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Money in a Theory of Banking

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Abstract

We examine the role of banks in the transmission of monetary policy. In economies where banks financed by real demand deposits intermediate credit, fluctuations in the timing of production can force banks to scramble for real liquidity, or even fail, which can greatly affect credit and aggregate output. The adverse effect on output can be reduced if banks finance with nominal deposits. Nominal deposits also open a “financial liquidity” channel for monetary policy to affect real activity. However, the banking system may be better off issuing real deposits (e.g., foreign exchange denominated) under some circumstances.

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What is the connection between money, banks, and aggregate credit? When can expansionary monetary policy lead to expanded bank credit? And when can monetary policy help avert bank failures? These are the questions that motivate this paper.

We start with a “real” model where all contracts are denominated in goods. A bank is an intermediary, which has special skills that enable it to lend to firms that are hard to collect from. The bank finances these loans by issuing demandable claims. In Douglas Diamond and Raghuram Rajan (2001), we show why such an arrangement allows banks to fund potentially long term projects while allowing investors to consume when needed.

Unfortunately, demandable claims expose the banking system to real shocks that create a mismatch between the production of consumption goods and the real amount promised to the depositors. Even if the shortage of consumption goods, also termed a real “liquidity shortage”, is merely due to delay in production and not because of any reduction in the production possibilities of the economy, it can be amplified by the banking system. Banks will cut short long term projects, that is, curtail credit. Banks may fail even if depositors have the most optimistic beliefs possible (unlike, say, in models like Douglas Diamond and Philip Dybvig (1983)).

All this is shown in an economy where all contracts, including demand deposits, are denominated in goods. The repayment on deposits is assumed fixed over the next instant and, because of difficulties in contracting, cannot be an explicit function of realizations of individual bank states or detailed aggregate conditions. In practice, however, bank deposits and loans typically promise to repay money, not goods—though deposits are denominated in foreign currencies in some countries, which essentially makes them real from the perspective of that country’s citizens. What would happen if deposit contracts were instead *nominal*, that is, denominated and repayable in domestic currency? Would the system smooth real shocks?

To examine this, we focus on two sources of value for money. First, money, and any maturing government liability, is a claim on the government and may have intrinsic value from the ability of citizens to pay taxes with it. Second, money, specifically currency, facilitates transactions; Some goods may be sold only for cash as in cash-in-advance models (see Robert

Clower (1967) and Robert Lucas, Jr. and Nancy Stokey (1987)). Examples are illegal goods like drugs, services that are sold in transactions where the seller seeks to keep his identity hidden from tax authorities, or goods encountered serendipitously where the relative cost of establishing a credit transaction may be too high. Depending on circumstances, the “fiscal” value of money may exceed or be dominated by its “transactions” value.

Suppose now that banks issue nominal deposits. To the extent that the real value of a unit of money falls when the current aggregate supply of goods is low, deposits denominated in money offer banks a natural hedge by adjusting effective real demand to supply. And there are reasons why this might happen—the present value of taxes on production falls if production is delayed so the fiscal value of money falls. Similarly, it is plausible that transactions fall when aggregate production is delayed so the transactions value of money could also fall. If the quantities of currency and bonds are fixed, the real value required to be paid out on nominal deposits will then fall in consonance with an adverse shock to the supply of goods, reducing the real liquidity shortage and its effects on, and via, the banking system.

However, perfect state contingent adjustment of liquidity demand using nominal deposit contracts is a fairly special idealization. More generally, the transactions value of money may have large variation that is independent of aggregate production.¹ If so, banks that issue nominal demandable deposits are particularly vulnerable: if cash goods are temporarily cheap (for example, because the supply of money is low relative to available cash goods) banks will be forced to increase interest rates offered on demandable deposits to keep depositors from withdrawing. In turn, this will increase the real repayment obligations of the banks, potentially

¹ If illegal goods and informal services require cash-in-advance, there is no reason they should be positively correlated with aggregate formal production. In fact, if more workers lose formal employment and enter the informal sector, there could be a case for arguing for negative correlation between aggregate production and cash transactions. Similarly, if a temporary downturn prompts asset sales, a negative correlation could again emerge.

even causing bank failures. When the value of money is not “well-behaved”, far from reducing aggregate real liquidity demand when aggregate supply falls, nominal deposits may actually increase it, exacerbating real liquidity shortages, and reducing credit.

Monetary intervention could play a useful role here by making the value of money correspond better to aggregate real liquidity conditions. By increasing the money supply available for transactions when the transactions demand is high (and by committing to provide monetary support in the future when needed), the monetary authority offsets aberrant monetary or real shocks and keeps the price level stable. This limits depositor incentives to withdraw and the future real repayment obligations of banks. Banks then will respond by continuing, rather than curtailing, credit to long-term projects, thus increasing aggregate economic activity.

Our view of the monetary transmission mechanism could then be termed a version of the *bank lending channel view* (see Ben Bernanke and Mark Gertler (1995) or Anil Kashyap and Jeremy Stein (1997) for comprehensive surveys) but with a difference. According to the traditional lending channel view, monetary policy affects bank loan supply, which in turn affects aggregate economic activity. Three assumptions have been thought to be key to the centrality of banks in the transmission process: (i) binding reserve requirements tie the issuance of bank demand deposits to the availability of reserves (ii) banks cannot substitute between demand deposits and other forms of finance easily so they have to cut down on lending when the central bank curtails reserves (iii) client firms cannot substitute between bank loans and other forms of finance, so they have to cut down on economic activity.

The concern with the traditional view of the bank lending channel is that as reserve requirements have been eliminated for almost all bank liabilities except demand deposits, the argument that banks will find it difficult or expensive to raise alternative forms of financing to demand deposits becomes less persuasive—see, for example, the critique by Cristina Romer and David Romer (1990), though see Jeremy Stein (1998) who argues that demand deposits are still special because they are insured. But there does seem to be strong evidence that monetary policy has effects on bank loan supply (Anil Kashyap et al. (1993), Sydney Ludvigson (1998)), has

greater effect on banks at times when their balance sheets look worse (Michael Gibson (1997)) and has the greatest effect on the policies of the smallest and least liquid banks (Kashyap and Stein (2000)).

In contrast to traditional models of the lending channel, our model does not rely on reserve requirements or on deposit insurance, or even on sticky prices. An expansionary open market operation (buying bonds with money) increases financial liquidity, decreases the real value that banks must pay in the future to retain deposits in the present, which alleviates the real liquidity demands on banks, which then allows them to fund more long-term projects to fruition. These effects will be most pronounced for constrained banks in bad times. Thus it is perhaps best to term ours the *liquidity* version of the lending channel of transmission.

The rest of the paper is as follows. In section I, we describe the framework, in section II we describe the problems with real deposit contracts and the circumstances under which nominal contracts can improve upon them. In section III, we introduce money and examine how aggregate activity and bank credit is affected by shortages of real and financial liquidity. In section IV we examine how monetary policy is transmitted in our model, then in section V, we examine robustness, after which we conclude.

I. The Framework

A. Agents, Assets, Endowments, Preferences, Technology.

Consider an economy with four types of risk neutral agents: investors, entrepreneurs, bankers, and dealers, a date when contracts are written and five future dates: 0, 1, 2, 3, and 4. Dates 1 and 3 are event rather than calendar dates, and are best assumed close in calendar time to dates 2 and 4 respectively.

There are goods, and financial assets consisting of cash and government bonds. Only investors are initially endowed with resources. Each investor has one unit of good, M_0 of cash, and B_2 face value of government bonds maturing at date 2 (a table of notation is on page 39 in the

appendix).² When the bonds mature, the government extinguishes them by repaying their face value in new money or issuing fresh bonds maturing at date 4.

Investors are impatient in that their utility is only the sum of consumption on, or before, date 2. Because all consumption on or before date 2 is perfectly substitutable, we will refer to all such consumption as *early* consumption. All other agents also get equal utility from consumption after date 2 (*late* consumption), so their utility is the sum of early and late consumption. The impatience of investors limits the response of consumption demand to interest rates (equivalently, it limits the substitutability of consumption across time), which is needed for liquidity to matter.

Each entrepreneur has a project, which requires the investment of a unit of good before date 0. It produces C in goods at date 2 if the project is *early* or C at date 4 if the project is delayed and produces *late*. Alternatively, goods can be stored at a gross real return of 1. We assume that there is a shortage of endowments of goods initially relative to projects that can be invested in. Let the gross real interest rate between date j and date k be r_{jk} and the gross nominal interest rate be i_{jk} .

B. Projects and the non-transferability of skills

The primary friction underlying the model is that those with specific skills cannot commit to using their human capital on behalf of others, that is, they cannot commit to repay all the value they generate to outsiders. Creditors will lend only to the extent that they can compel borrowers to repay—either by threatening to seize the borrower’s assets and generate value with them or by committing to harm the borrower if the borrower defaults.

Specifically, since entrepreneurs have no endowments, they need to borrow to invest. Each entrepreneur has access to a banker who has, or can acquire during the course of lending, knowledge about an alternative, but less effective, way to run the project. The banker’s specific knowledge allows him to (make the credible threats that will enable him to) collect γC from an

² Because we have three calendar dates in the model, 0, 2, and 4, a bond maturing at date 2 is a one period (short term) bond. We do not consider long term bonds in this paper.

entrepreneur whose project just matures. No one else has the knowledge to collect from the entrepreneur. Because the entrepreneur's skills are critical to the project, the project is illiquid in that the entrepreneur can pay at most γC of the C that he generates.

Regardless of whether a project is early or late, the banker can also *restructure* the project at any time to yield c in early consumption goods—intuitively, restructuring implies stopping half finished projects and salvaging all possible goods from them. We assume $c < 1 < \gamma C < C$.

