A New Explanation for Underdiversification *

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Abstract

Contrary to the prediction of the standard portfolio diversification theory, most investors invest a large fraction of their stock investment in a small number of stocks and less wealthy investors concentrate on even fewer stocks. We provide a new and somewhat surprising explanation that it can be exactly the need for risk reduction that causes underdiversification. Specifically, we assume that investors save more than what is necessary for survival and do not buy on margin or short sell. We show that investors always underdiversify when wealth is low and investors with CRRA or mean-variance preferences may always underdiversify no matter how wealthy they are. Furthermore, investors select stocks only by expected returns and any higher moments such as variance and skewness are irrelevant for this choice. In addition, less wealthy and less risk averse investors underdiversify more, and a less diversified portfolio (inclusive of the risk-free investment) may be less risky than a more diversified one. These main results still hold in equilibrium. Moreover, our equilibrium model shows that a less diversified stock portfolio has a higher expected return, a higher volatility, and maybe also a higher skewness and a lower Sharpe ratio, all consistent with existing empirical findings. Finally, underdiversification in equilibrium implies that idiosyncratic risks can be priced.

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1. Introduction

In contrast to the standard portfolio theory on the risk reduction effect of diversification, it is widely found that investors invest a large portion of their stock investment only in a small number of stocks. As Campbell (2006) stated in his American Finance Association presidential address, "Analyses of the Survey of Consumer Finances find that among households that hold individual stocks directly, the median number of stocks held was two until 2001, when it rose to three (Blume and Friend 1975, Kelly 1995, and Polkovnichenko 2005)." In addition, less wealthy investors underdiversify more and even the wealthy may underdiversify (e.g., Goetzmann and Kumar 2005, Polkovnichenko 2005). The existing literature has proposed many possible explanations, including various trading costs (e.g., Brennan 1975, Merton 1987, Nieuwerburgh and Veldkamp 2005, and Campbell 2006),[1] psychological factors and behavioral biases (e.g., Huberman 2001), and special preferences (e.g, Polkovnichenko 2005, Barberis and Huang 2005, Mitton and Vorkink 2007).

In this paper, we propose a new and somewhat surprising explanation that it can be exactly the need for risk reduction that causes underdiversification (defined as investing in only a proper subset of available stocks). This explanation does not require the existence of any type of stock trading costs, behavioral biases, or special preferences. Specifically, we consider a discrete-time model where an investor can trade one risk-free asset and multiple risky stocks. We assume that the investor cannot tolerate any lower utility than a certain level $\underline{u}$ and stock payoffs are unbounded from above and can be arbitrarily close to 0. Therefore, to guarantee the minimum utility

[1]While Vissing-Jørgensen (2002) concludes that small participation costs can generate significant underdiversification, Goetzmann and Kumar (2005) find that transaction costs and search costs do not affect investors' diversification choices.

level (and solvency), the investor do not borrow or short sell.[2] In addition, we assume that the minimum utility level $\underline{u}$ is strictly above the subsistence utility, which implies that the marginal utility at $\underline{u}$ is finite.[3]

Without the minimum utility requirement or the solvency constraint, as shown in the standard diversification theory, the investor always invests in all the available stocks, independent of his wealth level and cross-sectional return distributions. With the minimum utility requirement, however, we show that the investor always underdiversifies if wealth is sufficiently low. Moreover, under some reasonable conditions on cross-sectional return distributions, an investor with constant relative risk aversion (CRRA) or mean-variance preferences always underdiversifies no matter how wealthy he is. This finding suggests that our model can potentially explain the underdiversification puzzle for both the poor and the rich.

Furthermore, consistent with empirical findings, our model implies that a less wealthy investor holds a smaller of stocks. Somewhat surprisingly, an investor always selects stocks with highest expected returns, regardless of any higher moments such as variance and skewness.[4] This finding implies in particular that Sharpe ratios are

[2]Following most of the existing portfolio choice literature with stock trading constraints (e.g., Black 1972, Ross 1977, Jarrow 1980, Detemple and Murthy 1997), we implicitly assume that investors cannot trade derivative securities to undo the portfolio constraints. This assumption is consistent with the empirical evidence that the participation rate in option markets is much lower than that in stock markets. In addition, in an economy where stock payoffs are unbounded from above, can be arbitrarily close to 0, and investors can only trade in discrete times, to avoid possible insolvency no one will sell derivative securities such as call options that can help others get around these constraints.

[3]As Liu and Zhou (2006) point out, the riskless saving level can be positive and potentially high relative to the current wealth (especially for the poor and the underinsured), if significant future wealth shocks, such as those from disability and unemployment, can occur with a positive probability. The positive saving level can also exist due to habit formation (e.g., Constantinides 1990 and Dybvig 1995) among other causes. The minimum utility level can also be subjective and can vary across individuals and across time for the same individual. In particular, the wealth required to achieve the minimum level for the rich may be greater than that for the poor.

[4]See Footnote 12 for an explanation of why most investors do not hold out-of-money options even though they have high expected returns.

irrelevant for stock selection. In addition, we show that a more diversified portfolio (including the risk-free investment) is more risky in terms of a higher return volatility.[5]

We show that our main results hold in an equilibrium model that is solved in closed-form. In particular, less wealthy investors underdiversify. Since some investors underdiversify in equilibrium, idiosyncratic risks are priced in this economy, consistent with some existing empirical evidence (e.g., Campbell, Lettau, Malkiel and Xu 2001). In addition, this equilibrium model implies that as the degree of underdiversification increases, both the expected return and the volatility of the stock portfolio (excluding the risk-free asset) also increase. Moreover, under some reasonable conditions, a less diversified stock portfolio also has a higher skewness and a lower Sharpe ratio. A less diversified stock portfolio can also have a higher volatility-scaled-skewness (i.e., skewness/volatility). Furthermore, this equilibrium model predicts that less wealthy and less risk averse investors underdiversify more. All these implications are consistent with the empirical findings of Goetzmann and Kumar (2005) and Mitton and Vorkink (2007). Different from Mitton and Vorkink (2007), we do not assume that some investors prefer skewness and we show that skewness is irrelevant for stock selection. Our model shows that it is not necessary to assume skewness preference for an explanation of the empirical skewness pattern.

To understand the essential intuitions behind the above results, we examine how the optimal portfolio changes as wealth increases from the minimum consumption level $\underline{C}$ that is necessary for achieving the minimum utility. When the wealth is equal to the minimum level $\underline{C}$, the investor can only invest in the risk-free asset.

