Market Based Regulation and the Informational Content of Prices[1]

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Abstract

A recurrent policy proposal is that governments should make use of the information reflected in market prices. We identify a key problem with this proposal: once the government announces its intention to make use of market prices, prices adjust to reflect this, and might become less revealing. We study a leading example of such a proposal, namely that bank supervision should make use of the market prices of traded bank securities. We show that the feasibility of this proposal depends critically on the information gap between the market and the government. When the gap is small, the government is able to implement its preferred intervention rule as a unique equilibrium. When the gap is moderate, additional equilibria exist in which the government intervenes either too much, or too little. When the gap is large, the government is unable to implement its preferred intervention strategy in equilibrium. We show that charging bank security holders for the cost of intervention, tracking the prices of several bank-related securities, and other policy measures can increase the ability of the government to make use of market information.

1 Introduction

A basic premise in economics is that prices in financial markets aggregate useful information. This premise underlies several recent laws and policy proposals that call for government agencies to use the information in market prices when making various decisions. One example is the Sarbanes-Oxley Act of 2002 that made changes to how publicly traded corporations are governed in the United States. Section 408 of the act calls for the Securities and Exchange Commission to consider market data, namely, share price volatility and price to earnings ratios, when deciding whether to review the legality of a firm's disclosures. Another example is class action securities litigation. Courts in the United States use share price changes as a guide for determining damages.[1] We argue in this paper that despite the attraction of such laws and proposals, they should confront the following problem: once the government announces its intention to make use of market prices, prices adjust to reflect this, and might become less revealing. We analyze the extent to which the feedback from government actions to market prices prevents effective inference in equilibrium.

Our analysis focuses on a specific – perhaps the most seriously discussed – policy proposal of this type, namely that government supervision of banks should make use of the information conveyed by the market prices of traded bank securities. We demonstrate that the feasibility of this policy proposal depends critically on the information gap between the market and the bank supervisor.

An important responsibility of bank supervisors is to assess the probability of a bank failure and, if needed and possible, to take actions to reduce this probability. Their main tool for assessing the risk of failure is direct examination. Supervisors evaluate all the important facets of a bank, including its loans, its balance, and its management. Small banks and medium sized banks are examined periodically. The largest banks are examined continuously. This direct supervision is expensive. In the United States, the federal and state governments spent nearly 3 billion dollars in 2005 supervising banks and similar institutions like thrifts and credit unions.[2] There are also limits to direct supervision.

[1] See, e.g., Cooper Alexander (1994).

[2] This is based on authors' calculations of direct costs only. Including indirect costs will increase this

Evaluating banks balance sheets is now more complex than it used to be, and the technical skills for such evaluation are in short supply. In addition, it might be hard for the government to obtain relevant information, which may be held by other market participants. If market prices contain information on banks' fundamentals, then, by learning from market prices, the government may be able to avoid substantial expenses and improve the quality of its information.

For these reasons, many recent proposals call for the use of information conveyed by market prices of traded bank securities in bank supervision. For example, one of the three pillars of the Basel II reform of capital regulations is the use of market discipline, one form of which is the use of information in market prices.[3] More specifically, proposals by Evanoff and Wall (2000) and others cited in their paper suggest that bank supervisors should monitor the price of subordinated bank debt as a means of evaluating the health of the issuing bank.[4] Finally, many policymakers emphasize the benefits of using information conveyed by market prices. Speaking in 2001, Gary Stern, President of the Federal Reserve Bank of Minneapolis, summarized these benefits as follows:[5]

> "Market data are generated by a very large number of participants. Market participants have their funds at risk of loss. A monetary incentive provides a perspective on risk taking that is difficult to replicate in a supervisory context. Unlike accounting-based measures, market data are generated on a nearly continuous basis and to a considerable extent anticipates future performance and conditions. Raw market prices are nearly free to supervisors. This characteristic seems particularly important given that supervisory resources are limited and

number substantially.

[3] More generally, market discipline is the use of private counterparty supervision to monitor and limit bank risk. Information produced by market participants and reflected in market prices provides one form of market dicipline.

[4] Relatedly, the Gramm-Leach-Bliley Act (1999) mandates that large banks wishing to engage in certain activities possess at least one outstanding issue of debt receiving a high rating from a credit rating agency (Section 5136A). This implies that the government is seeking information from market participants, such as rating agencies.

[5] See: http://www.minneapolisfed.org/pubs/region/01-09/stern.cfm

are diminishing in comparison to the complexity of large banking organizations."

According to Alan Greenspan (2001):[6]

> "The Federal Reserve and other regulatory agencies already monitor subordinated debt yields and issuance patterns in evaluating the condition of large banking organizations.[7] ... This use of subordinated debt is one example of the effort supervisors should undertake to employ data from a variety of markets."

We study a rational expectations model, with a financial market that trades the securities of a bank, and with a supervisor that can take costly actions that alter a bank's risk, and ultimately affect the value of the bank's securities. The price that is set in the financial market reflects the expected value of the securities given the information available in the market. The bank supervisor decides on his action based on his own information about the bank's fundamentals, and also based on the information that he can infer from the prices of the bank's securities.

Our theoretical analysis highlights a key problem in the implementation of a supervision policy that is based on market prices. The problem is that the supervisor wishes to infer information about the fundamentals of the bank from the prices of its securities, but the prices do not only reflect information about the fundamentals; they also take into account the effect of the resulting supervisor's action on the value of the securities. Thus, the inference from the price is non-trivial. For example, a low price may indicate that the fundamentals are bad, and thus call for the supervisor's intervention. It may also indicate

[6] See: http://www.minneapolisfed.org/pubs/region/01-09/greenspan.cfm

[7] Greenspan's assertion is supported by Burton and Seale (2005) and Feldman and Schmidt (2003). The former study describes the use of market signals by the FDIC, and reports that these signals are frequently used in off-site surveillance of banks and can be used to help target more detailed exams. The latter study surveyed Federal Reserve supervisors to determine whether they used market data. It found that market signals were frequently used by supervisors to help form their overall opinion of a bank. It also found the signals were also used to assess the quality of a bank's borrowers. For both institutions, however, the degree to which market signals were used varied across supervisory personnel.

that the fundamentals are not bad enough to justify intervention, in which case the price is low just because no intervention is expected (this assumes that intervention has a positive effect on the value of the security). This issue is further complicated by the fact that, in equilibrium, the price has to be consistent with the expected supervisor's action.

Overall, the fact that prices *affect* and *reflect* the supervisor's action at the same time makes the analysis of equilibrium outcomes (i.e., finding a fixed point) quite hard. We show that the ability of the supervisor to make use of market information depends critically on the information gap between the market and the bank supervisor. When the gap is small, the supervisor is able to implement its preferred intervention rule as a unique equilibrium. When the gap is moderate, additional equilibria exist in which the supervisor intervenes either too much, or too little. In this range, the type of equilibrium depends on the type of the traded security, e.g., equity vs. debt. When the gap is large, the supervisor is unable to implement its preferred intervention strategy in equilibrium. Thus, successful implementation of market based supervision requires the government to have reasonably precise information of its own. This suggests that the government should not completely abandon direct examination, and should use it together with learning from market information.

We also discuss measures that may reduce the extent of the problems mentioned above, and help restore the equilibrium that fully reveals the market's information to the government, even when the information gap between the market and the government is not small. These measures include observing the prices of multiple traded securities, improving the transparency on the side of the government, introducing a security that pays off in the event that the government intervenes, and taxing security holders to change the effect of the government's action on the value of their securities.

As mentioned in our opening paragraph, the theoretical analysis in this paper is based on the idea that market prices provide useful information to supervisors. The usual justification for this is that markets gather information from many different participants, who trade on their private information on bank fundamentals, and who do not communicate with the supervisors outside the trading process. This idea goes back to Hayek (1945), who argues that markets provide an efficient mechanism for information production and aggregation. The ability of financial markets to produce information that accurately predicts future events has also been demonstrated empirically. For example, Roll (1984) shows that private information of citrus futures traders regarding weather conditions gets impounded into citrus futures' prices, so that prices improve even public predictions of the weather. Furthermore, there is a large empirical literature that documents that bank security prices reflect underlying risk (see Flannery (1998) and Furlong and Williams (2006) for surveys), and that markets do have information that supervisors do not have. For example, works by Krainer and Lopez (2004) and others find that market prices can forecast ratings downgrades by bank supervisors.[8]

Our analysis is also based on the idea that market prices reflect the expected result of the subsequent supervisory action. This is consistent with the empirical finding that the connection between market prices and risk seems to depend on the supervisory regime. Gropp, Vesala, and Vulpes (2004) find that the correlation between security prices and rating downgrades depends on the likelihood of government support. They argue that bond holders are more likely to receive a bail out so bond prices are less sensitive than equity prices in countries where the government is more likely to intervene. Related, Covitz, Hancock, and Kwast (2004) find that in the United States bond spreads are more positively associated with bank risk measures in periods with less potential for government intervention than periods with more potential.

Our paper is related to several recent papers in the corporate finance literature that explore the theoretical implications of the feedback effect from stock prices to firms' investment decisions. Dow and Gorton (1997) and Subrahmanyam and Titman (1999) develop models that emphasize the increase in investment efficiency due to the information in stock prices. Goldstein and Guembel (2005) show that the feedback from stock price to real investments can generate manipulation of stock prices. Bond and Eraslan (2006) argue

[8] This effect is not found, however, in all markets; Gilbert, Vaughan, and Meyer (2003) find that prices in the jumbo CD market does not contain information not already contained in the supervisory surveillance model.

that the feedback from stock price to actions in the real sector can create a motive for trade. Dow, Goldstein and Guembel (2006) show that the feedback effect may affect the incentives of speculators to produce information and create incentives for firms to overinvest overinvestment. Empirically, Chen, Goldstein and Jiang (2006) and Luo (2005) provide evidence that such feedback exists, and is created by managerial learning from stock price.

Our paper is also related to the paper by Bernanke and Woodford (1997), who analyze the proposal that central banks should make use of private sector inflation forecasts in setting monetary policy. They establish that if the central bank targets monetary policy to private sector forecasts without having any private information, then the central bank cannot make full use of these forecasts without destroying their information content. Their result is based on a strong assumption that, once the government knows the private sector inflation forecast, it can costlessly achieve any level of expected inflation. Given this very strong policy instrument it is easy for the government, in their model, to destroy the information content of private sector forecasts. Aside from studying a different policy proposal, our paper differs from theirs in that we allow the government to acquire some information of its own; and we endow the government with a costly tool to affect bank health. Most of our results, such as those on the amount of government information needed to successfully use market information, and the comparative efficacy of using debt and equity prices, stem directly from these differences.

Finally, two recent papers analyze the effect of market prices on bank supervision. In a preliminary note, Birchler and Facchinetti (2004) point to the possibility of equilibrium non existence. Unlike our paper, however, their note considers policy rules that do not involve optimal learning from the price. As such, their analysis avoids many of the issues raised in our paper. Lehar, Seppi, and Strobl (2005) and Rochet (2004) emphasize very different implications of the effect of market prices on bank supervision. They study the implications of such an effect on the ability of the government to commit to an optimal supervisory policy.[9]

[9] Also related is Faure-Grimaud's (2002) analysis of using stock price information for regulation. He points out that information can lead a regulator to expropriate the firms it is overseeing; and that the availability

The remainder of the paper is organized as follows. In Section 2, we present the model. Section 3 defines an equilibrium in our model. In Section 4, we characterize equilibrium outcomes when the government learns from the market price of one traded security. In this section, we highlight the problems that result from a supervision policy that is based on market prices. Section 5 discusses measures that may restore optimal intervention in equilibrium. Section 6 concludes. All proofs are relegated to the appendix.

2 The model

The model has one bank, a government, and a financial market that trades the bank's securities. There are three dates, $t = 0, 1, 2$. At date 0, the prices of bank securities are determined in the market. At date 1, the government may intervene in the bank's operations. Finally, at date 2, all security holders are paid. We now describe the model in more detail.