Since no one other than the bank has the specific skills to collect from the entrepreneur, the loan to the entrepreneur is also illiquid in that the banker will get less than γC if he has to sell the loan before the project matures. Any buyer will realize that the banker will extract a future rent for collecting the loan, and the buyer will reduce the price he pays for the loan accordingly. In fact, bank loans are so dependent on the banker's specific skills for collection that the banker prefers restructuring projects to selling them.³ The illiquidity of both projects and bank loans stems from the inalienability of human capital (see Oliver Hart and John Moore (1994)).

Given the shortage of endowment relative to projects, competition will force the select few entrepreneurs who get a loan to promise to repay the maximum possible on demand, γC , to obtain the loan. This simplifying assumption is relaxed later.

C. Financing Banks

Since bankers have no resources initially, they have to raise them from investors. But investors have no collection skills (and, consequently, bank loans are worthless in their hands), so how do banks commit to repaying investors? By issuing demand deposits! In our previous work (Diamond and Rajan (2000, 2001)), we argued that the demandable nature of deposit contracts introduces a collective action problem for depositors that makes them run to demand repayment whenever they anticipate the banker cannot, or will not, pay the promised amount. Runs destroy

³ See Diamond and Rajan (2005) for the easily satisfied conditions that lead to this outcome.

the banker's rents. Because depositors are committed to harming the banker if he reneges on his promise to pay, the banker will repay the promised amount on deposits whenever he can.

Deposit financing introduces rigidity into the bank's required repayments. Ex ante, this enables the banker to commit to repay if he can (that is, avoid strategic defaults by passing through whatever he collects to depositors). However, it exposes the bank to destructive runs if he truly cannot pay (it makes non-strategic default more costly): when depositors demand repayment before projects have matured and the bank does not have the means of payment, it will be forced to restructure projects to get c immediately instead of allowing them to mature and generate γC .

All banks face a perfectly competitive deposit market where deposits flow freely to any bank that can credibly repay the market clearing rate of return.

D. Uncertainty

Each bank faces an identical pool of entrepreneurs before date 0. At date 0, the state s is realized and the banks become differentiated. A bank could turn out to be type G with all entrepreneurs having early projects or type B with $\alpha^{B,s} < 1$ entrepreneurs with early projects. The fraction of banks of type G in state s is $\theta^{G,s}$. In what follows, we will suppress the dependence on the state for notational convenience. Also, instead of introducing the whole model at one go, we will introduce the essential features of the "real" model, leaving the details of money for later.

E. Timing.

Before date 0.

Investors deposit goods in competitive banks in return for claims that make them better off in expectation than storage.⁴ Let us start by assuming that banks issue real deposits, that is, each

⁴ While we have made assumptions here forcing the investor to lend via the bank, we show in Diamond and Rajan (2001) that banks and their fragile liability structures arise endogenously to facilitate the flow of credit from investors with uncertain consumption needs to entrepreneurs who have hard-to-pledge cash flows.

bank offers to repay d_0 early consumption goods (or claims of equivalent value) on demand for the resources they get from each identical investor. Banks lend the goods to entrepreneurs in return for a promise to repay γC on demand. Entrepreneurs invest the goods in projects. There is a competitive market for deposits, bonds, and goods at each date.

Date 0. Uncertainty is resolved: everyone learns which entrepreneur is early and who is late, and thus what fraction α^j of a bank i 's project portfolio is early. Depositors withdraw or renew. If they renew, they get $d_2 = d_0 * r_{02} = d_0$ (because everyone is indifferent between consumption at date 0 and at date 2 and no real investments between those dates offer a higher return, r_{02} is 1). We assume that depositors in a given bank run at date 0 only if they anticipate it cannot survive at date 2 given its realized distribution of projects and given the market clearing interest rate that will prevail at date 2. In other words, we do not consider panics where depositors run at date 0 only because they think other depositors will run, regardless of date-2 fundamentals—we allow collective action problems but not coordination failures.⁵ A run will force the bank to first pay out all the goods it has, and then restructure late projects, and finally early ones to generate the goods needed to pay depositors.

Date 2. Entrepreneurs with early projects will produce C , and repay the bank γC . This leaves them with $(1-\gamma)C$ to invest as they choose. If no run has occurred, the bank decides how to deal with each late project—whether to restructure it if proceeds are needed before date 4 or get more long run value by rescheduling the loan payment from date 2 to date 4 and keeping the project as a going concern. The bank obtains repayments from early entrepreneurs, proceeds from restructured late projects, and raises new funds by issuing deposits to early entrepreneurs and other bankers with surplus. It repays depositors d_2 out of all these resources.

⁵ Put another way, while we assume that each depositor expects the others in the same bank to choose a withdrawal that is an individual best response to others' actions (so we assume non-cooperative actions where individual incentives may not lead each depositor to maximize the welfare of the whole), they all agree to choose the set of Nash actions that make them best off.

Date 4. Late entrepreneurs repay banks and banks repay date-2 depositors. Entrepreneurs and bankers consume.

II. Aggregate Liquidity Shortages and Bank Credit.

In the normal course, banks can repay initial investors by borrowing from entrepreneurs and other banks that are flush with resources at date 2. But if aggregate production is significantly delayed (that is, the fraction of B type banks $(1 - \theta^G)$, or their fraction of late projects, $(1 - \alpha^B)$, increases), the liability structure of banks causes them to multiply the temporary delay, through bank credit contraction and bank failures, into a longer term, and more widespread adverse shock to production. We will sketch why this may be the case, then introduce a role for money.

A. Banks' maximization problem after uncertainty is revealed

It will be convenient to work on a per project (or equivalently, per investor) basis. It is easy to show that if banks lend any of the real goods they obtain before date 0 to entrepreneurs because the expected return on lending even for a single project dominates the return on storage, they will lend all of them and not store at all (see Diamond and Rajan (2005)).

The B type banker has the following decision problem; What fraction of late projects does he restructure at date 0 so as to maximize his consumption while constrained by the necessity to pay off all bank claimants? Let the B type banker restructure μ^B of his late projects (since all of a G type banker's projects are early, $\mu^G=0$). His maximization problem *if the bank is expected to survive* after uncertainty is revealed at date 0 is

$$(1) \quad \max_{\mu^B} \text{Real value of bank's financial assets} + \left[\alpha^B \gamma C + \mu^B (1 - \alpha^B) c + (1 - \mu^B)(1 - \alpha^B) \frac{\gamma C}{r_{24}} \right] \text{ s.t}$$

$$(2) \quad \text{Real value of financial assets} + \left[\alpha^B \gamma C + \mu^B (1 - \alpha^B) c + (1 - \mu^B)(1 - \alpha^B) \frac{\gamma C}{r_{24}} \right] \geq d_2$$

The constraint (2) is simply that the real amount the banker raises should be enough to pay outstanding deposits. On the left hand side of (2), the real value of the bank's financial assets is the value of the bank's money and bond holdings in terms of date-2 goods—we will derive

expressions for these shortly. The term in square brackets is the value in date-2 consumption goods that the B-type banker obtains from his project loans. The first term is the amount repaid by the α^B early entrepreneurs whose projects mature at date 2. The second term is the amount obtained by restructuring late projects. The third term is the amount the bank can raise in new deposits against late projects that are allowed to continue without interruption till date 4. Note that r_{24} , the gross real interest rate banks offer on deposits between dates 2 and 4 need not be 1 (unlike r_{02}), because initial investors prefer date 2 consumption over date 4 consumption.

The banker wants to maximize the present value of his total consumption, which is the residual amount he has left over after paying initial depositors d_2 . Since d_2 is a constant, the banker's objective is simply to maximize the real present value of his assets, the left hand side of (2). The solution to the banker's problem is

Lemma 1: Let $R = \frac{\gamma C}{c}$. The banker will restructure no late projects if $r_{24} < R$, be indifferent between continuing and restructuring late projects if $r_{24} = R$, and prefer restructuring all late projects if $r_{24} > R$.

Essentially, R is the implied real rate of return foregone in restructuring late projects, hence the lemma stems from comparing the return with the opportunity or market real rate of return, r_{24} .

As for other agents, the entrepreneur produces in due course if his project is not restructured by the bank. If he produces, he repays the bank. Early entrepreneurs deposit their residual goods (of $(1-\gamma)C$) in the B bank at date 2 if it can credibly promise to repay $r_{24} \geq 1$ or they store otherwise. G bankers do likewise with the value left after repaying their depositors.

B. Equilibrium Condition and aggregate credit.

Since initial investors can express their purchasing power only with their claims on the bank, the demand for consumption at date 2 is their (real) deposit claim on the bank. Thus market clearing implies the demand for real liquidity (that is, for date-2 goods) is less than the supply, so

$$(3) \quad d_2 \leq \text{Goods available for consumption on or before date-2}$$

where we will derive a specific expression for the right hand side shortly. Because demand deposits are real and investors are unwilling to substitute future consumption, goods prices at date 2 will not affect demand. Since all initial goods are invested in the bank, which further invests in projects, the supply of date-2 real consumption goods can only come from early projects or from restructured late projects. More supply can only come from more restructuring, so the only price that can adjust to clear the market is the real interest rate, r_{24} , which determines bank's incentives to restructure late projects.

The real side of our model should now be fairly clear. The adverse shocks in our model are merely delays in the timing of production—adverse shocks to total production would only exacerbate the problems. Even though the total production possibilities of the economy over dates 2 and 4 do not change with increases in late projects, the amount of consumption goods available at date 2 (aggregate real liquidity) falls. Given an excess of demand over supply for liquidity, the real interest rate will rise, increasing supply as banks restructure more late projects, until the market clears.⁶ The number of projects funded to maturity falls. We will expand on this shortly.

III. Money and Banking

We now introduce a role for money. Not only will this help us flesh out the date-2 real value of financial assets in (1) and (2), it will also let us determine the real value of nominal deposits and show when they can serve as a hedge.