[5] Although in the main text we assume a one-period static model, the period we consider can be interpreted as one of the periods in a multiperiod dynamic model. Accordingly, we can interpret a wealth change as either across investors or across time for the same investor. Therefore, the model can have both cross-sectional and time-series implications.

Suppose now wealth increases such that the investor can invest a small amount $\varepsilon > 0$ in stocks. Since the marginal utility at $\underline{C}$ of investing a small amount in a stock is positively proportional to the stock's expected return and does not depend on any higher moments, the marginal utility from investing in the stock with the highest expected return is the greatest. Therefore, the investor first adds the stock with the highest expected return to his portfolio and any higher moments such as volatility and skewness are irrelevant for this choice. The fundamental economic intuition is as follows. Since the marginal utility is finite at the choice point, the investor is effectively holding a certain amount of risk-free asset. When the dollar amount $\varepsilon > 0$ that can be invested in additional stocks is small, the total risk of investing in a stock, $\varepsilon^2\times$ volatility, is very small no matter how large the volatility is. Therefore the investor only cares about the expected return. This shows that only expected return matters for stock selection and in particular Sharpe Ratio and skewness are irrelevant.

As wealth increases further, the investor invests more in the first stock and thus the portfolio risk increases, which reduces the marginal utility from investing more in this stock. When wealth increases to a critical level at which the marginal utility of investing more in the first stock reduces to the marginal utility of investing in another stock, the investor adds a second stock. The same argument for adding the first stock implies that the second stock must have the second highest expected return. This process continues as wealth further increases. For high enough wealth, it may be optimal for the investor to invest in all the stocks and thus fully diversify. However, in some cases (e.g., CRRA or mean-variance preferences), because of the borrowing and short sales constraints, the marginal utility of investing more in some stocks may be always greater than that of investing in a new stock. In these cases, no matter how rich the investor is, he always underdiversifies.

The main reason for the finding that a more diversified optimal portfolio (including the risk-free investment) is riskier than a less diversified one is that a more diversified portfolio implies that the investor invests more in the risky assets, *cetera paribus*. Consider the extreme case where an investor can only afford the minimum consumption. His portfolio risk is obviously 0 since he can only invest in the risk-free asset. As wealth increases, the investor becomes more diversified and also invests more in each of the held stocks and thus portfolio risk increases. This finding may shed some light on the well-known diversification discount puzzle, i.e., more diversified firms trade at a discount relative to less diversified ones (e.g., Lang and Stulz 1994 and Berger and Ofek 1995). Our model suggests that if a firm can lever up then it will diversify more. Thus firms that diversify more may have a higher leverage ratio and are therefore riskier than those less diversified ones. This intuition is consistent with a recent study by Duan and Li (2006) who find that this widely documented diversification discount puzzle disappears in almost all sample years once firm leverage levels are controlled for.

Since a more risk averse investor cares more about risks, he holds a greater number of stocks to reduce risk given the same wealth level. Because as wealth increases the investor sequentially adds stocks with the highest expected returns, a more diversified stock portfolio has a lower expected return. Due to the diversification benefit, the return of a more diversified stock portfolio has a lower volatility. Finally, the main reason why our equilibrium model can produce the empirical skewness pattern found in Mitton and Vorkink (2007) without assuming skewness preferences is that under some conditions on stock payoff distributions, a portfolio with a high expected return also has a high skewness.

The important assumptions that drive our main results are (1) the investor requires a minimum utility level that is higher than the subsistence level, in other words, the marginal utility at the minimum consumption level $\underline{C}$ is finite; (2) the investor does not borrow or short sell. However, neither of these assumptions are necessary for all the main results. For example, for an investor with a CRRA preference who does not require a minimum utility level, if the unconstrained portfolio requires leverage, then in the presence of the borrowing and short sales constraints, the investor will also underdiversify no matter how wealthy he is. This shows that the first assumption is unnecessary. The second assumption can be relaxed too. Indeed, as long as the total amount that an investor can invest in stocks is limited and short sales are restricted to some extent (e.g., the presence of a margin requirement imposed by Regulation T; e.g., Cuoco and Liu 2000), the investor may still underdiversify under some conditions on asset return distributions.[6] For the existence of a wealth effect on the degree of underdiversification, however, the first assumption is necessary, as can be seen from the case with a CRRA preference. We adopt the two assumptions so that our main results apply to a broad class of preferences and asset return distributions.

These two assumptions are consistent with conclusions of many empirical studies on household finance. For example, Bucks, Kennickell, and Moore (2006) report that the value of financial assets (including stocks, mutual funds, retirement accounts, checking and saving accounts, etc.) is only 35.7% of the total assets in 2004 and the

[6]The margin requirement imposed by Regulation T and considered by Cuoco and Liu (2000) takes the form $\eta_1^\top \max(w, 0) + \eta_2^\top \max(-w, 0) \leq 1$, where w is the portfolio weight vector, $\eta_1 \in \mathbb{R}^n_{++}$, and $\eta_2 \in \mathbb{R}^n_{++}$. In addition, η_1 and η_2 can be reinterpreted as risk-weights assigned by investors to a stock and the portfolio constraints become risk management constraints (similar to capital requirement constraints imposed by the Basel committee on financial institutions). With this generalized setup, we can show that. *cetera paribus*, investors only hold stocks with the lowest risk-weights. Therefore to the extent that investors may believe they know "local" stocks better and thus assign lower risk-weights to them, our model can also shed some light on the "local bias" or "home bias" phenomenon well-documented in the literature (e.g., French and Poterba 1991, Huberman 2001).

total stock holdings (directly or indirectly held) is 47.4% of the financial assets. This suggests that overall households only allocate a small fraction of wealth to stock investment and thus even if they lose all their stock investment their remaining wealth can still maintain a living standard above the subsistence level. The results of Anderson (1999) suggest that for a vast majority of investors they do not buy on margin, their net financial assets are positive, and thus overall they do not borrow to buy stocks. It has also been well known that individual investors rarely short sell. For example, Boehmer, Jones, and Zhang (2005) show that only about 1.5% of the short sales come from individual investors.