2.1 The bank

In the absence of government, the bank's assets generate a gross cash flow of $\theta + \varepsilon - T$ at date 2. The component θ is stochastic and is realized at date 0. It represents the information available in the economy at date 0 regarding the bank's future cash flows. We will often refer to θ as the *fundamental* of the bank. The component ε is also stochastic and is realized at date 2. It represents the component of the bank's cash flow, about which no information is available at date 0. The component T is deterministic, and can be interpreted as an amount stolen by the bank's manager, or any other inefficiency involved in the management of the bank. Throughout, we assume that the fundamental θ is drawn uniformly from some interval $[\underline{\theta}, \bar{\theta}]$. The shock ε is drawn from a single-peaked density function g (the cumulative distribution function is G) with mean 0.

Generally speaking, the bank has two types of securities: insured deposits and uninsured

of free stock price information reduces the regulator's own monitoring efforts, potentially reducing total expropriation.

claims. We let D denote the face value of deposits. This amount is insured by the government, i.e., if the bank does not have enough resources to pay D to depositors, the government will bail out the bank, and provide the resources to ensure that depositors are paid in full. Other security holders are not insured. They receive payment from the bank at date 2 if the bank has resources left after paying D to depositors. The order in which they get paid is determined by a prespecified priority rule.

2.2 The government

At date 1, the government may intervene in the bank's business and increase the cash flows available to security holders. For tractability we will adopt the following simple specification of the effects of intervention: if the government intervenes, the bank's cash flow at date 2 increases by T. Specifically, if the government intervenes, the bank's date 2 cash flow is $\theta + \varepsilon$ instead of $\theta + \varepsilon - T$. This specification of intervention is consistent with an efficiency improvement or the prevention of a payment T to the bank's managers or other insiders.[10]

When making a decision whether to intervene or not, the government has to weigh the cost against the benefit. We assume a fixed cost of intervention C. The benefit of intervention is that it reduces the government's expected payment to insured depositors. Let $\Gamma(\theta)$ denote the expected payment to insured depositors *absent* government intervention as a function of the fundamental θ. It is given by

$$\Gamma(\theta) = G(T-\theta) D + \int_{T-\theta}^{D+T-\theta} (D - (\theta + \varepsilon - T)) g(\varepsilon) d\varepsilon. \tag{1}$$

Then, using $V(\theta)$ to denote the gain to the government from intervention, we get

$$V(\theta) \equiv \Gamma(\theta) - \Gamma(\theta + T). \tag{2}$$

If the government is fully informed about θ, it intervenes when $V(\theta) > C$ and does not

[10] We assume that the type of intervention performed by the government cannot be replicated by the banks' security holders. This assumption is quite reasonable given that the law provides the government various tools that enable it to control banks' actions. Governance by security holders, on the other hand, is quite constrained, as is pointed out in the vast literature on corporate governance.

intervene when $V(\theta) < C$. Lemma 1 establishes an important property of the function $V(\theta)$.

Lemma 1 *The value of government intervention $V(\theta)$ is first increasing and then decreasing in the fundamental θ.*

Intuitively, at low fundamentals, intervention is unlikely to prevent a bail out, while at high fundamentals, a bail out is unlikely to be needed. Thus, the value of intervention is maximized at intermediate values of θ, where intervention is most likely to affect the probability of a bail out. For tractability and to make the problem of economic interest, we assume that

$$V(\underline{\theta}) - C > 0 > V(\bar{\theta}) - C. \tag{3}$$

It follows that there exists a unique $\hat{\theta} \in [\underline{\theta}, \bar{\theta}]$ at which the government is indifferent between intervening and not intervening, i.e., $V(\hat{\theta}) = C$. For fundamentals below (above) $\hat{\theta}$, a fully informed government would strictly prefer to intervene (not intervene).

It is worth stressing that our main results would be qualitatively unchanged if the intervention cost C and/or the gross intervention benefit T were allowed to depend on θ. The key ingredient for our analysis is the existence of a threshold fundamental $\hat{\theta}$, above (below) which the net benefit of intervention to the government is negative (positive).

2.3 The value of bank securities

While the government may intervene in the bank to reduce its own bail out costs, the intervention has a spillover effect on the value of the uninsured securities. Some uninsured securities may be traded in the financial market at date 0, in which case their price reflects their expected equilibrium value. Before turning to the equilibrium analysis, we characterize the value of securities with and without government intervention.

We let $X_i(\theta)$ denote the expected value of security i absent government intervention. We use $U_i(\theta)$ to denote the value of government intervention for investors holding security i. Because the effect of government intervention is to leave the bank with an additional T

in resources, a general feature of our model is that,

$$X_i(\theta) + U_i(\theta) = X_i(\theta + T). \tag{4}$$

We focus on securities whose value $X_i(\theta)$ is strictly increasing in θ. A key determinant for our equilibrium results will be whether $X_i(\theta)$ is concave or convex.[11] For concreteness, it is useful to focus on two leading examples of uninsured bank securities: debt ($i = B$) and equity ($i = E$). Denoting the face value of debt as B and assuming that debt is junior to deposits, the values of equity and debt absent intervention are given as follows:

$$X_E(\theta) \equiv \int_{D+B+T-\theta} (\varepsilon - (D + B + T - \theta))\, g(\varepsilon)\, d\varepsilon, \tag{5}$$

$$X_B(\theta) \equiv \int_{D+T-\theta}^{D+B+T-\theta} \varepsilon - (D + T - \theta)\, g(\varepsilon)\, d\varepsilon + (1 - G(D + B + T - \theta))\, B. \tag{6}$$

Analyzing these expressions, we can see that $X_E(\theta)$ is convex (because equity holders are the residual claimants), while $X_B(\theta)$ has a convex then a concave shape (because debt is junior to deposits and senior to equity). In what follows, we will sometime refer to a convex security as equity, and to a concave security as debt (i.e., assuming that we are in the range where debt is concave). We conduct most of our analysis for the case where the government observes the price of only one traded security. In Section 5.1, we consider the case where multiple security prices can be observed.

A significant consequence of our specification is that government intervention helps both equity and debt holders. As such, one form of intervention that our basic model does *not* capture is an intervention that decreases the riskiness of the bank's projects and thus transfers value *from* equity *to* debt holders. In this, we are consistent with Akerlof and Romer's (1993) argument that an empirically relevant form of bank misbehavior is "looting," which presumably hurts debtholders and small shareholders at the same time. Importantly, our model could still apply to the case where intervention transfers value from equity to debt holders. In this case, if only debt is traded, the analysis will yield similar

[11] Our analysis applies equally to securities whose expected value is strictly decreasing in the fundamental θ. The results for a decreasing concave (convex) security would be equivalent to those we obtain for an increasing convex (concave) security.

results to what we currently have. Finally, it is important to realize that our current specification does not necessarily imply that bank intervention causes an increase in equity price — it is quite possible that intervention reveals to the market that bank fundamentals are poor, causing the share price to drop.

2.4 Information

A key point in our analysis is that the government does not know θ, and may learn it from the market. We assume that the realization of θ is known in the market at date 0, and that it serves as a basis for the price formation. In addition, at date 0, the government observes a private noisy signal of θ: $\phi_G = \theta + \xi$. We assume that ξ, the noise with which the government observes the fundamental, is uniformly distributed over $[-\kappa, \kappa]$.

One limitation of our informational structure is that it assumes that the government always knows less than the information collectively possessed by market participants (i.e., the information of market participants aggregates to θ, while the government only observes a noisy signal of θ). To assess the robustness of the model, we analyzed an extension, where we allow for the possibility that the government sometimes has more information than the market. This is modelled by assuming that, on top of the current information structure, the government may observe ε (which is not observed by the market) with some probability. Our analysis indicates that this extension does not affect the qualitative results of the model. Details of this analysis are available upon request.

3 Equilibrium

3.1 Market prices

When making its intervention decision, the government possesses two pieces of information: its own signal ϕ_G, and the observed prices of market securities P (if there are several traded securities P is a vector). An intervention policy is thus a function $I(P, \phi_G)$, where $I \in [0,1]$ is the probability of intervention.

For a given intervention policy $I(\cdot, \cdot)$, the price of a traded security reflects the intervention probability. Specifically, the equilibrium pricing function P satisfies the rational expectations equilibrium (REE) condition

$$P(\theta) = X(\theta) + E_{\phi_G}\left[I\left(P(\theta), \phi_G\right)|\theta\right] U(\theta). \tag{7}$$

The first component in this expression is the expected value of the security absent government intervention given the fundamental θ. The second component is the additional value stemming from the possibility of government intervention, the probability of which depends on the price $P(\theta)$ and the government's own signal ϕ_G.

3.2 Time consistency

For the analysis in the paper we also require the government's intervention policy to constitute a "best response" to the market price. That is, we require the intervention policy to be *time consistent*. This implies that the government intervenes with probability 1 when the expected benefit from intervention is greater than the cost, and intervenes with probability 0 when the expected benefit is smaller than the cost. Thus, under time consistency, I is either 0 or 1, except for the case where the expected benefit is exactly equal to the cost. In this case, the government may choose to play a mixed strategy.

For a given pricing function $P(\cdot)$, the observation of a particular price P tells the government that the market observed a fundamental θ' such that $P(\theta') = P$. So formally, we say that the intervention policy $I(\cdot, \cdot)$ is time consistent given a pricing function $P(\cdot)$ if for all equilibrium realizations $\left(\tilde{P}, \tilde{\phi}_G\right)$ of the price-signal pair, $I\left(\tilde{P}, \tilde{\phi}_G\right) = 1$ when

$$E_\theta\left[V(\theta)|P(\theta) = \tilde{P} \text{ and } \tilde{\phi}_G\right] > C, \tag{8}$$

$I\left(\tilde{P}, \tilde{\phi}_G\right) = 0$ when

$$E_\theta\left[V(\theta)|P(\theta) = \tilde{P} \text{ and } \tilde{\phi}_G\right] < C, \tag{9}$$

and $I\left(\tilde{P}, \tilde{\phi}_G\right) \in [0,1]$ when

$$E_\theta\left[V(\theta)|P(\theta) = \tilde{P} \text{ and } \tilde{\phi}_G\right] = C. \tag{10}$$

3.3 Equilibrium definition

The formal definition of an equilibrium is as follows:

Definition 1 *A pricing function $P(\cdot)$ and intervention policy $I(\cdot,\cdot)$ together constitute an equilibrium if they satisfy the REE condition (7). They constitute a time-consistent equilibrium if, in addition, (8), (9), and (10) hold.*

4 Market based intervention

We now explore the equilibrium outcomes when there is one traded security and the government attempts to learn the fundamental θ from the price of this security. Along the way, we will make a distinction between the equilibrium outcomes for a convex security (equity) and those for a concave security (debt). We start by defining an important class of equilibria, for which the government can perfectly infer the market's information.

Definition 2 *A fully revealing equilibrium is an equilibrium in which each price is associated with one fundamental, and thus the fundamental can be inferred from the price. An equilibrium is essentially fully revealing if when a price is associated with more than one fundamental the government can still distinguish among the different fundamentals based on its private information.*

In both fully revealing and essentially fully revealing equilibria, the government chooses the optimal action based on θ: it intervenes when θ is less than the cutoff value $\hat{\theta}$ and does not intervene when θ is above $\hat{\theta}$. Thus, we will sometimes refer to fully revealing and essentially fully revealing equilibria as equilibria with *optimal intervention*. Clearly, any other equilibrium does not feature optimal intervention, since in such equilibrium, the combination of the price and the government's private signal does not enable the government to infer θ perfectly.