We focus on two natural sources of value for money. First, money can serve as a store of value; we introduce this into our finite horizon economy by assuming that money (and any maturing government liability) can be used to pay future taxes. This generates a demand for nominal claims which we shall call the *fiscal* demand. Second, currency facilitates certain

⁶ In this model without bank capital and only one type of project, the interest rate will jump to R as soon as restructuring is needed. In a more continuous model with capital or heterogeneous projects, the interest rate moves smoothly up.

transactions that by their very nature are unexpected, opportunistic, small-volume, or worth concealing so that the use of formal credit is ruled out. This is the *transactions* demand for money. Both demands will be important in understanding the link between money and banking.

A. Transactions Demand

Start first with the *transactions* demand. Dealers, whom we referred to earlier but did not describe, receive an endowment of a perishable good, which can be sold only for cash (to fix ideas, the good is their labor, and they do not report income to the tax authorities so they accept only cash). “Early” dealers obtain an endowment q_1 of this *cash* good at date 1 while “late” dealers obtain q_3 at date 3 (recall that all quantities are per unit of initial project financed). One unit of this cash good is a perfect substitute for one unit of the production good. Unlike the cash good, both deposits and cash can be used to pay for the production good.

To introduce a motive for trade, we assume that no one can consume his own endowment or production. All trades require payment one period ahead in cash or deposits. This means that in order to consume a cash good that is produced at date j , the buyer has to pay cash to the seller at date $j-1$. If he wants to consume a production good, he *also* has the option of writing a check to the seller at date $j-1$, which will clear against the funds he has on deposit at date j . The seller can use the cash or deposit he receives at date j to buy goods for consumption at date $j+1$. This payment in advance constraint also applies to sales of bonds (to be described) and restructured loans. Finally, if a bank issues deposits (in exchange for cash, bonds or loans) at date j , they can be used to initiate transactions at date j .

Let P_{jk} denote the price in date j cash of a unit of date k consumption. For example, a transaction for date 4 goods initiated at date 3 in cash at price P_{34} yields the seller P_{34} units of cash at date 4.

B. The Fiscal Demand.

The government taxes sales of produced goods at the rate t . To maintain consistency with the expressions derived thus far, assume that the production quantities specified earlier are after-

tax, so total nominal taxes due on a project that matures at date j are $\frac{tC}{1-t} P_{j-1,j}$. Because cash goods may lie outside the formal economy, we assume they are not taxed—nothing significant depends on this. Taxes are due at the time of production and are payable in cash or through a check on a deposit (with the bank then transferring the cash to the government).

The odd-numbered dates, 1 and 3, simply make payment and settlement explicit. Since date 3 is close to date 4, we allow actions at date 4 to be committed to at date 3, so late entrepreneurs can borrow deposits at date 3 against what they will have at date 4 after repaying the bank loan ($= (1-\gamma)C P_{34}$). They can use the resulting deposits at date 3 to purchase goods for consumption at date 4. Similarly, the banker can also issue himself deposits at date 3 against his date-4 rents. This saves us the need to introduce another date to clear purchases initiated at date 4.

C. Money and Prices

Since cash goods and produced goods offer equivalent consumption on the dates agents want to consume, their relative prices will constrain the nominal interest rate deposits have to pay to prevent depositors arbitraging between cash and deposits. This is important in what follows.

Assume no new money or bonds are issued after date 2, so at date 4 there are M_2 units of money and bonds maturing into B_4 units of cash. Cash at this date is useful only to pay taxes, so it will be accepted in payment for produced goods because the seller wants to use them to pay taxes.

Let X_4 ($= (1-\theta^G)(1-\alpha)(1-\mu^B)C$) be the quantity of goods produced and sold for date 4 delivery. The nominal sales (all sales, including those paid with deposits) is $P_{34}X_4$, nominal tax owed is $tP_{34}X_4$, and the total supply of cash at date 4 is $M_2 + B_4$.⁷ As a result,

⁷ Implicit in this is that the price of produced goods in assignable date-4 deposits (what one could term P_{44}) is the same as its price in date-3 cash. Equivalently, the gross nominal interest rate on bonds and deposits between dates 3 and 4, i_{34} , equals 1. Suppose not and the rate banks paid on deposits were higher than 1. Then everyone would deposit their cash in banks and use deposits to

$$(4) \quad P_{34} = \frac{M_2 + B_4}{tX_4}$$

Intuitively, an examination of the government's balance sheet suggests if tax rates (and government spending) do not change but aggregate taxable output falls, the real value of government revenues fall, and so must the real value of its nominal liabilities. Prices must, therefore, increase (see Charles Calomiris (1988), John Cochrane (2001) and Michael Woodford (1995)).

At date 2, the purchase of q_3 of cash goods can be initiated with the outstanding date-2 cash, M_2 . Since agents who get utility from consumption after date 2 are indifferent between consumption at date 3 or date 4, a holder of date-2 cash will spend it at date 2 or 3 depending on where he can purchase greater consumption. So the real value of the money stock at date 2 will be the larger of its purchasing power in buying cash goods for delivery at date 3 or the value of holding it to purchase produced goods at date 3 for delivery at date 4. The purchasing power of the money stock is $Max\{q_3, \frac{M_2}{M_2 + B_4} tX_4\}$ where the second term is the quantity of produced

goods the current money stock can purchase at date 3 for delivery at date 4 ($= \frac{M_2}{P_{34}}$).⁸ As a result,

if P_{24} is the price of date 4 consumption in date-2 cash, the date-2 real value of the money stock is

buy goods. But for any bank to hold cash, the rate on deposits should be 1, else the bank would use any excess cash to pay down deposits. Similarly, for banks to hold cash and bonds, the rates of return on them should be equal. So the nominal rate, i_{34} , is 1 and cash, deposits, and bonds pay the same rate, as they will on all dates that cash has no special value. This also explains why i_{12} equals 1.

⁸ Another way to see this is that so long as the price of cash goods for transactions initiated at date 2 is below the price of produced goods at date 3, money will be fully used up in buying cash goods. But once there is enough money such that the price of cash goods equals the price of

$$\frac{M_2}{P_{24}} = \text{Max}\left\{q_3, \frac{M_2}{M_2 + B_4} tX_4\right\}, \text{ or equivalently: } P_{24} = \frac{M_2}{\text{Max}\left\{q_3, \frac{M_2}{M_2 + B_4} tX_4\right\}} = \text{Min}\left\{\frac{M_2}{q_3}, P_{34}\right\}$$

Comparing the two terms within the curly brackets in the expression for $\frac{M_2}{P_{24}}$, we see that when $q_3 > \frac{M_2}{M_2 + B_4} tX_4$, the transactions demand dominates in that money is valued more for its

role in paying for transactions at date 2 (it has a liquidity premium) than as a store of value.

Since a depositor can withdraw cash on date 2 to make payments, for someone to leave their money in the bank, deposits must offer a gross nominal interest rate of

$$i_{23} = \frac{P_{34}}{P_{24}} = \frac{q_3}{\frac{M_2}{M_2 + B_4} tX_4} > 1, \text{ and this is also the nominal rate on bonds (because banks can trade}$$

bonds with each other in a competitive market). Importantly, the quantity of currency at date 2 changes the relative price of cash goods and produced goods and thus influences the nominal interest rate, the opportunity cost of holding money.

Folding back to earlier periods, one can similarly derive the value of all nominal claims, cash and bonds. All nominal bond prices on date j of bonds maturing on date k are given by

$$b_{jK} = \frac{B_k}{i_{jk}}. \text{ Proposition 1 gives the equilibrium price levels and nominal interest rates for all}$$

dates, given real interest rates and real tax collections.

produced goods, any money left over after buying cash goods will be used as a store of value till it can be used for purchasing produced goods at date 3. Therefore, money will effectively be valued in terms of its date-3 purchasing power. This is the intuition behind the max function.

PROPOSITION 1.

The price levels on each date are given by:

$$P_{34} = \frac{M_2 + B_4}{tX_4}, \quad P_{24} = \text{Min}\left\{\frac{M_2}{q_3}, P_{34}\right\}.$$

$$P_{12} = \frac{M_0 + B_2}{M_2 + \frac{B_4}{i_{24}}}, \quad P_{02} = \frac{M_0}{\text{Max}\left\{q_1, \frac{M_0}{P_{12}}\right\}} = \text{Min}\left\{\frac{M_0}{q_1}, P_{12}\right\}$$

$$tX_2 + \frac{i_{24}}{r_{24}P_{24}}$$

Nominal interest rates are given by:

$$i_{01} = \frac{P_{12}}{P_{02}}, \quad i_{12} = 1, \quad i_{23} = i_{24} = \frac{P_{34}}{P_{24}}, \quad i_{34} = 1.$$

Proof: See appendix.

D. Revisiting the real model.

Augmenting the basic model with cash goods, prices, and the opportunity for depositors to withdraw cash to buy cash goods at dates 0 and 2, does not change the banker's maximization problem ((1) s.t. (2)). We substitute the date-2 real value of the bank's financial assets,

$$\frac{M_0}{P_{02}} + \frac{B_2}{P_{12}},$$

into those expressions (intuitively, money can buy goods at price P_{02} and deposits against maturing bonds can buy them at P_{12}).

Also, the total (pre-tax) supply of real liquidity – the goods available for consumption on or before date 2 in (3) -- is $q_1 + \frac{1}{1-t} \left[\theta^G C + (1-\theta^G) (\alpha^B C + (1-\alpha^B) \mu^B c) \right]$. This should be (weakly) greater than the real value of deposits, d_2 , to satisfy the demand for real liquidity. We show in the appendix that (3) is also the market clearing condition.

Now that we have the framework in place, we describe some comparative statics of the model with real deposits when the total supply of consumption goods is not enough to meet the total demand without some restructuring by B type banks. We focus on both *total credit*, which is

the fraction of projects that retain credit to maturity, $\theta^G + (1 - \theta^G)\{\alpha^B + (1 - \alpha^B)(1 - \mu^B)\}$ and the fraction of late projects continued, $(1 - \mu^B)$. We have

PROPOSITION 2:

For a given real level of deposits issued before date 0, if both types of banks survive and there is an aggregate liquidity shortage such that the B type banks have to restructure a positive fraction

of their late projects (that is $q_1 + \{\theta^G \frac{C}{(1-t)} + (1 - \theta^G)\alpha^B \frac{C}{(1-t)}\} < d_2$), then:

- (i) *Total credit and the fraction of late projects continued increase in the fraction of G type banks, θ^G ;*
- (ii) *Total credit and the fraction of late projects continued increase in the fraction of projects of B type banks that are early, α^B ;*
- (iii) *For a given θ^G and α^B , total credit and the fraction of late projects continued decrease in the outstanding level of real deposits, d_2 .*

Proof: See appendix.