There is a large literature on how habit-formation preferences affect portfolio selection (e.g., Constantinides 1990 and Dybvig 1995).[7] Similar to our model, an investor with a habit-formation preference also invests in such a way to guarantee a certain minimum consumption level. The key difference is that in our model the marginal utility at the minimum consumption level is finite and investors face portfolio constraints. As discussed above, both of these are important for our main results. Anderson (1999) shows that most investors do not buy on margin and the percentage of margin financed purchase is around 1.5%. In addition, the net financial assets of vast majority of population is positive, implying that overall investors are not borrowing to buy stocks. As Individual investors rarely short sale

Clearly this paper is not the first to show that portfolio constraints can induce an investor to invest in only a subset of available stocks (e.g., Ross 1977, Dybvig 1984, and Cuoco and Liu 2006). However, most previous works assume either special preferences or special asset return distributions and offer only partial equilibrium

[7]In contrast to our model, both Constantinides (1990) and Dybvig (1995) consider a financial market with only a single risky asset.

analysis. In addition, these works are done in different contexts and for different purposes. For example, Ross (1977) and Dybvig (1984) examine the shape of a mean-variance efficient frontier with short-sale constraints and there is no wealth effect on the optimal number of stocks. In considering the impact of the value-at-risk based capital requirements on the riskiness of a financial institution, Cuoco and Liu (2006) restrict to CRRA preferences and lognormally distributed stock prices.

The rest of the paper is organized as follows. In Section 2, we describe the model setup, state the main results, and explain the essential intuitions. In Section 3, we analyze some numerical and graphical examples to illustrate the main results. In Section 4, we show that the main results hold in an equilibrium model and derive some additional empirical predictions. Section 5 concludes. In the Appendix, we prove the main results.

2. The Model

To provide the simplest possible model to explain the main ideas, we consider a one-period discrete-time model, where an investor with initial wealth $W_0 > 0$ can invest in one risk-free asset and $n \geq 1$ risky stocks and maximizes his expected utility from the end-of-period wealth W_1. The risk-free interest rate is normalized to 0. Let $\tilde{P}$ denote the end-of-period gross return vector of the stocks.

Assumption 1 *The gross return $\tilde{P}$ is unbounded above and can get arbitrarily close to* 0*.*

This assumption implies that to ensure solvency at the end-of-period (in the absence of continuous trading), the investor cannot borrow or short-sell. Let $\tilde{z} = \tilde{P} - 1$ be the return vector, $\mu \equiv (\mu_1, \mu_2, ..., \mu_n)^\top = E[\tilde{z}]$ be the expected return vector, and $\sigma\sigma^\top = E[\tilde{z}\tilde{z}^\top]$ be the variance-covariance matrix. In addition, for expositional simplicity, we assume that

$$\mu_1 > \mu_2 > ... > \mu_n > 0$$

and $\sigma^2 = diag\{\sigma_1^2, \sigma_2^2, ..., \sigma_n^2\}$ is diagonal with strictly positive diagonal elements, which implies that stock returns are uncorrelated.[8]

The investor derives utility $u(W_1)$ from the end-of-period wealth W_1, where $u(\cdot)$ is strictly concave and increasing. Different from the standard literature, we assume that

Assumption 2 *The investor cannot tolerate any lower utility than* $\underline{u} \in (u(0), u(W_0)]$*, i.e.,* $u(W_1) \geq \underline{u}$*.*

This assumption implies that the investor must invest to ensure that $W_1 \geq \underline{C}$, where $\underline{C} = u^{-1}(\underline{u})$ is the wealth level necessary to achieve the minimum utility $\underline{u}$.[9] In addition, we assume that

Assumption 3 *The marginal utility at* $\underline{C}$ *is finite, i.e.,* $u'(\underline{C}) < \infty$*.*

One interpretation of this assumption is that $\underline{C}$ is above the subsistence consumption level. In other words, even if the investor loses the entire investable amount $W_0 - \underline{C}$, his living standard is still above the subsistence level.

Let θ denote the column vector of the dollar amount invested in the stocks. Then the investor's problem is to

$$\max_{\theta} E\left[u(W_1)\right] \tag{1}$$

[8]Assuming uncorrelated returns biases against us since diversification is most effective when returns are uncorrelated. See footnote 19 for a discussion on the correlated case. The assumption of positive expected excess returns is only for expositional simplicity. Because of the short-sales constraints, the main results on underdiversification hold obviously if some expected excess returns are negative.

[9]It is worth noting that the initial wealth W_0 may be interpreted as the net present value of all current assets and future endowment (e.g., houses, labor income) and the minimum level $\underline{C}$ may be interpreted as the present value of all future minimum consumption level.

subject to

$$W_1 = W_0 + \theta^{\top} \tilde{z} \geq \underline{C}. \tag{2}$$

Given that $\tilde{z}$ is unbounded above and can be arbitrarily close to -1, constraint (2) is equivalent to (in the absence of continuous trading):

$$\theta^{\top} \bar{1} \leq W_0 - \underline{C} \quad \text{and} \quad \theta \geq 0, \tag{3}$$

where $\bar{1}$ is the column vector of 1's. Therefore the investor's problem is equivalent to

$$\max_{\theta} E\left[u(W_1)\right], \quad \text{subject to (3).} \tag{4}$$

Now we collect our main analytical results in the following theorem.

Theorem 1 *We have:*

1. *For low enough initial wealth, the investor always underdiversifies. In addition, full diversification (i.e., investment in all available stocks) is optimal for all $W_0 > \underline{C}$ if and only if the expected returns are exactly the same across all stocks;*

2. *As the initial wealth increases, the investor holds more stocks;*

3. *Which stocks the investor holds in the optimal portfolio only depends on their expected returns, but not on any higher moments (e.g., variance, skewness) and thus in particular Sharpe Ratio is irrelevant for stock selection;*

4. *As the initial wealth increases, both the expected value and the variance of the end-of-period wealth increase. If the investor has a constant relative risk aversion (CRRA) utility, then as the initial wealth increases, both the expected return and the return variance of the optimal portfolio increase;*

5. *If the investor has a CRRA utility, then under some conditions on the asset returns, it is optimal for the investor to always* underdiversify*, no matter how wealthy he is.*

Proof: see Appendix.

Several observations are in order. First, Part 1 of Theorem 1 implies that it cannot be optimal for investors to fully diversify for *all* wealth levels. This is easy to see in the extreme case where $W_0 = \underline{C}$, since in this case the investor can only invest in the risk-free asset. Even if the initial wealth W_0 is strictly above the minimum wealth level $\underline{C}$, it is still rare that investors should always fully diversify, since this occurs only when the expected returns are exactly the same across all stocks.