The price function for the security under optimal intervention is given by:

$$P_i(\theta) = \begin{cases} X_i(\theta) + U_i(\theta) & if \quad \theta < \hat{\theta} \\ X_i(\theta) & if \quad \theta > \hat{\theta} \end{cases} . \tag{11}$$

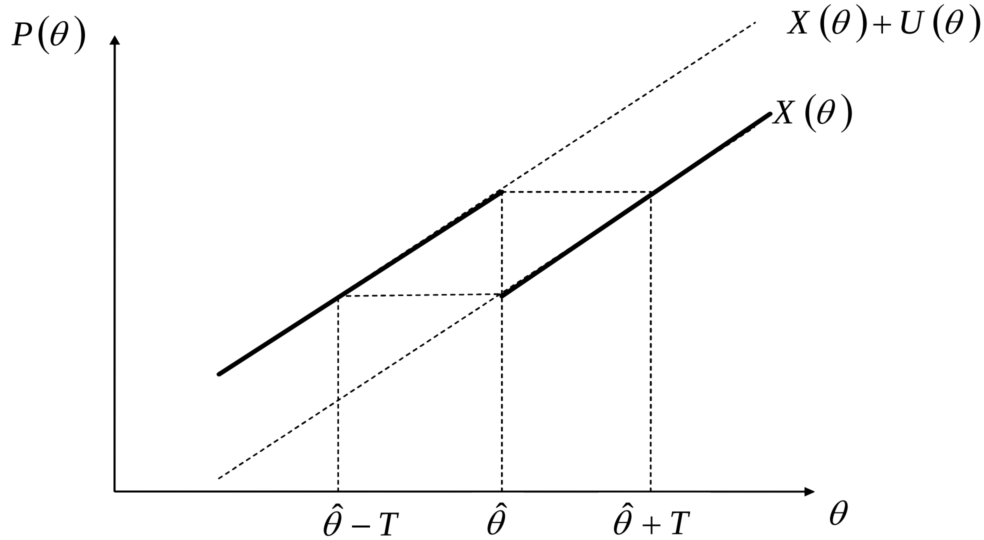

Figure 1: Security price under optimal intervention

It is depicted in Figure 1. (Note that Figure 1 and the other figures in the paper are only schematic. In particular, the functions X and $X+U$ are not linear.)

Inspection of Figure 1 reveals the difficulty in obtaining an equilibrium with optimal intervention in this model. The difficulty stems from the fact that under optimal intervention, the price function is non-monotone, and that the non-monotonicity occurs around $\hat{\theta}$. This is a result of the optimal intervention rule employed by the government – to intervene only when the fundamentals are below $\hat{\theta}$ – and of the fact that intervention increases the value of the security. As a result of this non-monotonicity, fundamentals on both sides of $\hat{\theta}$ have the same price, and so the government can infer neither the level of the fundamental, nor the the optimal action, from the price alone. Essentially, the fact that the price reflects the expected reaction of the government to the price makes learning from the price more difficult. Following this logic, the possibility of obtaining the optimal intervention rule in equilibrium depends on the precision of the government's private signal. A precise private signal will enable the government to distinguish between different fundamentals that have the same price. We now turn to a complete analysis of equilibrium outcomes based on the precision of the government's signal.

4.1 The government's signal is precise: unique equilibrium

We start with the case in which the government's signal ϕ_G is relatively precise. Our first result is that whenever $2\kappa < T$, there is an equilibrium with optimal intervention:

Proposition 1 *For $2\kappa < T$, an equilibrium with optimal intervention exists.*

The intuition behind this result is that under the optimal intervention rule, there are at most two fundamentals associated with each price. Moreover, these fundamentals are at a distance T from each other (see Figure 1). Since the government's signal is relatively precise, such that $2\kappa < T$, the government can use the signal to perfectly infer the realization of the fundamental when the price is consistent with two different fundamentals. Thus, it can follow the optimal intervention rule. It is important to stress that in this equilibrium, both the price and the private signal serve an important role: the price tells the government that one of two different fundamentals may have been realized, while the private signal enables the government to tell these two fundamentals apart. Thus, the government uses both the price and the private signal to infer the underlying fundamental.

Our next result shows that whenever the government's information is sufficiently precise, the optimal intervention equilibrium is the only equilibrium.

Proposition 2 *There is a $\bar{\kappa} > 0$ such that when $\kappa \leq \bar{\kappa}$ the optimal intervention equilibrium is the unique equilibrium.*

4.2 The government's signal is moderate: multiple equilibria

As the government's information precision worsens, in the sense that κ increases beyond the point $\bar{\kappa}$ defined by Proposition 2 and approaches $T/2$, the optimal intervention equilibrium remains an equilibrium. However, additional and less desirable equilibria emerge. In such equilibria, the government cannot perfectly infer the fundamental from market prices.

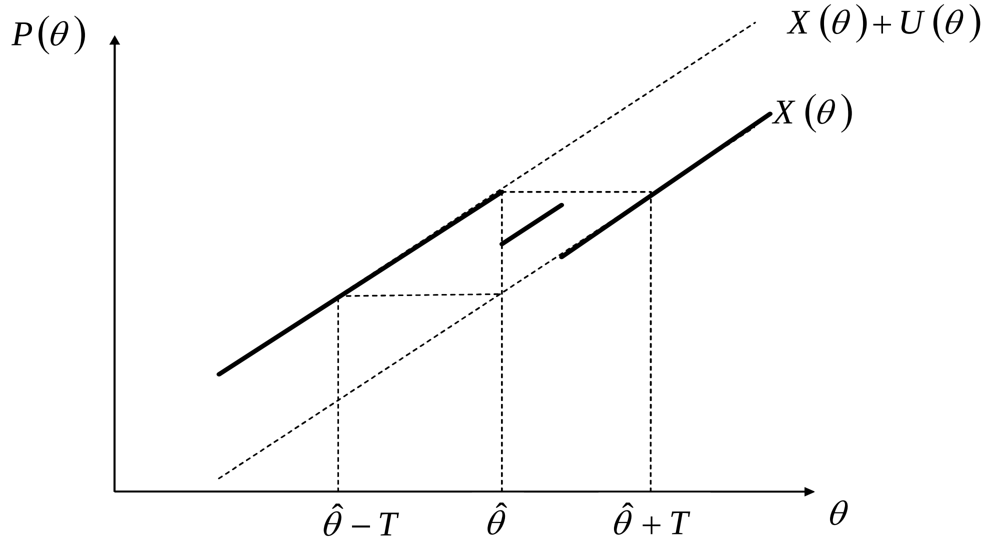

Figure 2: Security price in an equilibrium with too much intervention

4.2.1 The case of a convex security: equilibrium with too much intervention

We consider first the case in which the government observes the market price of a convex security. We establish that in this case there are equilibria in which the government intervenes too much relative to the optimal intervention equilibrium depicted in Figure 1. Figure 2 depicts an example of such an over-intervention equilibrium.

As we can see in Figure 2, in this equilibrium, the government always intervenes when the fundamentals are below $\hat{\theta}$ (the left line in the price function), never intervenes at *some* fundamentals above $\hat{\theta}$ (the right line in the price function), and intervenes with positive probability for other fundamentals above $\hat{\theta}$ (the middle line in the price function). This last feature reflects the fact that the government intervenes too much in this equilibrium. This happens because, at the fundamentals associated with the middle line, the price does not fully reveal that the fundamentals are above $\hat{\theta}$, even after the price is combined with the government's private signal. That is, the distance between the middle line and the left line is too small for the government to be able to tell fundamentals associated with these lines apart using its private signal, given that $\kappa > \bar{\kappa}$. Because at fundamentals associated with the middle line the government cannot rule out that the fundamental is below $\hat{\theta}$, it intervenes with positive probability. This increases the price at these fundamentals. In turn, it is this price increase that pushes the left and middle lines closer together, preventing the government from being able to distinguish them.

We now turn to formally prove the existence of an equilibrium with too much intervention. The main result here is Proposition 3, which establishes the existence of such an equilibrium and provides a full characterization of it. We start with some algebra leading to the proposition. In an equilibrium with too much intervention, over-intervention occurs for fundamentals lying in some interval to the right of $\hat{\theta}$. As a first step, consider what is required to generate too much intervention at a point $\hat{\theta}_+$ infinitesimally to the right of $\hat{\theta}$. Over-intervention occurs if the market price at $\hat{\theta}_+$ is the same as the market price at some $\theta \in \left(\hat{\theta}_+ - T, \hat{\theta}\right)$. The probability of intervention at $\hat{\theta}_+$ in this case is the probability that the government receives a signal ϕ_G that is consistent with the fundamental θ:

$$\Pr\left(\phi_G \leq \theta + \kappa | \hat{\theta}_+\right) = \frac{\theta + \kappa - \hat{\theta}_+ - (-\kappa)}{2\kappa} = 1 - \frac{\hat{\theta}_+ - \theta}{2\kappa}.$$

As such, the market prices at $\theta < \hat{\theta}$ and $\hat{\theta}_+$ coincide if and only if

$$X_E\left(\hat{\theta}_+\right) + \left(1 - \frac{\hat{\theta}_+ - \theta}{2\kappa}\right) U_E\left(\hat{\theta}_+\right) = X_E(\theta) + U_E(\theta). \tag{12}$$

We first show that there is a fundamental θ satisfying (12) whenever the security is convex, $2\kappa < T$, and 2κ is sufficiently close to T:

Lemma 2 *For $\kappa < T/2$ sufficiently close to $T/2$, there exists a unique $\theta_{01} < \hat{\theta}$ such that (12) holds.*

Lemma 2 establishes that too much intervention is possible at a point immediately to the right of $\hat{\theta}$. The convexity of the security is important here. To see why, let us inspect equation (12). Obviously, the LHS in (12) is equal to the RHS when $\theta = \hat{\theta}_+$. Moreover, because $2\kappa < T$, the LHS is smaller than the RHS when $\theta = \hat{\theta}_+ - 2\kappa$.[12] Since the LHS

[12] The RHS equals $X_E\left(\hat{\theta}_+ - 2\kappa\right) + U_E\left(\hat{\theta}_+ - 2\kappa\right) = X_E\left(\hat{\theta}_+ - 2\kappa + T\right)$, while the LHS equals $X_E\left(\hat{\theta}_+\right)$, which is smaller.

is linear and increasing in θ, a necessary condition for (12) to have a solution below $\hat{\theta}$ is that its RHS is convex in θ. This implies that the security has to be convex for too much intervention to occur.

Using the result in Lemma 2, the following proposition fully establishes and characterizes the equilibrium. To better understand the proposition, it is useful to look again at Figure 2, which depicts an equilibrium of the type described here.

Proposition 3 *Suppose the government observes the price of a convex security. For $\kappa < T/2$ sufficiently close to $T/2$, there exist equilibria in which the government intervenes with positive probability at some $\theta > \hat{\theta}$. In particular, there exists an interval $(\theta_{01}, \theta_{11})$ where θ_{01} is as defined in Lemma 2 and $\theta_{11} < \hat{\theta}$ such that the following holds:*

1. *For any set $Y_1 \subset [\theta_{01}, \theta_{11}]$, there exists a strictly increasing function $\theta_2^* : Y_1 \to \Re$ such that $\theta_2^* (\theta_{01}) = \hat{\theta}$, and such that the following is an equilibrium:*

2. *[Optimal intervention below $\hat{\theta}$] If $\theta \leq \hat{\theta}$, the government intervenes with probability* 1*, and the price is $X_E (\theta) + U_E (\theta)$.*

3. *[Over-intervention for some $\theta > \hat{\theta}$] If $\theta \in \theta_2^* (Y_1)$ the government intervenes with probability $1 - \frac{\theta - \theta_2^{*-1}(\theta)}{2\kappa} > 0$, and the price is $X_E (\theta) + \left(1 - \frac{\theta - \theta_2^{*-1}(\theta)}{2\kappa}\right) U_E (\theta)$.*

4. *[Optimal intervention for some $\theta > \hat{\theta}$] If $\theta > \hat{\theta}$ and $\theta \notin \theta_2^* (Y_1)$, the government never intervenes, and the price is $X_E (\theta)$.*

The result in Proposition 3 begs the question of whether, when the government observes the price of a convex security, there also exist equilibria in which too little intervention occurs. The following proposition provides a negative answer to this question. Any equilibrium entails too much intervention in the following sense:

Proposition 4 *Suppose the government observes the price of a convex security, and $2\kappa < T$. Then any equilibrium other than the optimal intervention equilibrium entails a strictly positive probability of intervention at some fundamental $\theta > \hat{\theta}$.*

