MEST)	0.00118	**0.00182**	-0.00728	**0.189**	**0.0428**	**0.0866**
pvalue	0.45	**0**	0.198	**0**	**0**	**0**
Adj. R^2	0.0178	0.0116	0.0967	0.168	0.17	0.168
AFD/PRICE	**0.0297**	**0.0127**	**-0.0726**	**1.68**	**0.726**	**1.46**
pvalue	**0.002**	**0**	**0.006**	**0**	**0**	**0**
Adj. R^2	0.0189	0.0115	0.0971	0.17	0.176	0.174
AFD/STDEPS	0.00318	0.00111	**-0.0355**	**-0.159**	**0.151**	**0.303**
pvalue	0.249	0.072	**0.005**	**0.009**	**0**	**0**
Adj. R^2	0.0176	0.00998	0.0965	0.168	0.171	0.169

12-Month Horizon	E(R)	Var(R)	AC(R)	C(|R|V)	E(V)	Var(V)
AFD	0.0236	0.00171	**-0.0922**	**0.588**	**0.413**	**0.833**
pvalue	0.133	0.478	**0.003**	**0**	**0**	**0**
Adj. R^2	0.0401	0.0177	0.0494	0.174	0.204	0.204
AFD/abs(MEST)	**0.0138**	0.00344	**-0.0329**	**0.225**	**0.0852**	**0.173**
pvalue	**0.043**	0.052	**0.035**	**0**	**0**	**0**
Adj. R^2	0.0398	0.0202	0.0499	0.171	0.19	0.19
AFD/PRICE	**0.105**	**0.0272**	**-0.222**	**1.41**	**1.07**	**2.16**
pvalue	**0.035**	**0.013**	**0.022**	**0**	**0**	**0**
Adj. R^2	0.0415	0.0201	0.0506	0.173	0.2	0.2
AFD/STDEPS	0.0157	0.00237	**-0.0689**	**0.125**	**0.205**	**0.417**
pvalue	0.054	0.251	**0.009**	**0.002**	**0**	**0**
Adj. R^2	0.0383	0.0172	0.0493	0.169	0.192	0.192

Table 5: Regression Results with Controls on Low Dispersion Subsample: The table shows the regression coefficient on the dispersion measure in monthly Fama-MacBeth regressions specified in (14) - (19). The sample of firms is restricted to those with AFD/STDEPS lower than the median in the given cross-section. Control variables include scaling variables for the dispersion measures, size, market to book ratio and number of analysts. Newey-West standard errors are used to correct for serial correlation and coefficients significant at the 5% level are in bold.

1-Month Horizon	E(R)	Var(R)	AC(R)	C(|R| V)	E(V)	Var(V)
AFD	**0.0111**	**0.00363**	**0.121**	**3**	**0.598**	**1.2**
pvalue	**0.005**	**0.001**	**0**	**0**	**0**	**0**
Adj. R^2	0.0407	0.047	0.0267	0.191	0.153	0.153
AFD/abs(MEST)	0.000804	-0.00016	0.0185	**0.85**	**0.0864**	**0.173**
pvalue	0.644	0.828	0.175	**0**	**0**	**0**
Adj. R^2	0.04	0.0468	0.0263	0.19	0.146	0.146
AFD/PRICE	0.011	**0.0129**	0.143	**4.72**	**1.22**	**2.43**
pvalue	0.229	**0**	0.069	**0**	**0**	**0**
Adj. R^2	0.0411	0.0482	0.0272	0.191	0.15	0.15
AFD/STDEPS	0.0357	-0.00016	0.143	**6.64**	**1.6**	**3.21**
pvalue	0.06	0.986	0.341	**0**	**0**	**0**
Adj. R^2	0.0395	0.0453	0.0257	0.189	0.148	0.148

3-Month Horizon	E(R)	Var(R)	AC(R)	C(|R|V)	E(V)	Var(V)
AFD	**0.031**	**0.0152**	-0.0321	**1.52**	**0.602**	**1.2**
pvalue	**0.004**	**0**	0.378	**0**	**0**	**0**
Adj. R^2	0.0428	0.094	0.133	0.184	0.176	0.174
AFD/abs(MEST)	-3.20E-05	0.00107	-0.0267	**0.513**	**0.0725**	**0.143**
pvalue	0.994	0.455	0.15	**0**	**0**	**0**
Adj. R^2	0.0419	0.0925	0.133	0.182	0.167	0.166
AFD/PRICE	**0.0519**	**0.0443**	-0.119	**2.73**	**1.16**	**2.3**
pvalue	**0.04**	**0**	0.127	**0**	**0**	**0**
Adj. R^2	0.043	0.0945	0.133	0.183	0.172	0.17
AFD/STDEPS	-0.0143	0.0207	**-0.468**	**4.44**	**1.49**	**2.94**
pvalue	0.73	0.252	**0.01**	**0**	**0**	**0**
Adj. R^2	0.0412	0.0913	0.133	0.182	0.169	0.167

12-Month Horizon	E(R)	Var(R)	AC(R)	C(|R|V)	E(V)	Var(V)
AFD	0.0227	**0.0713**	-0.0358	**0.686**	**0.715**	**1.43**
pvalue	0.633	**0**	0.622	**0.001**	**0**	**0**
Adj. R^2	0.0514	0.196	0.0578	0.177	0.203	0.202
AFD/abs(MEST)	-0.0427	0.00584	0.0512	**0.506**	**0.147**	**0.29**
pvalue	0.187	0.668	0.609	**0**	**0.006**	**0.007**
Adj. R^2	0.0501	0.193	0.0586	0.174	0.19	0.19
AFD/PRICE	0.217	**0.226**	-0.128	**2.28**	**1.47**	**2.93**
pvalue	0.151	**0**	0.538	**0.001**	**0**	**0**
Adj. R^2	0.0525	0.196	0.0586	0.178	0.197	0.197
AFD/STDEPS	-0.182	**0.325**	-0.148	**2.92**	**1.64**	**3.22**
pvalue	0.302	**0**	0.802	**0**	**0**	**0**
Adj. R^2	0.0485	0.19	0.0585	0.173	0.19	0.19