Second, the proof of Theorem 1 reveals how the optimal portfolio compositions change as the initial wealth increases. As stated above, if $W_0 = \underline{C}$, then the investor can only invest in the riskless asset. When the initial wealth is slightly above the minimum level, the investor adds the stock with the highest expected return (i.e., Stock 1) to his portfolio. He increases his investment in Stock 1 as the initial wealth increases until it reaches a critical level, say $\hat{W}_2$, when he adds the stock with the second highest expected return (i.e., Stock 2). Then he increases his investment in both Stock 1 and Stock 2 as the initial wealth increases until it reaches the next critical level, say $\hat{W}_3$, when he adds the stock with the third highest expected return (i.e., Stock 3). This process continues as the initial wealth further increases. If the borrowing and short-sales constraints are not binding when the wealth is high enough, then eventually the investor invests in all the n stocks and thus fully diversifies.[10] The above process shows that, somewhat surprisingly, the order in which stocks are

[10]Theorem 1 implies that an investor can add multiple stocks at the same time if these stocks have the same expected returns. See Figure 3 for an illustration.

added only depends on the expected returns, but not on the volatilities or any higher moments.[11] To understand this result, suppose wealth increases from the minimum level such that the investor can invest a small amount $\varepsilon > 0$ in stocks. Since the final wealth W_1 is affine in asset returns, the marginal utility at $\underline{C}$ of investing a small amount in a stock is positively proportional to the stock's expected return and does not depend on any higher moments. Thus, the investor first adds the stock with the highest expected return to his portfolio and any higher moments such as volatility and skewness are irrelevant for this choice. The same argument applies to the subsequent additions of other stocks, as illustrated in Example 1 in the next section. The fundamental intuition is as follows. Since the marginal utility is finite at the choice point, the investor is effectively holding a certain amount of risk-free asset. When the dollar amount $\varepsilon > 0$ that can be invested in additional stocks is small, the total risk of investing in a stock (e.g., $\varepsilon^2\sigma_i^2$) is very small no matter how large the stock volatility (e.g., σ_i) is. Therefore the investor only cares about the expected return and adds the stock with the next highest expected return irrespective of any higher moments. This shows that only expected return matters for stock selection and in particular Sharpe Ratio and skewness are irrelevant. On the other hand, it is important to note that while risk is irrelevant for stock selection, it significantly affects how much is invested in a stock once the stock is selected. For a highly risky asset, the investor may add it first, but the amount invested in the asset can be very small and thus a small trading cost would prevent him from holding it at all.[12]

[11] If stock returns were correlated, then the covariance would also affect such order, as discussed in footnote 19.

[12] This may reconcile our model predictions with the fact that most investors do not hold out-of-money options even though they have high expected returns, since these options are highly risky and option trading costs are nontrivial.

Third, Parts 1 and 4 of Theorem 1 implies that as the number of stocks that investor optimally holds decreases, the riskiness of the portfolio also decreases. This shows that a less diversified portfolio is *less* risky than a more diversified one, which is in contrast to the conventional wisdom that greater diversification implies less risk. Theorem 1 suggests that the imposition of the portfolio constraints does achieve the goal of reducing risk. The reduction in risk stems from the reduction of the total amount invested in the stocks. This finding can potentially shed some light on the well-known diversification discount puzzle. Our model suggests that more diversified firms may be borrowing more and thus have a higher leverage ratio and greater risks than less diversified firms. Therefore, for firms that diversify more, if the gain in the expected return is not sufficient to justify the increase in the risk, investors rationally discount these firms. This intuition is consistent with a recent study by Duan and Li (2006) who find that the widely documented diversification discount disappears in almost all sample years once firm leverage levels are controlled for.

Finally, Part 5 of Theorem 1 suggests under some conditions on preferences and return distributions, no matter how wealthy an investor is, he never fully diversifies. To understand this result, consider the case with a CRRA utility. In this case, the optimal fraction w_i of the initial wealth W_0 invested in stock i is independent of W_0 in the absence of portfolio constraints, for $1 \leq i \leq n$. If this portfolio requires leverage, i.e., $\sum_{i=1}^{n} w_i > 1$, then when the investor is subject to the constraints (3), he can never hold this portfolio. If in addition, some of the expected returns are low enough, then since he cannot short sell either, he will not hold these stocks with the lowest expected returns. This result is consistent with the empirical evidence that even the rich may underdiversify. On the other hand, it is in sharp contrast with the result in the standard unconstrained model, where the investor always fully diversifies, no matter how poor he is. This explanation of underdiversification does not require any types of costs for trading, behavioral biases, or special preferences, as proposed by the existing literature.

3. Some examples

In this section, we provide some numerical and graphical examples to illustrate the main ideas behind our main results. For simplicity, we specialize to the mean-variance preference case, which provides the most transparent economic intuitions.[13]

Suppose there are $n > 0$ uncorrelated stocks in addition to the risk-free asset that the investor can hold and the interest rate is normalized to 0. Let $w \equiv \theta/W_0$ be the initial portfolio weight vector of dimension $n \times 1$. Then (3) is equivalent to

$$w^\top \bar{1} \leq 1 - \frac{C}{W_0}, \quad w \geq 0. \tag{5}$$

The mean-variance investor then solves the following problem:

$$\max_w \left[w^\top \mu - \frac{1}{2} A w^\top (\sigma\sigma^\top) w \right],$$

subject to (5), where $A > 0$ measures the investor's risk aversion.

The mean-variance efficient portfolio is

$$w^{MV} = \frac{1}{A}(\sigma\sigma^\top)^{-1}\mu. \tag{6}$$

Therefore in the absence of portfolio constraints (5), as implied by the standard diversification theory, no matter how poor the investor is and no matter how low the Sharpe ratios are for some stocks, he always invests in the tangency portfolio on the mean-variance efficient frontier that includes *all* of the available stocks. As shown

[13] A detailed analysis with a closed-form solution to this case is available from the author.

below, these results no longer hold in the presence of portfolio constraints (5) implied by Assumptions 1 and 2.

Example 1. $\mu_1 = 0.1$, $\sigma_1 = 0.5$, $\mu_2 = 0.09$, $\sigma_2 = 0.25$, $A = 2$, and $\underline{C} = 1$. The Sharpe ratios of Stocks 1 and 2 are therefore 0.2 and 0.36, respectively. The tangency portfolio weights are $w_1^{MV} = \mu_1/(A\sigma_1^2) = 0.2$ and $w_2^{MV} = \mu_2/(A\sigma_2^2) = 0.72$.

1. Since $w_1^{MV} + w_2^{MV} = 0.92 < 1$, the investor invests in the mean-variance efficient portfolio if his wealth $W_0 \geq 1/(1 - 0.92) = 12.5$. The portfolio risk (i.e., the return variance of the portfolio that includes the risk-free investment) is $\left(w_1^{MV}\right)^2 \sigma_1^2 + \left(w_2^{MV}\right)^2 \sigma_2^2 = 0.0424$, which yields a volatility of 0.206.