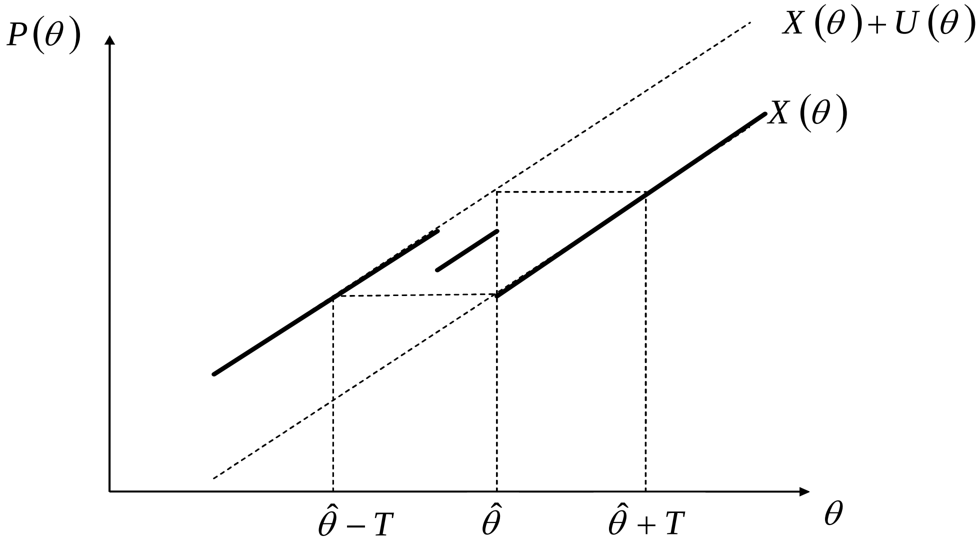

Figure 3: Security price in an equilibrium with too little intervention

4.2.2 The case of a concave security: equilibrium with too little intervention

We now consider the case in which the government observes the market price of a concave security. It turns out that parallel but opposite results hold in this case relative to the case of a convex security. Specifically, we establish that in this case there are equilibria in which the government intervenes too little relative to the optimal intervention equilibrium depicted in Figure 1. Figure 3 depicts an example of such an under-intervention equilibrium.

As we can see, the equilibrium depicted in Figure 3 exhibits too little intervention. The government intervenes optimally at fundamentals associated with the left line and the right line of the pricing function, but intervenes too little at fundamentals associated with the middle line. This happens because at these fundamentals, the price does not fully reveal that the fundamental is below $\hat{\theta}$, even after it is combined with the government's private signal.

Proposition 5 establishes the existence of equilibria with too little intervention and provides a full characterization of them. The proof is parallel to the proof of Proposition

3. To clarify the mathematical intuition and the role of concavity for equilibria with too little intervention, we now go over the first steps leading to the proof. Consider what is required to generate too little intervention at a point $\hat{\theta}_-$ infinitesimally to the left of $\hat{\theta}$. Under-intervention occurs if the market price at $\hat{\theta}_-$ is the same as the market price at some $\theta \in \left(\hat{\theta}, \hat{\theta}_- + T,\right)$. The probability of intervention at $\hat{\theta}_-$ in this case is the probability that the government receives a signal ϕ_G that is not consistent with the fundamental θ:

$$\Pr\left(\phi_G \leq \theta - \kappa | \hat{\theta}_-\right) = \frac{\theta - \kappa - \hat{\theta}_- - (-\kappa)}{2\kappa} = \frac{\theta - \hat{\theta}_-}{2\kappa}.$$

As such, the market prices at $\theta > \hat{\theta}$ and $\hat{\theta}_-$ coincide if and only if

$$X_B\left(\hat{\theta}_-\right) + \left(\frac{\theta - \hat{\theta}_-}{2\kappa}\right) U_B\left(\hat{\theta}_-\right) = X_B(\theta). \tag{13}$$

For this equation to hold, the value of the security has to be concave. To see why note that the LHS in (13) is equal to the RHS when $\theta = \hat{\theta}_-$. Moreover, because $2\kappa < T$, the LHS is larger than the RHS when $\theta = \hat{\theta}_- + 2\kappa$. Since the LHS is linear and increasing in θ, a necessary condition for (13) to have a solution above $\hat{\theta}$ is that its RHS is concave in θ. This implies that the security has to be concave for too little intervention to occur. We now turn to the proposition.

Proposition 5 *Suppose the government observes only the price of a concave security. For $2\kappa < T$ such that 2κ is sufficiently close to T, there exist equilibria, in which the government intervenes with probability less than one at some $\theta < \hat{\theta}$. In particular, there exists an interval $(\theta_{02}, \theta_{12})$ where $\theta_{02} > \hat{\theta}$ such that the following holds:*

1. *For any set $Y_2 \subset [\theta_{02}, \theta_{12}]$, there exists a strictly increasing function $\theta_1^* : Y_2 \to \Re$ such that $\theta_1^*(\theta_{12}) = \hat{\theta}$, and such that the following is an equilibrium:*

2. *[Optimal intervention above $\hat{\theta}$] If $\theta \geq \hat{\theta}$, the government intervenes with probability* 0*, and the price is $X_B(\theta)$.*

3. *[Under-intervention for some $\theta < \hat{\theta}$] If $\theta \in \theta_1^*(Y_2)$ the government intervenes with probability $\frac{\theta_1^{*-1}(\theta) - \theta}{2\kappa} > 0$, and the price is $X_B(\theta) + \frac{\theta_1^{*-1}(\theta) - \theta}{2\kappa} U_B(\theta)$.*

4. *[Optimal intervention for some $\theta < \hat{\theta}$] If $\theta < \hat{\theta}$ and $\theta \notin \theta_1^* (Y_2)$, the government always intervenes, and the price is $X_B (\theta) + U_B (\theta)$.*

To complete the analysis of a concave security, the next proposition provides a result that is parallel to the one in Proposition 4. Essentially, for the case of a security with a concave payoff, any equilibrium that does not exhibit optimal intervention has too little intervention.

Proposition 6 *Suppose the government observes only the price of a concave security, and $2\kappa < T$. Then any equilibrium other than the optimal intervention equilibrium entails an intervention probability strictly less than 1 at some fundamental $\theta < \hat{\theta}$.*

4.3 The government's signal is imprecise: no equilibrium

Finally, consider the case when $2\kappa > T$, that is, when the government's signal is imprecise so that the information gap between the market and the government is large. The first thing to note is that when $2\kappa > T$, optimal intervention cannot occur in equilibrium. To see this, go back to Figure 1. As we can see in the figure, in an equilibrium with optimal intervention, there are fundamentals at a distance of T from each other on both sides of $\hat{\theta}$ that have the same price. Since the government's signal is imprecise, i.e., since $2\kappa > T$, the signal does not enable the government to always distinguish between two fundamentals that have the same price. Thus, given a price that is associated with two fundamentals, it is impossible for the government to always intervene at one fundamental and never intervene at the other fundamental, and therefore optimal intervention cannot occur.

Our main result in this subsection is in fact much stronger. Proposition 7 shows that when $2\kappa > T$, not only there is no equilibrium with optimal intervention, but there is also no other rational-expectations time-consistent equilibrium.

Proposition 7 *Suppose that the government's information is relatively poor ($2\kappa > T$) and that it observes the price of a single security. Regardless of whether the security's expected payoff is convex or concave in the fundamental θ, no equilibrium exists.*

The intuition underlying this result is most easily conveyed in the limiting case in which the government receives no information at all (i.e., $\kappa \to \infty$). First, we claim that the only candidate equilibrium in this case is one with fully revealing prices. To see this, suppose instead that there is an equilibrium in which two fundamentals θ_1 and $\theta_2 \neq \theta_1$ are associated with the same price. Since the government has no information, its intervention policy must be the same at θ_1 and θ_2. But then the prices are not equal, giving a contradiction.[13] However, there is no fully revealing equilibrium either. For in such an equilibrium, time-consistency implies that there is intervention with probability 1 for $\theta < \hat{\theta}$, and intervention with probability 0 for $\theta > \hat{\theta}$. But then the price must be the same for some pair of fundamentals lying opposite sides of $\hat{\theta}$ (as in Figure 1), giving a contradiction. This result extends much more widely to the case of $2\kappa > T$, as is shown in the proof of Proposition 7.

4.4 Discussion

Overall, the results in this section demonstrate the difficulty in implementing an intervention policy that is based on the market price of a bank's security. The problem stems from the fact that the market price adjusts to reflect the expected government's action, and this reduces the extent to which the fundamental, which is what the government is trying to learn, can be inferred from the price.

A key determinant of whether optimal intervention can be implemented based on market price is then the quality of the private signal that is observed by the government in addition to the price. We show that when the government has a very precise signal, optimal intervention is obtained as a unique equilibrium. When the government has a moderately precise signal, optimal intervention is still an equilibrium, but there are also other equilibria with suboptimal intervention. Finally, when the government's signal is imprecise, there is no equilibrium in the model. Of course, one has to be cautious when interpreting a no-equilibrium result. At the very least, however, we believe it points to the problem associated with implementing a market based intervention when the quality of the government's own

[13] This same argument implies that even if the government can commit to an intervention policy (i.e., time-consistency is not imposed) there is no non-fully revealing equilibrium.

signal is poor.

The implication coming from the results in this section is that a successful implementation of market based intervention requires the government to have a reasonably precise signal of its own. Thus, while learning from the market can be very helpful in government policy, it should complement, rather than completely substitute, the government's own information. Therefore, policy proposals that call for learning from the market need to realize that the government should still maintain some private sources of information.

In the next section we discuss other policy measures that may help implementing optimal intervention based on market prices.

5 Policy implications: improving market based intervention

We now consider measures that may help the government implement the optimal intervention policy in equilibrium. The first measure is to learn from the prices of multiple securities. The second measure is to improve transparency by disclosing the government's signal to the market. The third measure is to issue a security that directly predicts whether the government is going to intervene. The fourth measure is to impose taxes on security holders that change their payoffs in case of government intervention.

5.1 Multiple securities

Thus far we have restricted attention to the case in which the government observes either only the price of a convex security (equity), *or* only the price of a concave security (debt). A key question is whether it helps if both these securities trade publicly, and the government learns from the prices of both.

It turns out that observing the prices of both securities resolves the problem of multiple equilibria when the government's signal is moderately precise, but does not solve the problem of no equilibrium when the government's signal is imprecise. We start by proving the first result.

Proposition 8 *Suppose that $2\kappa < T$ and that the government observes the price of both a concave and a convex security. Then the optimal intervention equilibrium is the unique pure strategy equilibrium.*

The explanation for this result is the following: in the previous section, we have shown that when the government's information is moderately precise, there may exist equilibria with too much or too little intervention, in addition to the equilibrium with optimal intervention. We have also shown that an equilibrium with too much intervention requires that the security, whose information the government observes, be convex, while an equilibrium with too little intervention requires that the security be concave. Thus, in this range, observing both the price of a concave security and the price of a convex security eliminates the equilibria with suboptimal intervention and leaves the optimal intervention equilibrium as the unique equilibrium. We now prove the second result.

Proposition 9 *Suppose that $2\kappa > T$ and that the government observes the price of both a concave and a convex security. Then no equilibrium exists.*

Essentially, once equilibrium fails to exist with one traded security, it will not be generated by adding another security.

5.2 Transparency

Going back to the case of one traded security, suppose that the government makes public its own signal ϕ_G *before* the market price is formed. Our analysis implies that this form of transparency by the government improves the government's ability to make use of market information to some extent. In particular, transparency resolves the problem of multiple equilibria when the government's signal is moderately precise, but does not solve the problem of no equilibrium when the government's signal is imprecise.

To see this, note that if the government discloses ϕ_G before the market price is formed, the price will be a function of both θ and ϕ_G. This implies that the government has no private information beyond what is reflected in the price. Then, if there is an equilibrium, it must be one with fully revealing prices. This is because, if, for a given ϕ_G, there is an equilibrium in which two fundamentals θ_1 and $\theta_2 \neq \theta_1$ are associated with the same price, then, since the government has no information beyond what is reflected in the price, its intervention policy must be the same at θ_1 and θ_2. But then the prices are not equal, giving a contradiction. By time consistency, we get that, if there is an equilibrium, it must exhibit optimal intervention. Thus, the suboptimal intervention equilibria when $2\kappa < T$ are ruled out. The intuition is that these equilibria were based on the fact that the market does not know the government's action, a problem that is solved once the government discloses its signal truthfully.