The proposition indicates that a decrease in the intrinsic supply of real liquidity (early goods) via a decrease in either θ^G and α^B , or an increase in demand (an increase in d_2), leads to a curtailment in credit. Essentially, in a situation of aggregate liquidity shortage, banks are squeezed between a rock (non-negotiable deposits) and a hard place (hard to sell loans). They survive only by calling loans and restructuring projects. Ironically, when aggregate liquidity is plentiful, the same features of demand deposits commit the bank to collect, enabling it to issue new deposits against late projects and thus meet the individual liquidity needs of depositors and borrowers.

E. Bank failures.

If the liquidity shortage is severe enough, there are two reasons banks with real deposits can fail. First, there can be insufficient aggregate output to allow all deposits to be repaid even if

all late loans are called and projects restructured. Second, it is possible that type B banks are insolvent at the real interest rate that must prevail for banks to have incentives to call in loans and force projects to be restructured.

Recall that if a bank is expected to fail, depositors run and demand payment immediately on the revelation of uncertainty at date 0. Since projects pay at date 2 at the earliest, and since the bank obtains more from restructuring rather than selling the illiquid project loans to satisfy depositor demands at date 0, all projects of a failing bank are restructured, including the early ones that would pay off at date 2. Failure is inefficient because early projects could have produced C in a timely manner to satisfy early consumption needs, but now produce only c . The specificity of bank relationships means that bank failures can cause a persistent drop in real activity, as in Bernanke (1983). The collective action problem inherent in demand deposits is now destructive for it forces the costly production of consumption goods at a time when they are not really needed.

When bank contracts are real, the real interest rate, r_{24} , is the only price which can adjust to clear markets. The system may have insufficient degrees of freedom to adjust to an adverse shock without the stark consequences we have documented—major changes in credit and possibly bank failures. This real model is not without practical interest—when a country's banking system has deposits denominated in foreign exchange (or if the country were on the Gold standard), it is as if the banks issue real deposits. But our primary purpose is to make clear that credit contraction and failure are essentially real phenomena and occur when the bank is squeezed between non-renegotiable demand deposits and a limited production of consumption goods.

If fully state-contingent deposit contracts were available, where the real face value of each deposit depended on the individual bank's situation and the aggregate state of nature, then the banking system would produce a given amount of liquidity (expected date 2 consumption) at the minimum cost of foregone future (date 4) consumption. The outcomes with such contracts are easily characterized. First, early projects should never be restructured because the cost of

providing liquidity increases with no offsetting benefit. A necessary and sufficient condition for avoiding restructuring of early projects is that there be no runs. Second, the minimum number of late projects should be restructured, in order to just meet the consumption needs of investors; none should be restructured if goods are to be stored, or consumed by those who are willing to consume late. This condition will also be met if there are no runs. Last, in order to minimize the date 4 cost of a given amount of expected date 2 consumption, if late projects must be restructured in some states of nature, then the contingent deposit payment in all other states should at least be all of the return from the early projects.

Explicit and full state contingency may not be possible for the usual technological and institutional reasons. The real interest rate, with just two values in our model, offers limited additional contingency even if observable and verifiable. However, deposits could be made implicitly contingent by denominating them in cash, so that their real value can fluctuate with the price level. If the price level rises with production delays, the real payout on deposits will fall, reducing liquidity demand. Nominal deposits will thus offer a hedge against the consequences of aggregate shortages. Because banks are of heterogeneous types ex-post, automatic movements in the state contingent value of money cannot possibly replicate complete state contingent contracts, but they can be “contingent enough” to hedge the banking system as a whole against runs.

In what follows, we describe conditions under which nominal deposits can hedge the banking system as a whole against real shocks. But we go on to show that it also exposes the system to monetary shocks. This then suggests a role for monetary policy: to smooth over such monetary shocks (which may have real roots). Not only does our model then highlight a distinctive channel for the transmission of monetary policy, it also suggests that an objective of monetary policy is to ensure that a banking system with nominal deposits is not destabilized.

F. Nominal Deposits as a Hedge against Aggregate Liquidity Shortages.

Instead of real deposits, let banks have nominal deposits outstanding at date 0, which let the depositor withdraw δ_0 units of cash on demand. Deposits will return the nominal rate, i_{02} , if rolled

over until date 2, so they will pay $\delta_2 = \delta_0 * i_{02}$.

Fiscal demand dominates.

First consider a situation where the fiscal demand dominates—for example, when there is plenty of money relative to cash goods (most simply, if there exist no cash goods) so at the margin money is valued only for its role in paying taxes. It is easy to see from Proposition 1 that

$$(5) \quad P_{02} = P_{12} = \frac{M_0 + B_2}{t \left[X_2 + \frac{X_4}{r_{24}} \right]}$$

So prices are inversely proportional to the present value of taxes, which is a constant function of discounted production. Now nominal deposits serve as a hedge; Intuitively, the real date-2 payment they entail adjusts via the price level to be a fixed fraction of the present value of total output. If the bank's assets are also a relatively fixed fraction of the present value of total output, repayment, if feasible for any realization of output, will always be feasible both in terms of aggregate liquidity and bank solvency.

To see this, consider a “representative” bank, with $\alpha^i = \bar{\alpha} = \theta^G * 1 + (1 - \theta^G) * \alpha^B$ -- for instance, if we imagine that all banks are merged into one. Net of what they can buy with their financial assets, banks have to find additional real goods at date 2 of

$$\frac{\delta_0}{P_{02}} - \left(\frac{M_0}{P_{02}} + \frac{B_2}{P_{12}} \right) = \frac{\delta_2 - (M_0 + B_2)}{M_0 + B_2} \left(tX_2 + \frac{tX_4}{r_{24}} \right) \text{ to pay off their depositors, where } P_{02} \text{ and } P_{12} \text{ are}$$

from (5) and $\delta_2 = \delta_0 * i_{02} = \delta_0$ because $i_{02} = 1$ when the transaction demand is dominated.

The banking system's ability to pay depositors on date 2 is increasing in the fraction of projects that are early. If all projects are early, then $\alpha^B = 1$, $X_2 = \frac{C}{(1-t)}$, $X_4 = 0$, and the bank

collects γC on its loans. For the bank to be able to repay when all projects are early, we require

$$\text{that } \gamma C \geq \frac{\delta_2 - (M_0 + B_2)}{M_0 + B_2} \frac{tC}{(1-t)}, \text{ or simply that}$$

$$(6) \quad \frac{\delta_2 - (M_0 + B_2)}{M_0 + B_2} \frac{t}{(1-t)} \leq \gamma < 1$$

Interestingly, once there is some α^B at which the representative bank survives, we can show the representative bank will never fail, no matter what the aggregate liquidity shock, that is, no matter what the aggregate $\bar{\alpha}$. To see this, note that for the bank to be solvent, we require

$$\bar{\alpha}\gamma C + (1-\bar{\alpha})[\bar{\mu}c + (1-\bar{\mu})\frac{\gamma C}{r_{24}}] \geq \frac{\delta_2 - (M_0 + B_2)}{P_{02}} = \frac{\delta_2 - (M_0 + B_2)}{M_0 + B_2} (tX_2 + \frac{tX_4}{r_{24}})$$

where the left hand side of the inequality is the date-2 value of the representative bank's real assets based on the aggregate amount of restructuring, $\bar{\mu}$. Expanding the right hand side,

$$\bar{\alpha}\gamma C + (1-\bar{\alpha})[\bar{\mu}c + (1-\bar{\mu})\frac{\gamma C}{r_{24}}] \geq \frac{\delta_2 - (M_0 + B_2)}{M_0 + B_2} \frac{t}{1-t} \{ \bar{\alpha}C + (1-\bar{\alpha})[\bar{\mu}c + (1-\bar{\mu})\frac{C}{r_{24}}] \}$$

Given (6), this is certainly true for $\bar{\mu} = 1$. Since there is at least one feasible level of restructuring that leaves the bank solvent (and arguing similarly, also liquid⁹) the bank will not fail because it will select a $\bar{\mu} \in [0,1]$ such that it is solvent and liquid. Nominal deposits can eliminate the inefficient restructuring of late projects caused by runs.

PROPOSITION 3: *If the fiscal demand determines the value of money, the bank issues nominal deposits, and there is some $\bar{\alpha}$ such that the representative bank survives, then the representative bank will not fail no matter what the actual realization of $\bar{\alpha}$.*

We do require some conditions: First, each bank's loan portfolio has to be representative of aggregate economic activity, that is, bank portfolios should be similar. If not, then banks with low α^i can fail, even when they have issued nominal deposits. Of course, given our results that

⁹ In order for the bank to be liquid when all projects are early, it must be that

$$\frac{C}{1-t} + q_1 > \frac{\delta_2}{P_{02}} = \frac{\delta_2 * \frac{1}{1-t} C}{(M_0 + B_2)} \text{ or } \frac{t\delta_2}{M_0 + B_2} \leq \{1 + \frac{(1-t)q_1}{C}\}. \text{ If there are no cash goods, this condition}$$

is sufficient to ensure that the banking system is liquid regardless of $\bar{\alpha}$ when all late projects are

$$\text{restructured, for it ensures that } \bar{\alpha} \frac{C}{1-t} + (1-\bar{\alpha}) \frac{c}{1-t} > \frac{\delta_2}{P_{02}} = \frac{\delta_2 * \frac{1}{1-t} (\bar{\alpha}C + (1-\bar{\alpha})c)}{(M_0 + B_2)}.$$

prices adjust to aggregate liquidity, there exists a set of cross-subsidies from high α banks to low α banks that will keep all the banks alive. These cross-subsidies may not be privately rational for a healthy bank but may be in the collective interest.