2. At the other extreme with $W_0 = 1$, the investor can only invest in the risk-free asset. So the risk of his portfolio is zero. This extreme case illustrates the basic intuition behind one of the main results that a less diversified portfolio can have a lower risk than a more diversified one: While an investor underdiversifies, he also reduces his investment in the risky assets and thus the total risk is reduced.

3. Now suppose the investors wealth starts to increase such that he can invest a small amount $\varepsilon > 0$ in the stocks (i.e., wealth increases to $1/(1 - \varepsilon)$). Will he hold both stocks? If not, will he hold only the stock with the highest Sharpe ratio (i.e., Stock 2)? To answer these questions, suppose he invests a fraction w of ε in Stock 1 and a fraction $1 - w$ of ε in Stock 2. Then his utility is

$$U = [w\varepsilon\mu_1 + (1 - w)\varepsilon\mu_2] - \left[\frac{1}{2}A\varepsilon^2(w^2\sigma_1^2 + (1 - w)^2\sigma_2^2)\right]. \tag{7}$$

   It is obvious that for small enough ε, the optimal solution is always $w^* = 1$ as long as $\mu_1 > \mu_2$, because the second bracketed term is dominated by the first one. This implies that when ε is small enough, the investor will only invest in one stock and it is the stock with the highest expected return. In particular, the volatility of the stock (and thus the Sharpe ratio) is irrelevant for this choice. This analysis illustrates the basic intuition behind another main result that the order in which stocks are added to (or dropped from) the optimal portfolio only depends on their expected returns but not on their risks. Essentially, when the amount that can be invested in a stock is small, the expected return has a first-order effect, while the variance has only a second-order effect.

Equation (7) also implies that if $\mu_1 = \mu_2$ then the investor always invests in both stocks no matter how small ε is, because of the diversification benefit.

4. Then at what wealth level will the investor add Stock 2 to his portfolio? Suppose it is at the point $W_0 = \hat{W}_2$. Then it must be that the utility from investing $\alpha \equiv (1 - 1/\hat{W}_2)$ in Stock 1 is almost the same as the utility of investing $\varepsilon > 0$ in Stock 2 and $\alpha - \varepsilon$ in Stock 1. To be precise, we must have: As ε goes to 0,

$$\frac{\left(\mu_1\alpha - \frac{1}{2}A\alpha^2\sigma_1^2\right) - \left(\mu_1(\alpha-\varepsilon) + \mu_2\varepsilon - \frac{1}{2}A[(\alpha-\varepsilon)^2\sigma_1^2 + \varepsilon^2\sigma_2^2]\right)}{\varepsilon} \to 0,$$

which simplifies to

$$(\mu_1 - \mu_2) - A\alpha\sigma_1^2 = 0.$$

Therefore we must have

$$\alpha = \frac{\mu_1 - \mu_2}{A\sigma_1^2}, \tag{8}$$

which is equal to 0.02 in this example and implies that $\hat{W}_2 = 1.02$; i.e., when wealth is 2% above the minimum level, he adds Stock 2.

At $\hat{W}_2$, the volatility of the portfolio return is

$$\alpha\sigma_1 = 0.01,$$

which is greater than the risk in Part 2 and less than the risk in Part 1. This shows that as the number of the stocks held decreases, the portfolio risk also decreases.

Also note that α in Equation (8) and thus $\hat{W}_2$ depends only on the expected return of Stock 2, but not on its risk. so the order in which stocks are added is independent of return volatilities.

5. As wealth grows beyond $\hat{W}$, the investor increases the holdings of both stocks, but at different speeds. The increase in the holding of Stock 2 is faster than that of Stock 1, until the wealth reaches 12.5, above which he always invests in the tangency portfolio as specified above. Note that in the tangency portfolio, he invests more in Stock 2. So he invests more in Stock 2 when he is rich and invests more in Stock 1 when he is poor. In this sense, the poor cares more about expected returns and less about risks.

Example 2. Suppose $\mu_1 = 0.15$, $\sigma_1 = 0.2$, $\mu_2 = 0.05$, $\sigma_2 = 0.15$, $A = 2$, and $\underline{C} = 1$. The mean-variance efficient portfolio then changes to $w_1^{MV} = 1.875$ and $w_2^{MV} = 1.111$.

Similar arguments to those in Example 1 imply that as wealth increases from 1, the investor first adds Stock 1 to his portfolio. However, different from Example 1, the investor will *never* hold Stock 2, no matter how wealthy he becomes. Solving Equation (8) yields that $\alpha = 1.25$, which is greater than 1. Since the investor can never lever (i.e., $1 - 1/W_0 \leq 1$), no matter how wealthy he is, the risk from investing in Stock 1 can never be large enough to offset the benefit of higher expected return from Stock 1. Therefore, the investor will never hold Stock 2. This result shows that consistent with empirical evidence, even wealthy investors can underdiversify.

Equation (8) also implies that in this example as long as the expected return of

Stock 2 is below $\mu_1 - A\sigma_1^2 = 0.07$, no matter how high its Sharpe ratio is and no matter how wealthy the investor is, he will never hold Stock 2. Suppose now there are $n > 0$ uncorrelated stocks in addition to Stock 1. Then the same argument implies that as long as the highest expected return of the additional n stocks is below 0.07, the investor will only hold Stock 1, no matter how wealthy he is and no matter how high the Sharpe ratios of the additional stocks are. This example suggests using Sharpe ratios to measure the attractiveness of a stock can be quite misleading.

Next, we provide some graphical examples to illustrate our main results, again assuming mean-variance preferences for expositional simplicity. Figure 1 plots the optimal portfolio weights as functions of the initial wealth W_0. The figure shows that as the initial wealth increases from $\underline{C}$, the investor first adds the stock with the highest expected return, i.e., Stock 1. As the wealth further increases, he increases the fraction of wealth invested in Stock 1. When the wealth reaches 1.07 (7% above the minimum level $\underline{C}$), he adds stocks 2 and 3 simultaneously, because they have the second highest expected returns. As the wealth increases beyond 1.07, the investor first increases the investment in Stocks 1, 2, and 3 and then adds Stock 4 when it reaches 1.35.

Figure 1 suggests that the order of the dollar amount invested in different stocks can be different for the rich and the poor. For example, for a rich investor, he invests more in the second stock than in the first stock. However, for a poor investor, he invests more in the first stock than in the second stock and may not even invest in the second stock at all. This is because, as discussed before, for the order in which a stock is selected, only the expected return matters and the second stock has a lower expected return than the first one.