To complete the argument, we need to show that fully revealing equilibrium is possible for every ϕ_G, when $2\kappa < T$, but not when $2\kappa > T$. The reason for that is simple. Once ϕ_G is disclosed, the range of fundamentals consistent with it is between $\phi_G - \kappa$ and $\phi_G + \kappa$, i.e., it has a size of 2κ. A fully revealing equilibrium requires that each fundamental in this range is associated with a different price, while optimal intervention implies that fundamentals at distance T from each other may have the same price. This might generate a contradiction when $2\kappa > T$, because then there are fundamentals in the relevant range at distance T from each other, but not when $2\kappa < T$, because then all fundamentals in the relevant range are at distance smaller than T from each other.

Of course, transparency raises other issues that are beyond the scope of this paper. For example, if a bank knows that the government will make its information public, it may be less inclined to grant easy access to the government in the first place. In this sense, it is possible that transparency would serve to increase κ, potentially making the government's inference problem worse instead of better.

5.3 Government security

Suppose that government intervention is a publicly observed event and that, in addition to regular bank securities that split up the bank's return (as discussed so far), the market trades a security that pays 1 if the government intervenes and 0 otherwise. Let $Q(\theta)$ be the price of this security. In equilibrium, the price of the security must satisfy

$$Q(\theta) = I(P(\theta), Q(\theta), \theta). \tag{14}$$

In addition, the market trades one bank security (equity or debt).

The following proposition shows that when a government security is traded in addition to a regular bank security, then optimal intervention constitutes the only equilibrium of the model, no matter what the quality of the government's information is, i.e., for all κ.

Proposition 10 *If the market trades a bank security and the government security, then the unique equilibrium of the model features optimal intervention.*

The intuition behind this result is the following: a regular bank security may have the same price for different fundamentals because the probability of intervention is different across these fundamentals. But, once the government security is traded, the probability of intervention can be inferred from its price, and thus the fundamental can be inferred from the combination of its price and the prices of regular bank securities. This implies that the government will choose optimal intervention in equilibrium.

Note that the government security specified here is not a standard security, i.e., it is not a security typically traded in financial markets. This is because the price of this security is not linked to the value that goes to any class of bank claim holders, rather it just aggregates information regarding whether the government is going to intervene in the bank or not.[14] The result in Proposition 10 implies that issuing such a unique security is a very powerful tool: with this security in place, the unique equilibrium of the model entails optimal intervention for all κ.

5.4 Taxation

The last policy tool we will consider here is taxation. Suppose that there is one security, whose value absent intervention is $X_i(\theta)$ and with intervention is $X_i(\theta + T)$. The problem in implementing an optimal intervention policy by learning from the price of this security is that the value of the security under optimal intervention is not monotonic in θ. In principle, the government could restore monotonicity by taxing the investors who hold this

[14] In this sense, it resembles securities traded in precition markets, which try to predict the probability of an event.

security in case it intervenes. A simple way to do this is to tax the investors at the amount of $X_i(\theta + T) - X_i(\theta)$ after intervention has occurred and the value has been realized. This will ensure that the price of the security in the market is $X_i(\theta)$, which is monotone in θ, and will enable the government to learn the fundamentals perfectly from the price and to intervene optimally.

Another issue is that the optimal intervention policy will change once taxation becomes possible. So far, we assumed that the government's optimal policy minimizes the payments the government needs to make on intervention and bail out. If the government can impose some of these payments on investors by taxing them, then this optimal policy will change. In fact, the government will then be able to implement the socially optimal policy by simply letting every agent pay for what he gains from intervention. This issue is not unique to our model, and thus is not developed here in more detail.

5.5 Summary

We analyzed four policy measures that improve the ability of the government to use market information for bank supervision. Two of these measures – learning from multiple securities and disclosing the government's information – resolve the problem of multiple equilibria when the government's information is moderately precise, but do not solve the problem of no equilibrium when the government's information is imprecise. The other two – issuing a government security and taxation – solve both problems. As we noted, these policy measures raise other issues that are beyond the scope of this paper.

6 Conclusions

We study a rational-expectations model of governmental supervision of banks based on the market prices of bank securities. Because prices reflect bank fundamentals and expectations of government actions at the same time, the government cannot always extract information from the price to make an efficient intervention decision. The ability of the government to extract such information depends on the gap between the information available to the market and the information available to the government. When the gap is large, our model has no rational-expectations equilibrium. When the gap is moderate there are multiple equilibria, some of which exhibit sub-optimal intervention. When the gap is small, there is a unique equilibrium with optimal intervention. We discuss measures that improve the ability of the government to make use of market prices. These include improving transparency on the side of the government, expanding the set of securities, and taxing investors.

Overall, we believe that the analysis in this paper is relevant for many situations, in which the government, or other agents, take an action that is based on market prices and that affects market prices at the same time. A general insight is that the information from the market should complement, rather than completely substitute information held by acting agents. It is important to note that the nature of the analysis stems from the fact that under the optimal intervention policy (from the point of view of the agent who takes the intervention decision), the price function of the traded security is non monotone (see Figure 1). Thus, other applications of the analysis in this paper have to include this feature as well.

7 Mathematical appendix

Proof of Lemma 1: Observe first that

$$\begin{aligned} \Gamma'(\theta) &= -\left(G(D+T-\theta) - G(T-\theta)\right) < 0. \\ \Gamma''(\theta) &= g(D+T-\theta) - g(T-\theta). \end{aligned}$$

This implies that $-\Gamma'(\theta)$ is single-peaked since (given that g is itself single-peaked) $-\Gamma''(\theta)$ is negative if and only if $D+T-\theta$ is below some cutoff level, i.e., if and only if θ is above some cutoff level. Second, since

$$V'(\theta) = -\Gamma'(\theta+T) - \left(-\Gamma'(\theta)\right)$$

it follows that V is itself single-peaked, since $V'(\theta)$ is first positive and then negative as a function of θ. ∎

Proof of Proposition 1: In an optimal-intervention equilibrium there is a cutoff fundamental, $\hat{\theta}$, for which the government only intervenes for $\theta \leq \hat{\theta}$. What we need to check is that this policy is feasible. Under this intervention policy for $i = B, E$, $P_i(\theta) = X_i(\theta) + U_i(\theta)$ for $\theta \leq \hat{\theta}$, and $P_i(\theta) = X_i(\theta)$ for $\theta > \hat{\theta}$. As such, there are at most two fundamentals related to each price. For prices that are related to just one fundamental, the equilibrium price is trivially fully revealing. For prices that are related to two fundamentals, e.g., $\theta_1 < \widehat{\theta} < \theta_2$,

$$X_i(\theta_1) + U_i(\theta_1) = X_i(\theta_2).$$

But from (4), this means that $X_i(\theta_1 + T) = X_i(\theta_2)$, and thus $\theta_2 = \theta_1 + T$. Since $T > 2\kappa$, the government can distinguish between θ_1 and θ_2 with its own signal and follow its optimal intervention rule. ∎

Proof of Proposition 2: We prove this for the case in which X is convex. The X concave case follows similarly.

We know that there is an equilibrium in which the government intervenes with probability 1 when $\theta < \hat{\theta}$ and probability 0 when $\theta > \hat{\theta}$. We will show that this is the only equilibrium. Suppose to the contrary that there exists an equilibrium in which for some $\theta^* \neq \hat{\theta}$ the intervention probability at θ^* is neither 0 nor 1. Let p be the associated price, and let Θ be the set of fundamentals associated with this price.

As a preliminary, observe that if $\theta < X^{-1}\left(\hat{\theta}\right)$ then the price reveals that the fundamental is below $\hat{\theta}$ and intervention always occurs; while if $\theta > X^{-1}\left(\hat{\theta} + T\right)$ the price reveals that the fundamental is above $\hat{\theta}$ and intervention never occurs. As such, we know that $\theta^* \in \left[X^{-1}\left(\hat{\theta}\right), X^{-1}\left(\hat{\theta} + T\right)\right]$. Define $\bar{\kappa} > 0$ by $\bar{\kappa} = \min\left\{\frac{U(\tilde{\theta})}{2X'(\tilde{\theta}+T)} : \tilde{\theta} \in \left[X^{-1}\left(\hat{\theta}\right), X^{-1}\left(\hat{\theta} + T\right)\right]\right\}$.

The government intervenes after observing the price p if and only if it sees a signal $\phi_G \leq \phi$, where

$$\phi \in \{\theta + \kappa | \theta \in \Theta\} \cup \{\theta - \kappa | \theta \in \Theta\}.$$

The intervention probability at $\theta \in \Theta$ is $\Pr(\theta + \xi \leq \phi) = \min\left\{\max\left\{\frac{\phi - \theta + \kappa}{2\kappa}, 0\right\}, 1\right\}$.

Suppose first that $\phi = \theta_0 - \kappa$, some $\theta_0 \in \Theta$. So the intervention probability for any $\theta \in \Theta$ such that $\theta \geq \theta_0$ is 0. So in particular the intervention probability at θ_0 is zero, and all members of Θ fall below θ_0. Since the intervention probability at θ^* lies in $(0, 1)$ it follows that $\theta^* > \theta_0 - 2\kappa$. Since θ^* and θ_0 are associated with the same price,

$$X(\theta^*) + \left(\frac{\theta_0 - \kappa - \theta^* + \kappa}{2\kappa}\right) U(\theta^*) - X(\theta_0) = 0.$$

However, the function $Z(\theta) = X(\theta^*) + \left(\frac{\theta - \kappa - \theta^* + \kappa}{2\kappa}\right) U(\theta^*) - X(\theta)$ is concave, zero at $\theta = \theta^*$, and equal to $X(\theta^*) + U(\theta^*) - X(\theta^* + 2\kappa)$ at $\theta = \theta^* + 2\kappa$. This expression is positive since $2\kappa < T$. But then $Z(\theta)$ must be strictly positive for all $\theta \in (\theta^*, \theta^* + 2\kappa)$, contradicting the existence of θ_0.

Second, suppose that $\phi = \theta_0 + \kappa$. So the intervention probability for any $\theta \in \Theta$ such that $\theta \leq \theta_0$ is 1. So in particular the intervention probability at θ_0 is 1, and all members of Θ lie above θ_0. Since the intervention probability at θ^* lies in $(0, 1)$ it follows that $\theta^* < \theta_0 + 2\kappa$. Since θ^* and θ_0 are associated with the same price,

$$X(\theta^*) + \left(\frac{\theta_0 + \kappa - \theta^* + \kappa}{2\kappa}\right) U(\theta^*) - X(\theta_0) - U(\theta_0) = 0.$$

The function $Z(\theta) = X(\theta^*) + \left(\frac{\theta + \kappa - \theta^* + \kappa}{2\kappa}\right) U(\theta^*) - X(\theta) - U(\theta)$ is concave, zero at $\theta = \theta^*$, and equal to $X(\theta^*) - X(\theta^* - 2\kappa) - U(\theta^* - 2\kappa)$ at $\theta = \theta^* - 2\kappa$. This expression is negative since $2\kappa < T$. To complete the proof, we claim that Z is strictly increasing over the interval: for if this is the case, then Z is strictly negative for all $\theta \in (\theta^* - 2\kappa, \theta^*)$, contradicting the existence of θ_0.

Since Z is strictly concave, it suffices to show that the derivative of Z at $\theta = \theta^*$ is positive, i.e., $\frac{U(\theta^*)}{2\kappa} - X'(\theta^* + T) > 0$. This holds provided that $\kappa < \bar{\kappa}$, where $\bar{\kappa}$ is as defined above. ∎

Proof of Lemma 2: Define the function

$$\begin{aligned} Z(\theta_1, \theta_2) &= X_E(\theta_2) + \left(1 - \frac{\theta_2 - \theta_1}{2\kappa}\right) U_E(\theta_2) - X_E(\theta_1) - U_E(\theta_1) \\ &= X_E(\theta_2) + \left(1 - \frac{\theta_2 - \theta_1}{2\kappa}\right) U_E(\theta_2) - X_E(\theta_1 + T), \end{aligned}$$

where $\theta_2 \geq \theta_1$. Intuitively, this is the difference between the price at fundamental θ_2 given an intervention probability $1 - \frac{\theta_2 - \theta_1}{2\kappa}$, and the price at fundamental θ_1 given an intervention probability 1.