Second, for the fiscal demand to provide a value of nominal claims that is proportional to the discounted value of real production, taxation should also be proportional to aggregate production (for example, a linear sales tax).

A Well-Behaved Dominant Transactions Demand

All this was derived in the context of a dominant fiscal demand for money. We can obtain a similar result to Proposition 3 if the transactions demand were “well” behaved (there are no nominal shocks). If the quantities of cash goods are positively linearly related to the quantities of produced goods—if when the real economy flourishes, so does the illegal economy – then

$q_1 = \phi X_2$ and $q_2 = \phi X_4$ for $\phi > 0$, the price level again adjusts to offset real liquidity shocks. We

will then have $P_{24} = \frac{M_2}{\text{Max}\{\phi, \frac{tM_2}{M_2 + B_4}\}X_4}$

and $\frac{M_0}{P_{02}} = \text{Max}\{\phi X_2, \frac{M_0}{M_0 + B_2} [tX_2 + \frac{1}{r_{24}} \text{Max}\{\phi X_4 + \frac{B_4}{M_2 + B_4} tX_4, tX_4\}]\}$. This is bilinear in X_2

and X_4 . If the fiscal demand is small (t close to zero), then the date 0 price level

is $P_{02} = \frac{M_0}{\phi \text{Max}\{X_2, \frac{M_0}{M_0 + B_2} \frac{X_4}{r_{24}}\}}$. Following the logic above, this is a case where a nominal

deposit contract with a fixed money supply again provides a good automatic hedge against aggregate liquidity shocks.¹⁰

¹⁰ In a model that combines elements from this paper, Diamond and Dybvig (1983), Franklin Allan and Douglas Gale (1998), and David Skeie (2004) extends this result that nominal deposits can serve as an automatic hedge against other types of shocks or coordination failures between

In our model with a dominant fiscal value or a well behaved transactions demand, a banking system that issues nominal deposits has built in state contingency into its deposit liabilities even though the state of real liquidity may not be directly contracted on. Why then would any banking system issue real deposits and risk meltdowns? To see why, let us turn to a less “well-behaved” transactions demand for money, where instead of nominal deposits serving as a hedge, they may destabilize the banking system.

G. Nominal Deposits and Shocks to Transactions Demand.

Let the quantity of cash goods at date 1, q_1 , no longer be a constant fraction of date-2 production. Let the transactions demand dominate, implying the value of money is set by its value in purchasing cash goods. At date 0, depositors can withdraw up to δ_0 in cash to buy the cash good, but they can also roll over their deposit at the prevailing nominal rate, i_{02} . Since banks set

the nominal rate $i_{02} = i_{01} * 1 = \frac{P_{12}}{P_{02}}$ to make depositors indifferent between withdrawing cash to

buy cash goods at date 0 and paying with deposits for delivery at date 2, it must be that the real

deposit value a bank has to pay out at date 2 is $\frac{\delta_2}{P_{12}} = \frac{\delta_0 * i_{02}}{P_{12}} = \frac{\delta_0 * \frac{P_{12}}{P_{02}}}{P_{12}} = \frac{\delta_0}{P_{02}}$.

Now, instead of the real value of deposits being determined by the price of date-2 produced goods, the real value is determined by the price of cash goods at date 0. But this price may have no relationship to aggregate real liquidity conditions (largely determined by the production of date-2 goods) in the economy. Instead, it will depend on the quantity of money (financial liquidity) relative to available cash goods. For example, when M_0 is not too large or q_1

is high, $P_{02} = \frac{M_0}{q_1}$. We substitute $d_2 = \frac{\delta_0 q_1}{M_0}$ in (2) to get the bank’s solvency condition.

depositors when all banks have identical portfolios. The hedging follows from the quantity theory of money, rather than a fiscal demand for money.

Intuitively, when depositors are promised a fixed amount of cash over the next period rather than a fixed real value, the real claim they must be promised at date 2 depends on the real value of withdrawing cash *anytime* earlier. In our model, the only outside opportunity they have is to buy cash goods, so if the price of cash goods is low, the real burden of deposit claims on banks becomes very high. Moreover, the real burden of repayment is now a function of the ex ante contracted level of nominal deposits, the quantity of cash goods available for purchase at date 0, and the money supply, M_0 , none of which are necessarily sensitive to aggregate liquidity.

If these shocks to money demand due to transactions in cash goods are sufficiently large, the banking system may be worse off issuing nominal demand deposits. In the worst case, q_1 is not constant but has a negative correlation with $\bar{\alpha}$ (for example if the illegal economy expands when the legal economy is anticipated to slow), the real deposit burden on banks issuing nominal deposits will be high precisely when they have the least resources to pay. By contrast, the repayment burden if real deposits had been issued would be constant across states, and this will result in lower bank failures. An example may be useful in bringing all this together.

H. Example

Let the realized fraction of banks of type G be $\theta^G=0.3$ and those of type B be 0.7. Let $\alpha^B = 0.25$, $c = 0.8$, $C = 1.6$, $\gamma = 0.8$, $t = 0.15$. Let $M_0=0.2$, $B_2=0.4$, $q_1 = 0.3$. Plugging in values, the real interest rate that will induce banks to restructure projects is $R=1.2$. Let the level of outstanding real deposits per unit invested in the bank at date 0 be $d_0=1.3$.

In the absence of any restructuring, the total supply of goods for early consumption is just $q_1 + \frac{1}{1-t} [\theta^G C + (1 - \theta^G) \alpha^B C] = 1.19$. But outstanding deposits are 1.3, so at least some late projects have to be restructured to meet the liquidity demand. This implies that the real interest rate r_{24} must rise to 1.2 to provide incentives to restructure and that type B banks must restructure a fraction of the late projects equal to $\mu^B = 0.779$ for aggregate liquidity supply to equal the aggregate liquidity demand of 1.3 (solving $q_1 + \frac{1}{1-t} [\theta^G C + (1 - \theta^G) (\alpha^B C + (1 - \alpha^B) \mu^B c)] = 1.3$).

If α^B falls to 0.115, aggregate liquidity, $q_1 + \frac{1}{1-t} [\theta^e C + (1-\theta^e)(\alpha^B C + (1-\alpha^B)c)]$, is below 1.3 even if all late projects are restructured. Depositors will run at date 0, and all type B bank projects are restructured, including early ones.

Now consider nominal deposits. Let the level of nominal deposits be $\delta_0 = 0.8667$. With $M_0 = 0.2$ and $q_1 = 0.3$, the date 0 price of a unit of date 2 consumption of produced goods is $P_{02} = \frac{M_0}{q_1} = 0.66$, the real value of required deposit payments is $d_2 = \frac{\delta_0}{P_{02}} = 1.3$, and the outcomes are the same as with real deposits of this amount; no banks fail and $\mu^B = 0.779$.

Now if there is a money demand shock because available cash goods, q_1 , go up to 0.32, P_{02} falls to 0.625 and the real deposit repayment rises to 1.39. B type banks fail. By contrast, when banks have issued real deposits, an increase in q_1 only increases the available goods for date-2 consumption without increasing the real deposit burden. As a result, available credit increases, and restructuring is reduced. In sum then, when the transactions demand dominates and does not fluctuate in lock-step with aggregate production, banks that issue nominal deposits can be extremely vulnerable to money demand shocks if monetary policy simply keeps the supply of money and bonds fixed.

I. Discussion.

When a bank offers nominal deposit contracts, its real repayment burden depends on the value of moving into cash at each instant. We have modeled one reason this value could rise—too little money in the system can depress the price of cash goods and make cash transactions extremely lucrative – but there are others. Nominal deposits make the bank highly susceptible to fleeting opportunities available in the cash market (or, equivalently, other exogenous changes in the money demand function). This indicates the desirability of policies to offset temporary shocks to money demand that are unrelated to total output.

Important prior work has noted un-accommodated shocks to money demand can cause “panics” when banks issue nominal liabilities (see Champ et al. (1996)). There are two

particularly important differences in our paper. First, the demand for money in Champ et al. (1996) comes from easily identified “movers” who demand it inelastically to carry it as a store of value to another location. This implies that if there are too many movers, relative price changes cannot deter them from withdrawing currency, and the constant quantity of currency is shared pro-rata among movers. The reduced value obtained by those who withdraw implies that depositors who do not move do not want to withdraw, and do not have to be paid to stay in. By contrast, in our model, all depositors face the outside opportunity that causes them to want to withdraw. Given that demand for currency is elastic, the potential currency drain, or more generally, potential conversions of deposits to other uses (or even currencies), raises the rate that the bank has to pay. Even if the actual currency “drain” is small, bank health can be impaired.

The second difference follows from the first: a panic in their model is simply a loss of reserves from banks. No banks actually fail as a result of high demand for cash because withdrawers are rationed pro-rata. The critical feature in our model, by contrast, is that a high transaction demand for cash translates into a higher payout on all deposits, which can affect aggregate credit, and in extremis, impair the solvency of the bank. Thus not only will there be a premium on cash during panics, but also banks will fail, as the historic evidence indeed suggests.

IV. Monetary Policy and its Channels of Transmission

Thus far, we have examined what happens when the quantities of money and bonds are fixed, and endogenous price level adjustments cause the real value of nominal deposits to vary. If, however, the endogenous adjustments do not serve to stabilize the system, monetary policy, by changing the quantities of money and bonds, can influence the price levels and, thereby, the real obligations of banks, and thus cause a better match between the aggregate liquidity demand and supply. If monetary policy, in particular preventing inefficient bank failures, could be committed to, it could be seen as part of the optimal contract described in section III E.