Figure 2 plots the expected return and the variance of the optimal portfolio as functions of the initial wealth W_0. It shows that as wealth decreases, both the expected return and the variance decrease. Since as Figure 1 shows, the number of stocks in the optimal portfolio also decreases as wealth decreases, this in particular suggests that a less diversified portfolio has a lower risk than a more diversified one. The main reason for the decrease in the risk as wealth falls is that the constraints (3) become more binding and thus the investor can invest less in stocks. This suggests that imposing portfolio constraints like (3) can be effective in reducing portfolio risk.

Figure 3 plots the optimal number of stocks held in the portfolio for two risk aversion levels in the presence of 50 stocks. This figure shows that the number of stocks an investor holds is quite small with a maximum of 10 for the wealthy,[14] which is consistent with the empirical finding that the median number of stocks held by investors is small.

In addition, Figure 3 suggests that as the number of stocks increases, the marginal benefit of diversification decreases and therefore needs a greater wealth increase to hold one additional stock in the portfolio. Indeed, when the risk aversion coefficient $A = 3$, no matter how wealthy an investor is, he never invests in more than the 10 stocks with the highest expected returns. The model thus has another empirically testable implication that the number of stocks held is more stable for the rich than for the poor and also the poor trades more often in and out a stock than the rich.

Moreover, the figure also shows that as risk aversion increases, the number of stocks held in the optimal portfolio increases. So less risk averse investors underdiversify more. This is because the diversification benefit is more important for the more risk averse investors.

[14]As implied by the result that an investor always holds stocks with the highest expected returns, adding more stocks would not increase the maximum number of stocks held as long as these additional stocks have lower expected returns than the 11th stock.

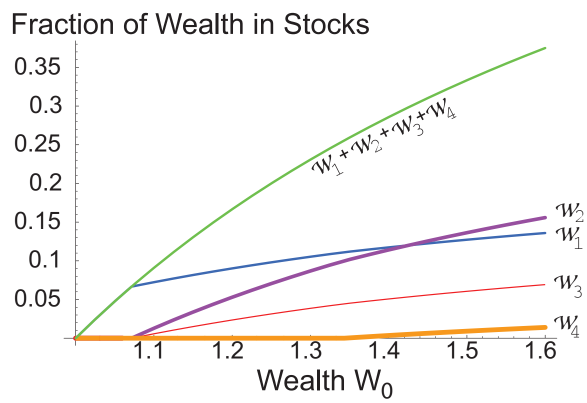

Figure 1: Portfolio weights against wealth W_0 for parameters $\mu_1 = 0.1$, $\sigma_1 = 0.3$, $\mu_2 = 0.07$, $\sigma_2 = 0.2$, $\mu_3 = 0.07$, $\sigma_3 = 0.3$, $\mu_4 = 0.05$, $\sigma_4 = 0.4$, and $A = 5$.

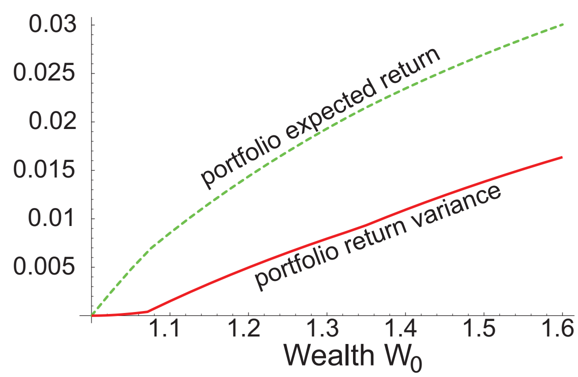

Figure 2: Portfolio expected return and variance against W_0 for parameters $\mu_1 = 0.1$, $\sigma_1 = 0.3$, $\mu_2 = 0.07$, $\sigma_2 = 0.2$, $\mu_3 = 0.07$, $\sigma_3 = 0.3$, $\mu_4 = 0.05$, $\sigma_4 = 0.4$, and $A = 5$.

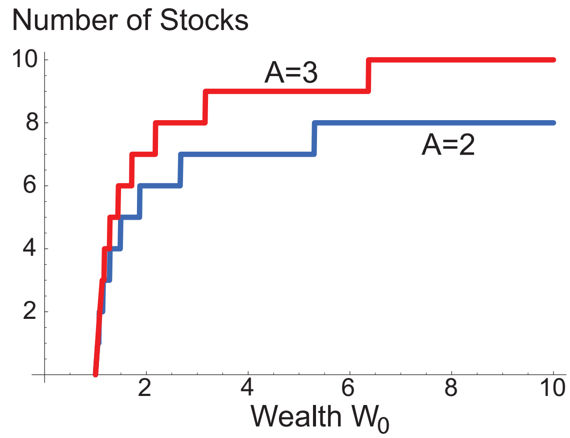

Figure 3: Number of stocks held against W_0 for the following parameters: for $i = 1, 2, ..., 50$, $\mu_i = 0.2-(0.2-0.01)(i-1)/50$, and $\sigma_i = 0.3-(0.3-0.2)(i-1)/50$.

4. An equilibrium model for underdiversification

In this section we show that underdiversification can be optimal in equilibrium. We again consider a one-period model where investors maximize the expected utility from the final wealth. For simplicity, we assume investors have the same constant absolute risk aversion (CARA) preferences,[15] i.e.,

$$u(W) = -e^{-AW},$$

where $A > 0$ is the CARA coefficient. There is one storable consumption good which is the only risk-free asset in the economy and there are $n > 0$ risky stocks. We choose the consumption good as the numeraire and thus the interest rate is normalized to 0. For $1 \leq i \leq n$, there are a total supply of $\bar{\omega}_i$ shares of Stock i, whose payoff per share $\tilde{P}_i$ is Gamma distributed with parameters $\alpha_i > 1$ and $\beta_i > 0$ such that its mean is $\eta_i = \alpha_i \beta_i$ and its variance is $\varphi_i^2 = \alpha_i \beta_i^2$.[16] The payoffs are independently

[15] Allowing different risk aversions is an straightforward extension and is not important for our purposes.

[16] Gamma distributions have similar properties to those of lognormal distributions and give us closed form solutions.

distributed across all stocks. In addition, there are two groups of investors: the rich, with mass 1 and the poor, with mass $\lambda \geq 0$. Both types of investors require the minimum consumption level of $\underline{C}$. The rich is endowed with $W_r \geq \underline{C}$ units of the consumption good and $\bar{\omega}_i > 0$ shares of Stock i, $1 \leq i \leq n$. The poor is endowed with $W_p \geq \underline{C}$ units of the consumption good but no stocks. Since stock payoffs are unbounded above and can get arbitrarily close to 0, to remain solvent, no one in the economy can borrow or short sell. Let p denote the initial equilibrium stock price vector. The rich solves

$$\max_{\omega} E\left[-e^{-AW_1}\right], \tag{9}$$

s.t.,

$$W_1 = W_r + (\bar{\omega}^\top - \omega^\top)p + \omega^\top \tilde{P} \geq \underline{C}, \tag{10}$$

where ω is the column vector of the number of shares held in stocks and $\tilde{P}$ is the column vector of stock payoffs.