Observe that Z has the following properties:

$$\begin{aligned} Z_{11}(\theta_1,\theta_2) &< 0, \\ Z_{12}(\theta_1,\theta_2) &= \frac{U_E'(\theta_2)}{2\kappa} > 0, \\ Z(\theta,\theta) &= 0. \end{aligned}$$

Moreover, for any θ,

$$Z(\theta-2\kappa,\theta) = X_E(\theta) - X_E(\theta-2\kappa+T) < 0.$$

We know that $Z\left(\hat{\theta}-2\kappa,\hat{\theta}\right) < 0$ and $Z\left(\hat{\theta},\hat{\theta}\right) = 0$. Since $Z_{11} < 0$, the result follows provided $Z_1\left(\hat{\theta},\hat{\theta}\right) < 0$. We know that,

$$\begin{aligned} Z_1\left(\hat{\theta},\hat{\theta}\right) &= \frac{U_E\left(\hat{\theta}\right)}{2\kappa} - X_E'\left(\hat{\theta}+T\right) \\ &= \frac{X_E\left(\hat{\theta}+T\right) - X_E\left(\hat{\theta}\right)}{2\kappa} - X_E'\left(\hat{\theta}+T\right) \\ &= \frac{1}{2\kappa}\int_{\theta=\hat{\theta}}^{\hat{\theta}+T}\left(X_E'(\theta) - \frac{2\kappa}{T}X_E'\left(\hat{\theta}+T\right)\right)d\theta. \end{aligned}$$

Since X_E is a convex function, $Z_1\left(\hat{\theta},\hat{\theta}\right) < 0$ for all 2κ close enough to T. ∎

Proof of Proposition 3: Define Z as in the proof of Lemma 2. Observe first that since $Z\left(\theta_{01},\hat{\theta}\right) = Z\left(\hat{\theta},\hat{\theta}\right) = 0$, and $Z_{11} < 0$, then $Z\left(\theta_1,\hat{\theta}\right) > 0$ for any $\theta_1 \in \left(\theta_{01},\hat{\theta}\right)$. Moreover, $Z\left(\cdot,\hat{\theta}\right)$ is single-peaked. Let $\hat{\theta}_{11} \in \left(\theta_{01},\hat{\theta}\right)$ be its maximum. Since for any $\theta_1 \in \left(\theta_{01},\hat{\theta}_{11}\right)$, $Z\left(\theta_1,\hat{\theta}\right) > 0$ and $Z(\theta_1,\theta_1+2\kappa) < 0$, by continuity there exists some $\theta_2 > \hat{\theta}$, for which $Z(\theta_1,\theta_2) = 0$. We define a function, $\theta_2^*(\theta_1)$, where θ_2^* is the smallest θ_2, above $\hat{\theta}$, for which $Z(\theta_1,\theta_2) = 0$. Economically, $\theta_2^*(\theta_1)$ is the fundamental which has the same market price as θ_1. We know that $\theta_2^*(\theta_{01}) = \hat{\theta}$.

We know $\theta_2^*(\theta_1)$ is a strictly increasing function. To see this, note that

$$Z(\theta_1,\theta_2) = Z\left(\theta_1,\hat{\theta}\right) + \int_{\hat{\theta}}^{\theta_2} Z_2(\theta_1,y)\,dy.$$

Since $Z\left(\theta_1,\hat{\theta}\right)$ is increasing over the range $[\theta_{01},\hat{\theta}_{11}]$, and $Z_{12} > 0$, it follows that for any $\theta_2 \geq \hat{\theta}$, $Z(\theta_1,\theta_2)$ is increasing in θ_1 over $[\theta_{01},\hat{\theta}_{11}]$. Thus, the smallest θ_2, at which

$Z(\theta_1, \theta_2) = 0$, is strictly increasing in θ_1, implying that $\theta_2^*(\theta_1)$ is a strictly increasing function.

Since $\theta_2^*(\theta_{01}) = \hat{\theta}$, the function $\frac{V(\theta_1)+V(\theta_2^*(\theta_1))}{2}$ is strictly positive at $\theta_1 = \hat{\theta}$. Define θ_{11} as the minimum of $\hat{\theta}_{11}$ and the infimum value such that $\frac{V(\theta_1)+V(\theta_2^*(\theta_1))}{2} \leq C$. As such, $\theta_2^*(\cdot)$ is increasing and $\frac{V(\cdot)+V(\theta_2^*(\cdot))}{2} > C$ over the interval $[\theta_{01}, \theta_{11}]$.

We have now defined the values θ_{01} and θ_{11} of the proposition statement. It remains to show that there is an equilibrium of the type described. This requires showing that the prices are rational given the intervention probabilities, and that the intervention probabilities result from the government's optimal behavior given the information in the price and its own private signal. It is immediate to show that the prices in the proposition statement are rational given the corresponding intervention probabilities. Thus, we turn to show that the intervention probabilities result from the government's optimal behavior. We will do this by analyzing different ranges of the fundamentals separately.

For a fundamental $\theta \leq \hat{\theta}$ and $\theta \notin Y_1$, the price is $X_E(\theta) + U_E(\theta) = X_E(\theta + T)$. The same price may be observed at the fundamental $\theta + T$. Since $2\kappa < T$, the government's private signal will indicate for sure that the fundamental is θ and not $\theta + T$. Hence, the government will optimally choose to intervene, generating intervention probability of 1, as is stated in the proposition. Note that the same price cannot be observed at any fundamental below $\theta + T$. Observing such a price at a fundamental below $\theta + T$ would imply that the fundamental belongs to the set $\theta_2^*(Y_1)$, but this contradicts the fact that $\theta \notin Y_1$.

For a fundamental $\theta \leq \hat{\theta}$ and $\theta \in Y_1$, the price is again $X_E(\theta) + U_E(\theta)$. As before, the same price may be observed at the fundamental $\theta + T$ without having an effect on the decision of the government to intervene at θ, given that $2\kappa < T$. Here, however, the same price will also be observed at the fundamental $\theta_2^*(\theta)$. This is because the fundamental $\theta_2^*(\theta) \in \theta_2^*(Y_1)$ generates a price of $X_E(\theta_2^*(\theta)) + \left(1 - \frac{\theta_2^*(\theta)-\theta}{2\kappa}\right) U_E(\theta_2^*(\theta))$, which by construction is equal to $X_E(\theta) + U_E(\theta)$. (Note that the same price will not be observed at any other fundamental in the set $\theta_2^*(Y_1)$, since $X_E(\theta) + U_E(\theta)$ and $\theta_2^*(\theta)$ are strictly increasing in θ.) Thus, at the fundamental θ, the government observes a price that is consistent with both θ and $\theta_2^*(\theta)$, and may observe a private signal that is also consistent with both of them. If this happens, given the uniform distribution of noise in the government's signal, the government will intervene as long as $\frac{V(\theta)+V(\theta_2^*(\theta))}{2} \geq C$. By construction, this is true for all $\theta \in Y_1$, and thus, at the fundamental θ, the government will intervene with probability 1, as is stated in the proposition.

For a fundamental $\theta > \hat{\theta}$ and $\theta \notin \theta_2^*(Y_1)$, the price is $X_e(\theta) = X_E(\theta - T) + U_E(\theta - T)$. The same price may be observed at the fundamental $\theta - T$ and also at some $\theta' > \hat{\theta}$; $\theta' \in \theta_2^*(Y_1)$. Since $2\kappa < T$, the government's private signal at the fundamental θ will indicate for sure that the fundamental is not $\theta - T$. Hence, the government will know that the fundamental is above $\hat{\theta}$, and will optimally choose not to intervene, generating intervention probability of 0, as is stated in the proposition.

Finally, for a fundamental $\theta > \hat{\theta}$ and $\theta \in \theta_2^*(Y_1)$, the price is $X_E(\theta) + \left(1 - \frac{\theta - \theta_2^{*-1}(\theta)}{2\kappa}\right) U_E(\theta)$. As follows from the arguments above, the same price will be observed at the fundamental $\theta_2^{*-1}(\theta)$, and also may be observed at some fundamental $\theta'' > \hat{\theta}$; $\theta'' \notin \theta_2^*(Y_1)$. (As argued before, two fundamentals in the set $\theta_2^*(Y_1)$ cannot have the same price.) As also follows from the arguments above, the government will optimally choose to intervene if and only if its signal is consistent with both θ and $\theta_2^{*-1}(\theta)$ (the signal cannot be consistent with both $\theta_2^{*-1}(\theta)$ and θ''). Due to the uniform distribution of noise in the government's signal, this generates an intervention probability of $1 - \frac{\theta - \theta_2^{*-1}(\theta)}{2\kappa}$, as is stated in the proposition. ∎

Proof of Proposition 4: Suppose to the contrary that there exists an equilibrium that is not the essentially fully revealing equilibrium, and such that the probability of intervention for all $\theta > \hat{\theta}$ is 0.

Since the equilibrium is not essentially fully revealing, there must exist a set of fundamentals Θ such that the price is the same for all members of Θ, and such that at least two members of the set lie within 2κ of each other. Since the intervention probability is 0 for all $\theta > \hat{\theta}$, there must exist $\theta_1 \in \Theta$ such that $\theta_1 \leq \hat{\theta}$ and such the intervention probability is strictly positive at θ_1.[15] Moreover, we claim that there exists $\theta_2 \in \Theta$ such that $\theta_2 > \hat{\theta}$: for if instead $\sup \Theta \leq \hat{\theta}$ then the intervention probability for any $\theta \in \Theta$ would be one, contradicting the fact that the price is the same for all members of the set. Since the price

[15] Note that $\theta_1 \geq \hat{\theta} - T$.

at all members of Θ is the same, and the intervention at any $\theta \in \Theta$ such that $\theta > \hat{\theta}$ is zero (by hypothesis), θ_2 is the unique member of Θ above $\hat{\theta}$.

Conditional on seeing the common price for fundamentals in Θ, when does the government intervene? After observing its signal ϕ_G, it regards some subset of fundamentals $\Theta(\phi_G) \subset \Theta$ as possible. Given our uniform distribution assumptions, it assigns equal weight to all members of $\Theta(\phi_G)$. It intervenes if and only if the average value of $V(\theta) - C$ over members of $\Theta(\phi_G)$ is non-negative. Observe that $\Theta(\phi_G)$ is increasing in ϕ_G, in the sense that if $\phi'_G > \phi_G$ then $\inf \Theta(\phi'_G) \geq \inf \Theta(\phi_G)$ and $\sup \Theta(\phi'_G) \geq \sup \Theta(\phi_G)$. It follows that the government intervenes if and only if $\phi_G \leq \phi_0$, for some ϕ_0. Moreover, either $\phi_0 = \theta_0 + \kappa$ for some $\theta_0 \in \Theta$, or $\phi_0 = \theta_0 - \kappa$ for some $\theta_0 \in \Theta$. We consider these two cases in turn, and show that both lead to a contradiction.

First, suppose that $\phi_0 = \theta_0 + \kappa$. The probability of intervention at θ_0 is thus one, and so the price at θ_0 is $X(\theta_0) + U(\theta_0) = X(\theta_0 + T)$. The probability of intervention at θ_2 is 0, and so $\theta_2 = \theta_0 + T > \theta_0 + 2\kappa$. By hypothesis the equilibrium is not the fully revealing equilibrium, and so there exists at least one further member of Θ, i.e., $\theta \in \Theta \backslash \{\theta_0, \theta_2\}$. Certainly $\theta \in (\theta_0, \hat{\theta}] \subset (\theta_0, \theta_2)$. As such, there exists a realization of $\xi \in [-\kappa, \kappa]$ such that $\theta + \xi \in (\theta_0 + \kappa, \theta_2 - \kappa)$. At the signal $\phi_G = \theta + \xi$ the government does not intervene. But this gives a contradiction, since at the signal $\phi_G = \theta + \xi$ the government knows that the fundamental lies in $\Theta \backslash \{\theta_2\}$, and all members of this set lie below $\hat{\theta}$.