A. Effects of Policy Changes

Given real parameters α^B and θ^G , bank solvency, liquidity, and credit decisions will be determined by the required real repayment on deposits at date 2. Both the date 0 price level (for

cash goods), P_{02} , and the date 1 price level, P_{12} , are critical here. If P_{02} is very low relative to P_{12} because of a shortage of money relative to cash goods, the real quantity depositors can get by withdrawing at date 0 goes up. In order to prevent all depositors from withdrawing, the bank will have to pay depositors a nominal interest rate that equalizes the real goods they can purchase by withdrawing at date 0 and the real goods they can purchase by writing checks at date 1 (since the gross real rate is 1 between these dates, they will then be indifferent between the two choices). But this means that when cash is at a premium, the date-0 price of cash goods determines the required date-2 real payout of the bank. The way to reduce the real obligations of the bank is then to raise the date-0 price by increasing the supply of money, M_0 . Open market operations that exchange money for bonds at date 0 will be effective in this case and lead to lower future real rates (r_{24}), greater credit, and more solvent banks.

Of course, once there is enough money supply such that net nominal interest rates are driven to zero ($P_{02} = P_{12}$), open market operations are no longer effective because the price level P_{02} is determined by the sum of money and bonds and not by money alone. Put another way, money no longer has a liquidity premium because cash goods are no longer cheaper than produced goods, the nominal interest rate is zero, and money and bonds are equivalent so exchanging one for the other has no effect. Even so, the government can still affect credit by printing more money or bonds and transferring them to agents (a “helicopter drop”) to inflate prices (alternatively, one could think about a credible reduction in the tax rate), thus reducing the real liabilities of the banking system. We examine the mechanics of all this in what follows.

B. The Liquidity Channel of Transmission of Monetary Policy

To see the channel working, consider an open market repurchase conducted by the monetary authority, which has the effect of increasing the date-0 money supply to $M_0 + \Delta$ and reducing the face value of outstanding date-2 bonds to $B_2 - i_{02}\Delta$ where Δ is a small number. To focus on the pure effect of the open market operation, let no other exogenous parameter be changed at this or other dates. The open market repurchase takes place after initial contracts are

negotiated and projects initiated, and is executed so that banks have the added money at date 0.

So long as $i_{02} > 1$, the effect of an increase in money will be to increase the price of cash goods

$P_{02} (= \frac{M_0 + \Delta}{q_1})$. This will lower the real value that the bank must pay at date 2 to retain its

nominal deposits to $d_2 = \frac{\delta_0 q_1}{M_0 + \Delta}$ (and lower both the nominal rate i_{02} and the premium on cash).

PROPOSITION 4: *If both types of banks survive without intervention, then so long as the gross nominal interest rate exceeds 1 and some late projects are being restructured, an open market repurchase of bonds with money at date 0 reduces the nominal interest rate, increases total credit and reduces the fraction of late projects restructured.*

Proof: See appendix.

Corollary 1: (i) *Open market operations may be effective but no longer feasible if the outstanding stock of bonds is fully bought back and the gross nominal interest rates still exceeds one. (ii) Further open market operations are ineffective if gross nominal interest rates fall to one. (iii) In either case, if late projects are being restructured, an unrequited transfer of money or bonds from the government to agents will increase total credit and reduce the fraction of late projects restructured, with a dollar of money being more effective than a dollar of bonds when the nominal interest rate exceeds 1.*

Proof: Available in extended version of paper on AER website.

One could ask if all situations of liquidity shortage can be alleviated by nominal deposits and state contingent monetary policy, so long as the ex-ante expected return constraint of depositors is satisfied. The answer is yes, for the price level at all dates can be driven up arbitrarily high, and the demand for liquidity made arbitrarily low simply by flooding the market with enough money and bonds. In fact, the banking system will continue all late projects if the real value of deposits is driven below the minimum of (i) the amount of liquidity available to a B type bank when no late project is restructured and (ii) the real value of the B type bank when no

late project is restructured and the real rate r_{24} is 1. After this point, further monetary expansion will have no effect on real activity.

C. Example continued

Consider again our base case example with $\alpha^B = 0.25$ and nominal deposits of $\delta = 0.9333$. If the money supply is fixed at 0.2, the type B banks fail (implying that all type B bank projects are restructured). A small increase in the money supply to 0.207 effected by buying down the quantity of outstanding bonds from 0.4 to 0.375 (see the proof of Corollary 1 for formulae) reduces the real value B type banks must pay out at date 2. This allows the type B banks to be just solvent at the real interest rate that provides incentives for restructuring ($r_{24} = R = 1.2$), and they restructure a fraction 0.26 of their late projects but can also see all early projects to maturity. So a small open market operation has a large effect on real activity. If the money supply at date 0 is increased to 0.234 by buying down bonds from 0.4 to 0.314, the type B banks survive without restructuring any projects ($\mu^B = 0$) and the date-2 real interest rate r_{24} falls to one.

D. Financial Contagion.

Our simple model of money allows us to see the effects of alternative assumptions easily. For instance, suppose depositors who run on a bank will not accept deposits on other banks but will only take money (for instance, because they need time to verify the quality of the bank they will deposit in). Bank failures can now be contagious through their effect on monetary conditions.

To see this, suppose that type B banks are not expected to meet their nominal deposit obligations at date 2 and fail. Then they will be run immediately at date 0. They will pay out their cash reserves to depositors, but once they run out, they will have to sell assets. If the only asset that their depositors will accept is cash, all bank assets must be sold for cash (and not for deposits in other banks). These transactions will lock up cash, leaving less cash to buy cash goods. The cash good price will fall further to

$$(7) \quad P_{02} = \frac{M_0}{q_1 + \theta^B (c + B_2)}$$

where the denominator in (7) now also includes the value of restructured loans and bonds that the B type banks sell. At this “fire-sale” price, the purchase of cash goods becomes even more lucrative, and forces the G type banks to pay a yet higher rate to keep their depositors from moving to cash.¹¹ Given the higher payout to deposits at date 2, these type G banks could also fail if their value falls below $\frac{\delta_0}{P_{02}}$. This resembles the contagious bank failures described by Friedman and Schwarz (1963); Depositors run on banks, forcing banks to sell more assets for cash, which renders the money supply inadequate for the quantum of real activity, forcing a further drop in the price level, still greater incentive to withdraw, and still more bank failures.

E. Notes.

Finally, a few notes. First, we do not need reserve requirements, capital requirements, or deposit insurance for monetary expansion to have real effects. Moreover, it is not necessary to fool the public or even surprise it. A fully anticipated pre-announced plan involving more appropriate state contingent monetary accommodation than another pre-announced plan can lead to greater credit today and credit and output in the future. As a stark example, consider a change in policy that prevents bank failures by reducing the real deposit burden by a small amount; This change will increase the ex-ante expected return on deposits, as well as credit and output.

Second, because a significant portion of bank liabilities is convertible on demand, banks are susceptible to temporary spikes in the transactions demand for money. By contrast, financial intermediaries with longer maturity liabilities are affected only if a substantial fraction of their liabilities mature together at a time of high transactions demand or shortage of liquidity supply. A financial intermediary with longer-term liabilities that are diversified across maturities will be

¹¹ Note that it is the increase in the interest rate required to keep money from being withdrawn, rather than the price drop of the assets sold in the fire sale (see Diamond (1997)) that is the source of the problem.

much less affected by fluctuations in monetary conditions, and they will be less central channels to the conduit of monetary policy.

Third, we have modeled the temporary shock as one to money demand, coming from a surge in supply of cash goods. It could equally well come from a direct increase in the demand for cash (for instance, a flight to cash) or from temporary fluctuations in the supply of money.

Finally, more traditional channels through which an exchange of money for bonds can affect the real activity of banks can also be seen in this model. For example, because demand deposits are special—even when uninsured—any kind of reserve requirement on demand deposits will immediately make banks a channel of transmission (also see Stein (1998)). Similarly, if firms have future revenues that are fixed in nominal terms, then a shortage of money can push up the nominal interest rate, reduce their net worth, and thus reduce lending (see Bernanke and Gertler (1989)).

V. Evidence and Robustness

A. Empirical Implications

The model has four important implications:

(i) A shortage of real liquidity supply relative to real liquidity demand causes bank fragility. One obvious situation where the demand and supply of liquidity could be mismatched is when neither is under the direct control of monetary authorities and market clearing price adjustments do not insulate banks from liquidity shocks—for example, when the banking system predominantly has foreign currency deposits. One would expect, *ceteris paribus*, a “dollarized” banking system to be more fragile and to have greater deposit volatility than one with domestic currency deposits. Gianni de Nicolo et al. (2003) find this to be the case. Banks in “dollarized” economies are, on average, more fragile: they are closer to default as measured by their Z-score or the ratio of non-performing loans to total loans. Deposit growth is also more volatile in these countries. An obvious caveat is that more fragile banking systems are more likely to issue foreign currency liabilities, and the authors use instrumental variables to correct for this.

An example of an adverse liquidity supply shock in such countries is a cessation of external capital inflows. There is a strong correlation between such a “sudden stop” to a country, the extent of dollarization of the country’s banking system, and the prevalence of banking crises (see Inter-American Development Bank (IADB) (2004)).

Finally, one example of a dramatic change in the value of money is a devaluation. In Argentina, by end 2000, an increasingly cash-strapped government with substantial debt service in dollars was tapping into the same dollar pool as the banking system, which had significant dollar liabilities of its own (see Raghuram Rajan and Ioannis Tokatlidis (2004) for details). As the recognition dawned that the dollar pool might not be enough at current exchange rates to service all the debt and a devaluation was likely, a domestic currency deposit run started on the banks—the current value of withdrawals (and conversion into goods or dollars) was greater than anticipated future value. Interestingly, dollar deposits were initially more stable, though eventually the knowledge that the dollar shortage would render all banks insolvent caused a more general run.

(ii) Unaccommodated temporary increases in the demand for money will cause banking system fragility. Reinhold Mueller (1997, p326) in his history of the Venetian money market, describes how the sailing of the trading galleys was fixed for July and August. Naturally, this was a time of enormously high transactions demand for money as merchants strove to buy the goods and bullion to stock the ships. Interestingly, nominal interest rates were very high during this time, causing tremendous pressure on the banks and concentrating bank failures around this time. Soon after the galleys sailed, rates collapsed and pressure eased on the surviving banks.