First we consider the case without the poor, i.e., $\lambda = 0$. In this case, since any equilibrium would be a no-trade equilibrium and $W_r \geq \underline{C}$, the constraints are not binding. Then (9) becomes

$$\max_{\omega} E\left[-e^{-A(W_r+\bar{\omega}^\top p+\omega^\top(\tilde{P}-p))}\right] = -e^{-A(W_r+\bar{\omega}^\top p)} \min_{\omega} \frac{e^{A\omega^\top p}}{\prod_{i=1}^{n}(1+A\omega_i\beta_i)^{\alpha_i}}. \tag{11}$$

The first order conditions then imply that

$$\omega_i = \left(\frac{\eta_i}{p_i} - 1\right)\frac{\eta_i}{A\varphi_i^2}, \tag{12}$$

which yields

$$p_i = \frac{\eta_i^2}{\eta_i + A\omega_i\varphi_i^2}. \tag{13}$$

The market clearing condition $\omega_i = \bar{\omega}_i$ then yields the equilibrium price

$$p_i = \frac{\eta_i^2}{\eta_i + A\bar{\omega}_i\varphi_i^2}, \tag{14}$$

which implies that the equilibrium expected gross return of Stock i is

$$\mu_i = \frac{\eta_i}{p_i} = 1 + A\bar{\omega}_i \frac{\varphi_i^2}{\eta_i}.$$

To facilitate exposition, we assume the parameters are such that

$$\mu_1 > \mu_2 > ... > \mu_n. \tag{15}$$

Now we consider the case with the poor in the economy, i.e., $\lambda > 0$. Suppose first that $W_p = \underline{C} + \varepsilon$ where $\varepsilon > 0$ is small enough so that the poor can only invest ε in the stocks. Since the marginal utility of investing in Stock i at $\underline{C}$ is

$$-Ae^{-A\underline{C}}\mu_i,$$

the poor will only buy Stock 1 which provides the greatest marginal utility. Since the poor will only buy Stock 1 and by (12) the rich's demand for a stock is independent of any other stocks, the equilibrium prices of other stocks will remain the same. Let $\hat{p}_1$ be the new equilibrium price of Stock 1. The market clearing condition for Stock 1 becomes

$$\lambda \frac{\varepsilon}{\hat{p}_1} + \left(\frac{\eta_1}{\hat{p}_1} - 1\right) \frac{\eta_1}{A\varphi_1^2} = \bar{\omega}_1,$$

which implies the new equilibrium price for Stock 1 is

$$\hat{p}_1 = \frac{\eta_1^2 + \lambda A \varphi_1^2 \varepsilon}{\eta_1 + A\bar{\omega}_1 \varphi_1^2} = p_1 \left(1 + \lambda A \varepsilon \frac{\varphi_1^2}{\eta_1^2}\right). \tag{16}$$

By (15), there exists a small enough $\varepsilon > 0$ such that (15) still holds with μ_1 replaced by $\hat{\mu}_1 \equiv \frac{\eta_1}{\hat{p}_1}$. Therefore for small enough $\varepsilon > 0$ the poor indeed only buys Stock 1 in equilibrium.

Now consider when the poor is about to invest in the ith stock. Suppose it is at $\hat{W}_i$. Let δ_j be the dollar amount invested in Stock j, $1 \leq j \leq i$. Let $\bar{p}_j$ be the new equilibrium price for Stock j, $1 \le j \le i$. Since at $\hat{W}_i$, the poor has not bought Stock i yet, the equilibrium price for Stock i remain unchanged from the case with $\lambda = 0$, i.e., $\bar{p}_i = p_i$. Let

$$V(\delta_1, \delta_2, ..., \delta_i) = E\left[-Exp\left[-A\left(\underline{C} + \sum_{j=1}^{i} \frac{\delta_j}{\bar{p}_j}\tilde{P}_j\right)\right]\right] \quad (17)$$

$$= -e^{-A\underline{C}} \prod_{j=1}^{i} \left(1 + \delta_j \frac{A\beta_j}{\bar{p}_j}\right)^{-\alpha_j} \quad (18)$$

be the value function. For it to be optimal to invest in the i stocks, we must have the marginal utility of investing in each is the same at $\delta_i = 0$. Therefore a little algebra implies that

$$\frac{\eta_j}{\bar{p}_j + A\delta_j\beta_j} = \frac{\eta_i}{p_i}, \quad j = 1, 2, ..., i-1. \quad (19)$$

Note that (19) implies that in the new equilibrium $\bar{\mu}_j \equiv \frac{\eta_j}{\bar{p}_j} > \mu_i \equiv \frac{\eta_i}{p_i}$, for $1 \le j < i$. So the poor will indeed invest in the first $i-1$ stocks at $\hat{W}_i$. Given the investment of δ_j in Stock j, similar argument to that for (16) implies that the new equilibrium price of Stock j is

$$\bar{p}_j = \frac{\eta_j^2 + \lambda A\varphi_j^2\delta_j}{\eta_j + A\bar{\omega}_j\varphi_j^2}, \quad j = 1, 2, ..., i-1. \quad (20)$$

Plugging (20) into (19), we have

$$\delta_j = \frac{\eta_j - \frac{\eta_i}{p_i}\left(\frac{\eta_j^2}{\eta_j + A\varphi_j^2\bar{\omega}_j}\right)}{A\frac{\eta_i}{p_i}\left(\frac{\lambda\varphi_j^2}{\eta_j + A\varphi_j^2\bar{\omega}_j} + \frac{\varphi_j^2}{\eta_j}\right)} = \frac{\eta_j^2(\mu_j/\mu_i - 1)}{A\varphi_j^2(\lambda + \mu_j)}(> 0), \quad (21)$$

which can be inserted into (20) to obtain the new equilibrium price:

$$\bar{p}_j = p_j \frac{\lambda/\mu_i + 1}{\lambda/\mu_j + 1}, \quad (22)$$

which implies that $\bar{\mu}_1 > \bar{\mu}_2 > ...\bar{\mu}_{i-1} > \mu_i > ... > \mu_n$. The threshold wealth level for adding Stock i is thus

$$\hat{W}_i = \underline{C} + \sum_{j=1}^{i-1} \delta_j.$$

Note that only the expected return μ_i affects δ_j and the threshold wealth level $\hat{W}_i$. Since as i increases, μ_i decreases and thus δ_j increases, the critical wealth level $\hat{W}_i$ also increases as i increases. Since the above derivation applies to any $i = 2, 3, ..., n$, we have shown that for $1 \leq i \leq n$, the poor holds the stocks with the highest $i - 1$ expected returns if and only if the initial wealth $W_0 \in (\hat{W}_{i-1}, \hat{W}_i]$.