Second, suppose that $\phi_0 = \theta_0 - \kappa$, some $\theta_0 \in \Theta$. Since the probability of intervention at θ_0 is zero, $\theta_0 = \theta_2$. The probability of intervention at θ_1 is $\Pr(\theta_1 + \xi \leq \theta_2 - \kappa) = \min\left\{\frac{\theta_2 - \theta_1}{2\kappa}, 1\right\}$ (by assumption this is strictly positive). Since the price is the same at θ_1 and θ_2, the expression

$$X(\theta_1) + \min\left\{\frac{\theta_2 - \theta_1}{2\kappa}, 1\right\} U(\theta_1) - X(\theta_2)$$

must equal 0. Observe that if $\theta_2 = \theta_1$ the expression is zero, if $\theta_2 = \theta_1 + 2\kappa$ the expression is $X(\theta_1) + U(\theta_1) - X(\theta_1 + 2\kappa) > 0$ (since $T > 2\kappa$), and the expression is concave in θ_2. As such, it is strictly positive for all values of θ_2 in $(\theta_1, \theta_1 + 2\kappa]$, and so the only possibility is that $\theta_2 > \theta_1 + 2\kappa$. However, in this case the government can perfectly distinguish θ_1 from θ_2, and intervenes with probability 1 at θ_1. Since θ_1 was chosen as an arbitrary member of

Θ below $\hat{\theta}$, this implies that $\Theta = \{\theta_2 - T, \theta_2\}$, and the equilibrium is fully revealing. This contradicts the hypothesis, and completes the proof. ∎

Proof of Proposition 5: The proof is omitted, and is available from the authors. It is parallel to the proof of Proposition 3. ∎

Proof of Proposition 6: The proof is omitted, and is available from the authors. It is parallel to the proof of Proposition 4. ∎

Proof of Proposition 7: We start by showing the non existence of equilibrium for the case of a concave security.

Suppose to the contrary that an equilibrium exists. Let $P(\cdot)$ be the equilibrium price function. We know that there cannot be a fully-revealing equilibrium (see the main text immediately prior to the proposition statement). Define Θ^* to be the non-empty set of fundamentals at which the price is not fully-revealing, i.e.,

$$\Theta^* = \left\{\theta : \exists \theta' \neq \theta \text{ such that } P(\theta) = P(\theta')\right\}.$$

Given Θ^*, define $\theta^* = \sup \Theta^*$. We prove the following claims.

Claim 1: If two fundamentals θ' and θ'' have the same price, i.e., $P(\theta') = P(\theta'')$, then $|\theta' - \theta''| \leq T < 2\kappa$.

Proof of Claim 1: Let θ' and $\theta'' > \theta'$ be two fundamentals with the same price. We know that $P(\theta'') \geq X(\theta'')$ and

$$X(\theta' + T) = X(\theta') + U(\theta') \geq X(\theta') + E(I|\theta')\, U(\theta') = P(\theta').$$

So if $\theta'' > \theta' + T$ then $P(\theta'') > X(\theta' + T) \geq P(\theta')$, a contradiction. Thus $\theta'' \leq \theta' + T$.

Claim 2: If $\theta > \max\left\{\theta^*, \hat{\theta}\right\}$ then $P(\theta) = X(\theta)$; and if $\theta \leq \max\left\{\theta^*, \hat{\theta}\right\}$ then $P(\theta) \leq X\left(\max\left\{\theta^*, \hat{\theta}\right\}\right)$.

Proof of Claim 2: By definition, if $\theta > \theta^*$ the price is fully-revealing. So if $\theta > \hat{\theta}$ also, the government does not intervene, and $P(\theta) = X(\theta)$. So for any $\theta \in \left(\max\left\{\theta^*, \hat{\theta}\right\}, \infty\right)$, the price is $X(\theta)$.

Next, suppose that contrary to the claim $P(\theta') > X\left(\max\left\{\theta^*, \hat{\theta}\right\}\right)$ for some $\theta' \leq \max\left\{\theta^*, \hat{\theta}\right\}$. But then there exists $\theta > \max\left\{\theta^*, \hat{\theta}\right\} \geq \theta^*$ such that $P(\theta) = P(\theta')$, contradicting the fact that $\theta^* = \sup \Theta^*$. This completes the proof of Claim 2.

Claim 3: $\theta^* > \hat{\theta}$.

Proof of Claim 3: Suppose to the contrary that $\theta^* \leq \hat{\theta}$, so that $\max\left\{\theta^*, \hat{\theta}\right\} = \hat{\theta}$. By Claim 2, $P(\theta) = X(\theta)$ if $\theta > \hat{\theta}$, and $P(\theta) \leq X\left(\hat{\theta}\right)$ for $\theta \leq \hat{\theta}$. As such, whenever the true fundamental is strictly below $\hat{\theta}$ the government knows either that the fundamental is strictly below $\hat{\theta}$; or that the fundamental is either strictly below $\hat{\theta}$ or equal to $\hat{\theta}$, with a positive probability of both. So the government intervenes with probability one for any $\theta < \hat{\theta}$. But then the price is not below $X\left(\hat{\theta}\right)$ for any θ close to $\hat{\theta}$. This contradiction completes the proof of the Claim 3.

Claim 4: $P(\theta^*) = X(\theta^*)$, and so $E(I|\theta^*) = 0$.

Proof of Claim 4: From Claims 2 and 3, $P(\theta) \leq X(\theta^*)$ for $\theta \leq \theta^*$. The claim follows since certainly $P(\theta^*) \geq X(\theta^*)$.

Now, consider first the case where $\theta^* \in \Theta^*$. By construction, there exists a fundamental $\theta' < \theta^*$ such that: $P\left(\theta'\right) = X\left(\theta'\right) + E\left(I|\theta'\right) U\left(\theta'\right) = X(\theta^*)$. Since $E(I|\theta^*) = 0$, the government does not intervene at signals above $\theta^* - \kappa$. Thus, $E\left(I|\theta'\right) \leq \Pr\left(\theta' + \xi \leq \theta^* - \kappa\right) = \frac{\theta^* - \theta'}{2\kappa}$. Define the function $Z\left(\theta', \theta^*\right)$ as follows:

$$Z\left(\theta', \theta^*\right) \equiv X\left(\theta'\right) + U\left(\theta'\right)\frac{\theta^* - \theta'}{2\kappa} - X(\theta^*).$$

By the above arguments, in the proposed equilibrium, $Z\left(\theta', \theta^*\right) \geq 0$. We know that $Z\left(\theta', \theta'\right) = 0$, and that $Z\left(\theta', \theta' + 2\kappa\right) = X\left(\theta' + T\right) - X\left(\theta' + 2\kappa\right) < 0$. Since the security is concave, $Z_{22} > 0$. Thus, there are no θ' and $\theta^* \in \left(\theta', \theta' + 2\kappa\right)$ for which $Z\left(\theta', \theta^*\right) \geq 0$. This is a contradiction to the proposed equilibrium.

Suppose now that $\theta^* \notin \Theta^*$. There exists some sequence $(\theta_i)_{i=0}^{\infty} \subset \Theta^*$ that converges to θ^*. Moreover, by Claims 2, 3, and 4, $E(I|\theta_i) \to 0$ as $i \to \infty$: for if this is not true, there is a $\theta_i \leq \theta^*$ at which the price is above $X(\theta^*)$. For each θ_i in this sequence there exists at least one fundamental, θ_i' say, at which the price is the same and which lies to the left of $\hat{\theta}$. (If instead all fundamentals with price $P(\theta_i)$ were to the right of $\hat{\theta}$, no intervention would occur, and they could not have the same price.) So $X\left(\theta_i'\right) + E\left(I|\theta_i'\right) U\left(\theta'\right) = X(\theta_i) + E(I|\theta_i) U(\theta_i)$. Note that $\theta_i - \theta_i'$ is bounded away from 0 as $i \to \infty$ since $\theta_i \to \theta^* > \hat{\theta}$.

We know that

$$
\begin{aligned}
E\left(I|\theta_i'\right) &= \int_{\theta_i'-\kappa}^{\theta_i'+\kappa} I\left(P\left(\theta_i\right),\phi_G\right)\frac{1}{2\kappa}d\phi_G \\
&\leq \int_{\theta_i'-\kappa}^{\theta_i-\kappa} I\left(P\left(\theta_i\right),\phi_G\right)\frac{1}{2\kappa}d\phi_G + \int_{\theta_i-\kappa}^{\theta_i+\kappa} I\left(P\left(\theta_i\right),\phi_G\right)\frac{1}{2\kappa}d\phi_G \\
&\leq \frac{\theta_i-\theta_i'}{2\kappa} + E\left(I|\theta_i\right).
\end{aligned}
$$

Define the function $Z\left(\theta_i',\theta_i\right)$ as follows:

$$
Z\left(\theta_i',\theta_i\right) \equiv X\left(\theta_i'\right) + U\left(\theta_i'\right)\frac{\theta_i-\theta_i'}{2\kappa} - X\left(\theta_i\right) + E\left(I|\theta_i\right)\left(U\left(\theta_i'\right) - U\left(\theta_i\right)\right).
$$

By the above arguments, in the proposed equilibrium, $Z\left(\theta_i',\theta_i\right) \geq 0$. We use ε_i to denote $E\left(I|\theta_i\right)\left(U\left(\theta_i'\right) - U\left(\theta_i\right)\right)$. We know that ε approaches 0 (the value of intervention, $U\left(\theta\right)$, is bounded above by T). We know that $Z\left(\theta_i',\theta_i'\right) = \varepsilon_i$, and that $Z\left(\theta_i',\theta_i'+2\kappa\right) = X\left(\theta_i'+T\right) - X\left(\theta_i'+2\kappa\right) + \varepsilon_i < 0$ for all i large enough. Since the security is concave, $Z_{22} > 0$. Thus, for any θ_i between θ_i' and $\theta_i'+2\kappa$, $Z\left(\theta_i',\theta_i\right) \leq \varepsilon_i + \frac{(\theta_i-\theta_i')(X(\theta_i'+T)-X(\theta_i'+2\kappa))}{2\kappa}$. This implies that $Z\left(\theta_i',\theta_i\right) \geq 0$ can hold only if $\theta_i' \leq \theta_i \leq \theta_i' + \frac{2\kappa\varepsilon_i}{X(\theta_i'+2\kappa)-X(\theta_i'+T)}$. Then, since ε_i approaches 0, there are no θ_i' and θ_i that are bounded away from each other, for which $Z\left(\theta_i',\theta_i\right) \geq 0$. This is a contradiction to the proposed equilibrium.

Now, suppose that the security is convex. Define $\theta^{**} = \inf\Theta^*$. We prove the following claims.

Claim 2a: If $\theta < \min\left\{\theta^{**},\hat{\theta}\right\}$ then $P\left(\theta\right) = X\left(\theta+T\right)$; and if $\theta \geq \min\left\{\theta^{**},\hat{\theta}\right\}$ then $P\left(\theta\right) \geq X\left(\min\left\{\theta^{**},\hat{\theta}\right\}+T\right)$.

Proof of Claim 2a: By definition, if $\theta < \theta^{**}$ the price is fully-revealing. So if $\theta < \hat{\theta}$ also, the government intervenes, and $P\left(\theta\right) = X\left(\theta+T\right)$. So for any $\theta \in \left(-\infty,\min\left\{\theta^{**},\hat{\theta}\right\}\right)$, the price is $X\left(\theta+T\right)$.

Next, suppose that contrary to the claim $P\left(\theta'\right) < X\left(\min\left\{\theta^{**},\hat{\theta}\right\}+T\right)$ for some $\theta' \geq \min\left\{\theta^{**},\hat{\theta}\right\}$. But then there exists $\theta < \min\left\{\theta^{**},\hat{\theta}\right\} \leq \theta^{**}$ such that $P\left(\theta\right) = P\left(\theta'\right)$, contradicting the fact that $\theta^{**} = \inf\Theta^*$. This completes the proof of Claim 2a.

Claim 3a: $\theta^{**} < \hat{\theta}$.