(iii) A money supply which responds quickly and elastically to an increased demand for money will mitigate panics and bank failures. Bruce Champ et al. (1995) examine interest rates and strains on the banking system during periods of “crop moving” in the period 1880-1910 across two systems: the United States where all U.S. currency was either a direct liability of the US government or banknotes backed by government bonds and, therefore, relatively inelastic, and Canada where chartered banks were free to issue notes against their general assets. The authors

find the greater seasonal variation of Canadian currency (with a peak during the autumn crop moving) was associated with much smoother interest rates. Currency fluctuated less in the United States, resulting in greater fluctuations in interest rates, peaking around the crop moving period. The banking panics of 1893 and 1907 in the United States indeed occurred during the crop moving season.

(iv) Changes in monetary policy should most affect the lending decisions of the least liquid and least creditworthy banks (low α). Kashyap and Stein (2000) find that the effect of monetary policy on lending is indeed stronger for banks with less liquid balance sheets, that is, banks with lower ratios of securities to assets. Moreover, this effect is primarily due to the smallest (and thus least diversified) banks in their sample.

Other implications are less central. For instance, nominal deposits lend stability only if the monetary authority can be relied upon to be appropriately state contingent. However, if they do not have the necessary acumen or credibility, or have conflicting goals, nominal deposits are destabilizing. This implies that countries with histories of weak monetary management are likely to prefer real deposits (for example, through deposit dollarization) despite the attendant risks because the alternative could be worse (also see Olivier Jeanne (2003) and Ricardo Caballero and Arvind Krishnamurthy (2001)). De Nicolo et al. (2003) find that countries with weak institutions, high inflation, and high volatility of inflation “dollarize” more, but have more financial depth (M2/GDP) than similar countries that do not.

B. Alternative Assumptions: Sticky Prices

The pattern of response of aggregate output to monetary expansion has been well studied in the United States (see, for example, Bernanke and Gertler (1995)). One fact that is at odds with our model is that prices do not adjust rapidly. While we think the fact that our model does not require sticky prices to obtain monetary transmission is a virtue, we need to ask how we could get transmission if prices were indeed sticky. It turns out that a simple extension is sufficient.

Consider a very simple search model where dealers first post prices at the beginning of the period and buyers search for a seller. If the price cannot vary with supply or demand changes,

goods must be rationed. Assume the probability of a buyer meeting a dealer is an increasing function of the ratio of supply of cash goods to demand at the posted price, P_{02} . The nominal supply of goods at date 0 is $q_1 P_{02}$ and the nominal demand is the total amount of cash withdrawn at date 0 by depositors. An open market operation that increases the amount of cash that can be withdrawn by depositors will decrease the probability of each withdrawer finding a dealer. As this probability decreases, the return from withdrawing cash falls toward the return of holding cash to buy goods one period later, and the nominal interest rate on bank deposits between dates 0 and 1 will fall. The real payout burden on deposits at date 2 falls with an expansionary open market operation, much as in the case of flexible prices. While sticky prices prevent the real price level from changing immediately with changes in monetary policy, an immediate increase in the supply of money will reduce the real value of withdrawing to make an immediate purchase.

C. Alternative Assumptions: Nominal Loans

We have discussed real loans thus far, with nominal or real deposits. This is consistent with the assumption that resources are in short supply so the borrowing entrepreneur promises to pay out the maximum possible ex ante by agreeing to an extremely high nominal repayment on demand. As a result, actual repayments on project loans are renegotiated down to the underlying real collateral value of assets. By contrast, bank deposits are enforced through the threat of collective action, so their nominal face value matters, regardless of how high it is.

It is, however, interesting to discuss whether the system stabilizes automatically through the price level if the nominal repayment on the project loan, L , were set at a level that did not exceed the collateral value of underlying project assets (e.g., $L < CP_{jk}$). We have

PROPOSITION 5:

If the fiscal demand for money dominates and project loans are always nominal, then:

a) If deposits are also nominal, the representative bank will never fail at any $\bar{\alpha}$ if it is solvent and liquid at some $\bar{\alpha}'$.

b) If deposits are real, the representative bank becomes (weakly) less solvent as $\bar{\alpha}$ falls. Even if it is solvent at an $\bar{\alpha}'$, it can fail at some $\bar{\alpha} < \bar{\alpha}'$.

Proof: See appendix.

When financial assets are all nominal and effectively have the same maturity, changes in the price level have no effect on a bank's nominal solvency, and cannot push a bank from solvency to insolvency (or vice versa). However, price level changes will change the real value of deposits and influence the real excess demand for liquidity. So long as there is a binding liquidity constraint, policy induced increases in the price level will reduce the excess liquidity demand, and will increase aggregate credit.

When loans are nominal but deposits are real, an increase in the price level reduces the value of the bank's assets without affecting liabilities. Since, ceteris paribus, a fall in the economy wide fraction of early projects, $\bar{\alpha}$, (weakly) increases the price level, it can impair solvency. Automatic changes in the price level do not stabilize the banking system when it has this kind of asset-liability mismatch.

D. Alternative assumption: Capital.

For simplicity, banks in our model do not issue capital. Clearly, capital can buffer some of the effects of liquidity shortages but it will also reduce the ability of the bank to commit to pay depositors. An optimal capital structure will typically have the bank issuing a significant proportion of its capital structure in deposits (see Diamond and Rajan (2000)) and, as we show in the working paper version of this paper, liquidity shortages still are possible and all our results continue to hold. The issuance of capital (or heterogeneity in project returns) will, however, result in a much smoother and plausible path of real interest rates, r_{24} .¹²

¹² So long as all bank liabilities are held for liquidity purposes, bank capital also introduces an additional channel through which an exchange of money for outstanding bonds can expand credit. In particular, through such an open market operation, the government captures the liquidity premium that would otherwise go to traders, and thus reduces the real value of government

VI. Conclusion

Starting with the problem that banks are vulnerable to real liquidity shortages, we show that they may be able to mitigate it by issuing nominal deposits. However, because bank deposits can be converted on demand to money, this leaves them exposed to fluctuations in monetary conditions. This suggests a “liquidity” channel of transmission of monetary policy, and a role for monetary authorities in smoothing these conditions even in a world without many of the traditional assumed frictions. Our model suggests why policy would work largely through banks.

There is ample scope for future work in expanding our simple model. For example, we have only examined purchases with currency as the opportunity that causes strains on the banking system. More generally, any opportunity that requires immediate nominal liquidity would strain the system, and these deserve exploration. Also, our model ties the health of the banking system to monetary and fiscal policies. We have only touched on monetary policy and not explored fiscal policy (e.g., changes in the tax rate) at all. Also, we have made a number of strong assumptions to simplify our analysis. The effect of relaxing some are easy to see. For example, initial investors cannot substitute at all for consumption between dates. If they are willing to postpone consumption once the interest rate rises enough, the destructive effects of liquidity shortages will be capped. Similarly, allowing cash goods to be taxed will simply change the form of the fiscal demand for money. But the effects of relaxing others are harder to anticipate. For instance, we do not have long dated bonds nor do we incorporate concerns about monetary policy credibility or fiscal performance. Relaxing each of these is a realistic and potentially interesting extension that suggests scope for future work.

liabilities held on bank balance sheets. The reduced value of bank financial assets will translate into a lower value of bank liabilities (deposits plus capital) and hence a lower aggregate liquidity demand.

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List of Symbols

- C (After tax value of) goods produced by entrepreneur with project on maturity.
- γ Fraction of entrepreneur's production that can be produced by bank.
- c (After tax value of) goods that can be produced by anyone if the project is restructured.
- q_j Cash goods for sale on date j ($j=1,3$)
- $\theta^{G,s}$ Fraction of G type banks in state s .
- $\alpha^{B,s}$ Fraction of early projects in the portfolio of B type banks (for G banks, this is 1).
- $\bar{\alpha}$ Fraction of early projects in the portfolio of the representative bank.
- μ^B Fraction of late projects restructured by B banks (G banks have no late projects to restructure).
- $\bar{\mu}$ Fraction of late projects restructured by the representative bank.
- X_t Total taxable goods produced on date t ($t=2,4$)
- d_j Face value of real deposits due on date j
- δ_j Face value of nominal deposits due on date j
- M_j Quantity of money available to make purchases on date j .
- B_j Face value of bonds (repaying in money) maturing on date j .
- b_{jk} Cash value on date j of bond maturing on date k .
- r_{jk} Real interest rate between date j and date k
- i_{jk} Nominal interest rate between date j and date k
- R ($=\frac{\gamma C}{c}$) Real interest rate necessary to give B banks an incentive to restructure late projects
- P_{jk} The price in date i cash for a unit of date j consumption .
- t Tax rate on produced goods

Figure 1: Time line of transactions:

Before Date 0

Banks offer interest rates on deposits and issue deposits for bonds and cash, entrepreneurs receive loans (in bank claims), and entrepreneurs buy goods from initial investors with bank claims

Date 0

State of nature is realized. Depositors withdraw cash to buy cash goods (if no more expensive than produced goods) or to hold as an asset. If a bank faces withdrawals exceeding its cash, it sells bonds and restructured loans for date-1 delivery to meet withdrawal (similarly on all future dates)

Date 1

Cash goods sold at 0 delivered (similarly on all future dates). Cash from date 0 good sales available to seller to deposit or spend (similarly on all future dates). Cash from date 0 bank asset sales available for depositor to spend (similarly on all future dates). Early entrepreneurs sell produced goods for deposits or cash

Date 2

Early entrepreneurs repay loans with deposits or cash. Early entrepreneurs pay taxes with cash from sales and withdrawn bank deposits. Government repays maturing bonds in currency and issues new bonds. Cash is withdrawn to buy date-3 cash goods or to hold as an asset.