Proof of Claim 3a: Suppose to the contrary that $\theta^{**} \geq \hat{\theta}$, and so $\min\left\{\theta^{**},\hat{\theta}\right\} = \hat{\theta}$. By Claim 2a, $P\left(\theta\right) = X\left(\theta+T\right)$ if $\theta < \hat{\theta}$, and $P\left(\theta\right) \geq X\left(\hat{\theta}+T\right)$ for $\theta \geq \hat{\theta}$. As such, whenever the true fundamental is strictly above $\hat{\theta}$ the government knows either that the fundamental is strictly above $\hat{\theta}$; or that the fundamental is either strictly above $\hat{\theta}$ or equal to $\hat{\theta}$, with a positive probability of both. So the government intervenes with probability zero for any $\theta > \hat{\theta}$. But then the price is not above $X\left(\hat{\theta}+T\right)$ for any θ close to $\hat{\theta}$. This contradiction completes the proof of the Claim 3a.

Claim 4a: $P\left(\theta^{**}\right) = X\left(\theta^{**}+T\right)$, and so $E\left(I|\theta^{**}\right) = 1$.

Proof of Claim 4a: From Claims 2a and 3a, $P\left(\theta\right) \geq X\left(\theta^{**}+T\right)$ for $\theta \geq \theta^{**}$. The claim follows since certainly $P\left(\theta^{**}\right) \leq X\left(\theta^{**}+T\right)$.

Now, consider first the case where $\theta^{**} \in \Theta^{*}$. By construction, there exists a fundamental $\theta' > \theta^{**}$ such that: $P\left(\theta'\right) = X\left(\theta'\right) + E\left(I|\theta'\right)U\left(\theta'\right) = X\left(\theta^{**}+T\right)$. Since $E\left(I|\theta^{**}\right) = 1$, the government intervenes at signals below $\theta^{**}+\kappa$. Thus, $E\left(I|\theta'\right) \geq \Pr\left(\theta'+\xi \leq \theta^{**}+\kappa\right) = 1 - \frac{\theta'-\theta^{**}}{2\kappa}$. Define the function $Z\left(\theta',\theta^{**}\right)$ as follows:

$$Z\left(\theta',\theta^{**}\right) \equiv X\left(\theta'\right) + U\left(\theta'\right)\left(1 - \frac{\theta'-\theta^{**}}{2\kappa}\right) - X\left(\theta^{**}+T\right).$$

By the above arguments, in the proposed equilibrium, $Z\left(\theta',\theta^{**}\right) \leq 0$. We know that $Z\left(\theta',\theta'\right) = 0$, and that $Z\left(\theta',\theta'-2\kappa\right) = X\left(\theta'\right) - X\left(\theta'-2\kappa+T\right) > 0$. Since the security is convex, $Z_{22} < 0$. Thus, there are no θ' and $\theta^{**} \in \left(\theta'-2\kappa,\theta'\right)$ for which $Z\left(\theta',\theta^{**}\right) \leq 0$. This is a contradiction to the proposed equilibrium.

Suppose now that $\theta^{**} \notin \Theta^{*}$. There exists some sequence $\left(\theta_i\right)_{i=0}^{\infty} \subset \Theta^{*}$ that converges to θ^{**} (from above). Moreover, by Claims 2a, 3a, and 4a, $E\left(I|\theta_i\right) \to 1$ as $i \to \infty$: for if this is not true, there is a $\theta_i \geq \theta^{**}$ at which the price is below $X\left(\theta^{**}+T\right)$. For each θ_i in this sequence there exists at least one fundamental, θ_i' say, at which the price is the same and which lies to the right of $\hat{\theta}$. (If instead all fundamentals with price $P\left(\theta_i\right)$ were to the left of $\hat{\theta}$, intervention would occur with probability one, and they could not have the same price.) So $X\left(\theta_i'\right) + E\left(I|\theta_i'\right)U\left(\theta_i'\right) = X\left(\theta_i\right) + E\left(I|\theta_i\right)U\left(\theta_i\right)$. Note that $\left|\theta_i - \theta_i'\right|$ is bounded away from 0 as $i \to \infty$ since $\theta_i \to \theta^{**} < \hat{\theta}$. We know that

$$\begin{aligned} E\left(I|\theta_i'\right) &= \int_{\theta_i'-\kappa}^{\theta_i'+\kappa} I\left(P\left(\theta_i\right), \phi_G\right) \frac{1}{2\kappa} d\phi_G \\ &\geq \int_{\theta_i-\kappa}^{\theta_i+\kappa} I\left(P\left(\theta_i\right), \phi_G\right) \frac{1}{2\kappa} d\phi_G - \int_{\theta_i-\kappa}^{\theta_i'-\kappa} I\left(P\left(\theta_i\right), \phi_G\right) \frac{1}{2\kappa} d\phi_G \\ &\geq E\left(I|\theta_i\right) - \frac{\theta_i' - \theta_i}{2\kappa}. \end{aligned}$$

Define the function $Z\left(\theta_i', \theta_i\right)$ as follows:

$$\begin{aligned} Z\left(\theta_i', \theta_i\right) &\equiv X\left(\theta_i'\right) + U\left(\theta_i'\right)\left(E\left(I|\theta_i\right) - \frac{\theta_i' - \theta_i}{2\kappa}\right) - X\left(\theta_i\right) - E\left(I|\theta_i\right) U\left(\theta_i\right) \\ &= X\left(\theta_i'\right) + U\left(\theta_i'\right)\left(1 - \frac{\theta_i' - \theta_i}{2\kappa}\right) - X\left(\theta_i + T\right) + \left(E\left(I|\theta_i\right) - 1\right)\left(U\left(\theta_i'\right) - U\left(\theta_i\right)\right). \end{aligned}$$

By the above arguments, in the proposed equilibrium, $Z\left(\theta_i', \theta_i\right) \leq 0$. We use ε_i to denote $\left(E\left(I|\theta_i\right) - 1\right)\left(U\left(\theta_i'\right) - U\left(\theta_i\right)\right)$. We know that ε_i approaches 0. We know that $Z\left(\theta_i', \theta_i'\right) = \varepsilon_i$, and that $Z\left(\theta_i', \theta_i' - 2\kappa\right) = X\left(\theta_i'\right) - X\left(\theta_i' - 2\kappa + T\right) + \varepsilon_i > 0$ for i large. Since the security is convex $Z_{22} < 0$. Thus, for any θ_i between $\theta_i' - 2\kappa$ and θ_i', $Z\left(\theta_i', \theta_i\right) \geq \varepsilon_i + \frac{(\theta_i' - \theta_i)(X(\theta_i') - X(\theta_i' - 2\kappa + T))}{2\kappa}$. This implies that $Z\left(\theta_i', \theta_i\right) \leq 0$ can hold only if $\theta_i' + \frac{2\kappa\varepsilon_i}{X(\theta_i') - X(\theta_i' - 2\kappa + T)} \leq \theta_i \leq \theta_i'$. Then, since ε_i approaches 0, there are no θ_i' and θ_i that are bounded away from each other, for which $Z\left(\theta_i', \theta_i\right) \leq 0$. This is a contradiction to the proposed equilibrium. ∎

Proof of Proposition 8: Suppose to the contrary that there exists a pure-strategy equilibrium without optimal intervention. That is, there is a fundamental θ^* such that either $\theta^* < \hat{\theta}$ and $E[I|\theta^*] < 1$, or $\hat{\theta} > \theta^*$ and $E[I|\theta^*] > 0$. Time-consistency implies that in any equilibrium, for any $\theta < \hat{\theta}$ there is a strictly positive probability of intervention, because there is a positive probability of the government observing a signal strictly below $\hat{\theta} - \kappa$. Likewise, in any equilibrium the probability Of intervention is strictly less than unity for fundamentals $\theta > \hat{\theta}$. Thus $E[I|\theta^*] \in (0, 1)$.

Consider any security i. If $\theta^* + T < \hat{\theta}$ the price at θ^* certainly lies below $X_i\left(\hat{\theta}\right)$, and so the government can infer that $\theta^* < \hat{\theta}$. In this case, it will always intervene. Likewise, if $\theta^* > \hat{\theta} + T$ the price at θ^* certainly lies above $X_i\left(\theta^* + T\right)$, and so the government can infer that $\theta^* > \hat{\theta}$. In this case it will never intervene. Consequently, $\theta^* \in \left[\hat{\theta} - T, \hat{\theta} + T\right]$.

Let Θ be the set of all fundamentals associated with the same vector of security prices as θ^*. Define two points in Θ as *connected* if they lie within 2κ of each other. The connected subsets of Θ partition Θ. Let Θ^* be the connected subset containing θ^*. Note that Θ^* must have at least two members, since otherwise θ^* can be distinguished from all other fundamentals — in which case the intervention probability is either 0 or 1.

A pure-strategy equilibrium is one in which for any price vector and government signal ϕ, the intervention probability I is either 0 or 1. Time-consistency implies that for any price vector P the function $I(P, \cdot)$ is decreasing. Let P^* be the price vector associated with fundamental θ^*. Clearly $I(P^*, \cdot)$ cannot be constant over the interval $(\inf \Theta^* - \kappa, \sup \Theta^* + \kappa)$, for if it were then $E[I|\theta]$ would be identical for all members of Θ^*. So there must exist some ϕ^* such that $I(P^*, \phi) = 1$ if $\phi < \phi^*$ and $I(P^*, \phi) = 0$ if $\phi > \phi^*$. Moreover, ϕ^* is of the form of either $\theta_P - \kappa$ or $\theta_P + \kappa$, where $\theta_P \in \Theta^*$.

Case: $\phi^* = \theta_P - \kappa$.

In this case, $E[I|\theta_P] = 0$. So for each security i,

$$E[I|\theta^*] X_i(\theta^* + T) + (1 - E[I|\theta^*]) X_i(\theta^*) = X_i(\theta_P).$$

If security i is strictly concave between θ^* and $\theta^* + T$, it follows that

$$\theta_P < E[I|\theta^*](\theta^* + T) + (1 - E[I|\theta^*])\theta^*.$$

Conversely, if security i is strictly convex between θ^* and $\theta^* + T$ then

$$\theta_P > E[I|\theta^*](\theta^* + T) + (1 - E[I|\theta^*])\theta^*.$$

Clearly both inequalities cannot hold at once, and the resultant contradiction completes the proof.

Case: $\phi^* = \theta_P + \kappa$.

In this case, $E[I|\theta_P] = 1$. The remainder of the proof is identical. ∎

Proof of Proposition 9: Exactly as in Proposition **7** a fully-revealing equilibrium cannot exist. Suppose a non-fully revealing equilibrium exists. So at some set of fundamentals Θ^* the prices of both debt and equity must be the same for at least two distinct fundamentals.

That is, the set

$$\Theta^* \equiv \left\{\theta : \exists\theta' \neq \theta \text{ such that } P_E\left(\theta\right) = P_E\left(\theta'\right) \text{ and } P_D\left(\theta\right) = P_D\left(\theta'\right)\right\}$$

is non-empty. The proof of the convex half of Proposition **7** applies, and gives a contradiction. ∎

Proof of Proposition 10: First, in any equilibrium where there exist $\theta_1 < \theta_2$, for which $P(\theta_1) = P(\theta_2)$, $I\left(\theta_1\right)$ must be different from $I\left(\theta_2\right)$ (otherwise prices would not be identical). Given that the probability of intervention can be directly inferred from $Q(\theta)$, then the government can always infer θ based on $P(\theta)$ and $Q(\theta)$. Then, the government will choose to intervene when $\theta \leq \widehat{\theta}$, and not intervene otherwise. Thus, if there is an equilibrium, it must feature optimal intervention.

Now, let us show that optimal intervention is indeed an equilibrium. In such an equilibrium, bank security prices will be $X_j(\theta + T)$ for $\theta \leq \widehat{\theta}$ and $X_j(\theta)$ for $\theta > \widehat{\theta}$. The government security will have a price of 1 for $\theta \leq \widehat{\theta}$ and 0 for $\theta > \widehat{\theta}$. Then, independent of the government's private signal, the government will choose to intervene below $\widehat{\theta}$ and not intervene above $\widehat{\theta}$. This is indeed consistent with the prices, so optimal intervention is an equilibrium. ∎

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