Date 3

Cash goods sold at date 2 delivered. Late entrepreneurs sell goods for bank claims and cash

Date 4

Late entrepreneurs repay bank with currency and deposits. Banks repay remaining net deposits in currency. Government repays maturing bonds in currency. All currency goes to pay taxes

Appendix

Proof of Proposition 1:

We derived the price levels and interest rates after date 2 in the text. To determine the price level at dates prior to date 2, we have to determine the real value of all government liabilities at date 2. Since we already know the value of the money stock, we now determine the value of outstanding bonds.

Let the date-2 value in cash of bonds maturing to pay B_4 at date 4 be b_{24} . Then because of the competitive market for bonds,

$$b_{24} = \frac{B_4}{i_{24}} = \frac{B_4}{i_{23} * 1} = \frac{B_4}{\text{Max}\{1, \frac{q_3}{\frac{M_2}{M_2 + B_4} tX_4}\}} = B_4 \text{Min}\{1, \frac{tX_4}{q_3} (\frac{M_2}{M_2 + B_4})\}.$$

The real value (in date-4 consumption) of government liabilities leaving date 2 then is¹³:

¹³ We substitute for P_{24} and use $\text{Min}\{1, \frac{tX_4}{q_3} (\frac{M_2}{M_2 + B_4})\} = \frac{tX_4}{q_3} (\frac{M_2}{M_2 + B_4})$ and

$\text{Max}\{q_3, \frac{M_2}{M_2 + B_4} tX_4\} = q_3$ when the nominal interest rate exceeds 1 to simplify the expressions.

$$\begin{aligned} \frac{M_2 + b_{24}}{P_{24}} &= \frac{M_2 + B_4 \text{Min}\{1, \frac{tX_4}{q_3} (\frac{M_2}{M_2 + B_4})\}}{P_{24}} \\ &= \text{Max}\{q_3 + \frac{B_4}{M_2 + B_4} tX_4, tX_4\}. \end{aligned}$$

The quantity of money and bonds outstanding between date 0 to 2 are constant at M_0 and B_2 respectively (we will later allow monetary policy to alter these quantities). At date 2, the existing money stock and new money repaid on maturing date-2 bonds can be used to pay date-2 taxes as well as “buy” M_2 and B_4 . The value of maturing bonds and money in units of date 2 consumption

goods purchased at date 1 is $\frac{M_0 + B_2}{P_{12}} = tX_2 + \frac{1}{r_{24}} \text{Max}\{q_3 + \frac{B_4}{M_2 + B_4} tX_4, tX_4\}$

where we use the real interest rate to transform units of date-4 consumption into units of date-2 consumption. Substituting values, we can transform this into the expression for P_{12} in the proposition. At date 0, cash of M_0 can be used to purchase cash goods q_1 at date 1. So its real

value in terms of date 2 consumption is given by $\frac{M_0}{P_{02}} = \text{Max}\{q_1, \frac{M_0}{P_{12}}\}$. Again, if $q_1 > \frac{M_0}{P_{12}}$,

money is valued for its transaction services and the nominal interest rate paid by deposits and

bonds from date 0 to 1 is $i_{01} = \frac{q_1}{\frac{M_0}{P_{12}}} = \frac{P_{12}}{P_{02}} > 1$.

Sketch that (3) is the market clearing condition

The claim follows because (i) deposits are also claims to bonds and money (which together are worth the value of taxes and cash goods), and (ii) goods are stored when there is excess supply.

To see this, take the simple case where some late projects have to be restructured to meet aggregate liquidity demand, there are no cash goods, and aggregate bank assets just equal deposits. Since the real interest rate exceeds 1, there will be no storage. Now

$$d_2 = \frac{M_0 + B_2}{P_{12}} + \theta^G \gamma C + (1 - \theta^G) \left[\alpha^B \gamma C + \mu^B (1 - \alpha^B) c + (1 - \mu^B) (1 - \alpha^B) \frac{\gamma C}{r_{24}} \right]. \text{ We know from}$$

$$\text{Proposition 1 that } \frac{M_0 + B_2}{P_{12}} = \theta^G \frac{C}{1-t} + \frac{(1 - \theta^G)}{(1-t)} \left[\alpha^B C + \mu^B (1 - \alpha^B) c + (1 - \mu^B) (1 - \alpha^B) \frac{C}{r_{24}} \right].$$

Substituting this in the expression for d_2 and rearranging, we get

$$d_2 = \frac{1}{1-t} \left[\theta^G C + (1 - \theta^G) (\alpha^B C + (1 - \alpha^B) \mu^B c) \right] \\ - \theta^G (1 - \gamma) C - (1 - \theta^G) (1 - \gamma) \alpha^B C + (1 - \theta^G) (1 - \mu^B) (1 - \alpha^B) \frac{\gamma C}{r_{24}} + \frac{t}{(1-t)} (1 - \theta^G) (1 - \mu^B) (1 - \alpha^B) \frac{C}{r_{24}}.$$

On the right hand side, the second term and third term are the (negative of) goods left with early entrepreneurs (of G and B banks respectively) after repaying their banks. The fourth term is the value deposited in the B bank and the last term is the value of future claims on the government

that are sold at date 2 ($= \frac{tX_4}{r_{24}} = \frac{(M_2 + B_4)}{r_{24}P_{24}}$). For the early goods market to clear, the value of

future claims sold to early entrepreneurs should equal their holdings of early goods. So the terms on the second line of the right hand side sum to zero, leaving the condition on the first line that deposits should equal the pre-tax value of early goods produced, which is (3).

Proof of Proposition 2:

The aggregate liquidity market clearing condition is: $q_1 + \frac{1}{(1-t)} \left[\theta^G C + (1 - \theta^G) (\alpha^B C + (1 - \alpha^B) \mu^B c) \right] = d_2$.

This implies $\mu^B = \frac{(d_2 - q_1)(1-t) - C(\theta^G + \alpha^B(1 - \theta^G))}{c(1 - \alpha^B)(1 - \theta^G)}$. As a result,

$$\frac{\partial \mu^B}{\partial \theta^G} = -\frac{C - (d_2 - q_1)(1-t)}{c(1 - \alpha^B)(1 - \theta^G)^2} < 0 \text{ and } \frac{\partial \mu^B}{\partial \alpha^B} = -\frac{C - (d_2 - q_1)(1-t)}{c(1 - \alpha^B)^2(1 - \theta^G)} < 0.$$

These are negative because $C - (d_2 - q_1)(1-t) > 0$, which follows because if banks to avoid failure there must be sufficient liquidity to pay deposits in the best possible case, where all banks are of type G (which requires that $\frac{C}{1-t} + q_1 > d_2$). Of projects that are late, fewer are restructured

(more are continued, $1 - \mu^B$ increases) when there is more aggregate liquidity. In addition more

aggregate liquidity increases the total credit $\theta^G + (1 - \theta^G)\{\alpha^B + (1 - \alpha^B)(1 - \mu^B)\}$ because it is directly increasing in θ^G and α^B , and $1 - \mu^B$ is increasing in them.

Finally, $\frac{\partial \mu^B}{\partial d_2} = \frac{1-t}{c(1-\alpha^B)(1-\theta^G)} > 0$. An increase in real deposits requires increased

restructuring, this decreases total credit and the fraction of late projects that are continued.

Proof of Proposition 4:

When net nominal interest rates are positive, we have the real payout on deposits at date

2, $d_2 = \frac{\delta_0 q_1}{M_0}$. Clearly, this falls as M_0 increases. We showed in the proof of proposition 2 that a

fall in d_2 increases $(1 - \mu^B)$, the amount of credit extended at date 2 by the B type banks,

or $\frac{d\mu^B}{dd_2} > 0$, and therefore $\left. \frac{d(1-\mu^B)}{dM_0} \right|_{M_0+B_2=Const} > 0$.

Proof of Proposition 5:

(a) Suppose the nominal face value of a project loan is always L. This means that even when the project is late and the loan is restructured, the entrepreneur repays only L and keeps the residual of $P_{02}c - L$. Suppose also that the representative bank is solvent and liquid when $\bar{\alpha} = 1$. Because it is solvent, $L \geq \delta_2$ ($= \delta_0$). Then it is solvent at any other $\bar{\alpha}$. To see this, note that for any

realization $\bar{\alpha}$, the bank is solvent if $\bar{\alpha} \frac{L}{P_{02}} + (1 - \bar{\alpha})\mu \frac{L}{P_{02}} + (1 - \bar{\alpha})(1 - \mu) \frac{L^*i}{r_{24}P_{24}} \geq \frac{\delta_2}{P_{02}}$, where i is the

nominal interest rate at which the project loan is rolled over for late projects. But the inequality

holds if we set $\mu = 1$ because $L \geq \delta_2$. Since there is some μ at which the bank is solvent, and the

banker can always choose it rather than be run, the bank will always be solvent. Since the

liquidity condition does not depend on the actual repayment to the bank, the argument in footnote

9 is valid even for nominal loans. So if the representative bank is solvent and liquid at some $\bar{\alpha}'$, it

is solvent and liquid at any $\bar{\alpha}$.

(b) Sketch: Since the number of late projects, the fraction restructured, and the real interest rate

r_{24} all (weakly) increase as $\bar{\alpha}$ falls, the price level $P_{02} = P_{12} = \frac{M_0 + B_2}{t \left[X_2 + \frac{X_4}{r_{24}} \right]}$ will increase.

Now the solvency of the bank hinges on the rate i that loan repayments on late projects are rolled over at. Regardless of the precise bargaining process, a plausible assumption is that the value in date-2 consumption goods of the required payment on a continued loan does not increase as the value in date-2 consumption goods of a restructured loan falls. As $\bar{\alpha}$ falls, not only are more projects restructured but also the value in date-2 consumption goods of current nominal loan repayments of L will decrease because P_{02} increases (and hence, so (weakly) will the value in date-2 consumption goods of continued loans), as will the value of the financial assets on the bank's balance sheet. When the deposit repayments are fixed in real terms, the representative bank becomes weakly less solvent.