Since the threshold wealth levels for holding additional stocks decrease with the risk aversion coefficients, our model predicts that less risk averse investors underdiversify more. To the extent that younger and male investors are less risk averse, our model may help explain the finding by Mitton and Vorkink (2007) that younger and male investors tend to underdiversify more.

Equation (22) also implies that the expected return of Stock j decreases from μ_j to μ_i as λ increases from 0 to ∞, since the new equilibrium price increases from p_j to $p_j \frac{\mu_j}{\mu_i}$. This result shows that for a finite mass of the poor, the expected returns will always remain the same order as in the case with $\lambda = 0$.

The above equilibrium analysis shows that when the initial wealth of the poor is low, underdiversification is optimal. In addition, as the initial wealth of the poor increases, the poor adds sequentially the stocks with the highest expected returns and any higher moments are irrelevant for this choice.

Next we show that as the initial wealth of the poor increases the expected value and the variance of the end-of-period wealth both increase.

First, for $W_0 = \underline{C} + \varepsilon$ and small enough $\varepsilon > 0$ (i.e., $\varepsilon < \frac{\eta_1^2(\mu_1/\mu_2 - 1)}{A\varphi_1^2(\lambda + \mu_1)}$ by (21)), (16) implies that the expected end-of-period wealth is

$$\underline{C} + \varepsilon \frac{\eta_1}{\hat{p}_1} = \underline{C} + \varepsilon \frac{\eta_1(\eta_1 + A\bar{\omega}_1\varphi_1^2)}{\eta_1^2 + \lambda A\varphi_1^2\varepsilon},$$

and the variance of the end-of-period wealth is

$$\left(\frac{\varepsilon}{\hat{p}_1}\right)^2 \varphi_1^2 = \varepsilon^2 \frac{\varphi_1^2(\eta_1 + A\bar{\omega}_1\varphi_1^2)^2}{(\eta_1^2 + \lambda A\varphi_1^2\varepsilon)^2},$$

both of which increase as ε increases. Now suppose $W_0 \in (\hat{W}_i, \hat{W}_{i+1}]$ and so the poor optimally holds i stocks for some $1 \leq i \leq n$. The marginal utilities of investing in each of these i stocks must be equal, which by (17) implies that

$$\frac{\eta_j}{\check{p}_j + A\check{\delta}_j\beta_j} = k, \quad j = 1, 2, ..., i, \tag{23}$$

for some $\mu_i > k > \mu_{i+1}$, where $\check{p}_j$ is the equilibrium price of Stock j and $\check{\delta}_j$ is the dollar amount invested in Stock j in equilibrium. Similar derivation to that for (21) shows that

$$\check{\delta}_j = \frac{\eta_j^2(\mu_j - k)}{A\,k\varphi_j^2(\lambda + \mu_j)}, \tag{24}$$

and

$$\check{p}_j = p_j \frac{\lambda + k}{k(\lambda/\mu_j + 1)}. \tag{25}$$

Since $W_0 = \underline{C} + \sum_{j=1}^{i} \check{\delta}_j$, equation (24) implies that as W_0 increases, k decreases. The expected end-of-period wealth is

$$\underline{C} + \sum_{j=1}^{i} \frac{\check{\delta}_j}{\check{p}_j}\eta_j,$$

and the variance of the end-of-period wealth is

$$\sum_{j=1}^{i} \left(\frac{\check{\delta}_j}{\check{p}_j}\right)^2 \varphi_j^2.$$

Since $\frac{\check{\delta}_j}{\check{p}_j}$ increases as k decreases and k decreases as the initial wealth increases, we conclude that as the initial wealth increases both the expected wealth and the wealth variance increase.

This equilibrium model also yields the following additional results.

Proposition 1 *In this equilibrium model,*

1. both the expected return and the return volatility of a less diversified stock portfolio is higher than those of a more diversified one;

2. if for $1 \leq i \leq n$,

$$\frac{\sum_{j=1}^{i} C_{3j}\mu_j}{\sum_{j=1}^{i} C_{3j}} \geq \frac{\sum_{j=1}^{i} a_j\mu_j}{\sum_{j=1}^{i} a_j} \tag{26}$$

and

$$\frac{\sum_{j=1}^{i} C_{3j}\mu_j^3}{\sum_{j=1}^{i} C_{3j}\mu_j^2} + \frac{\sum_{j=1}^{i} C_{3j}\mu_j}{\sum_{j=1}^{i} C_{3j}} \geq 2\frac{\sum_{j=1}^{i} C_{3j}\mu_j^2}{\sum_{j=1}^{i} C_{3j}\mu_j}, \tag{27}$$

where $C_{3j} = \eta_j^4/\varphi_j^7$ *and* $a_j = \eta_j^2/((\lambda + \mu_j)\varphi_j^2)$, *then the return skewness of a less diversified stock portfolio is also higher than that of a more diversified one;*

2. if for $1 \leq i \leq n$,

$$\frac{\sum_{j=1}^{i} C_{1j}\mu_j^2}{\sum_{j=1}^{i} C_{1j}\mu_j} > \frac{\sum_{j=1}^{i}(C_{1j} - a_j)\mu_j}{\sum_{j=1}^{i}(C_{1j} - a_j)} \tag{28}$$

and λ *is small enough, the Sharpe ratio of a less diversified stock portfolio is lower than that of a more diversified one.*

Therefore our model predicts that a less diversified stock portfolio has a greater expected return and a higher volatility. In addition, under certain conditions such as (26), (27), and (26),[17] it also has a higher skewness and a lower Sharpe ratio. These predictions are supported by the findings of Mitton and Vorkink (2007). Different from Mitton and Vorkink (2007), we do not assume some investors prefer skewness.

Figure 4 shows that the number of stocks increases as the poor's initial wealth increases. Figure 5 confirms that a less diversified portfolio has a higher expected return and also a higher volatility. In addition, it shows that the Sharpe ratio of a

[17] To see that these conditions can be satisfied, note that the terms in these inequalities are just different weighted averages of the individual expected returns (μ_j's) in the absence of the poor. To satisfy these conditions, one can change these weights by varying parameters such as η_j, φ_j, and $\bar{\omega}_j$.

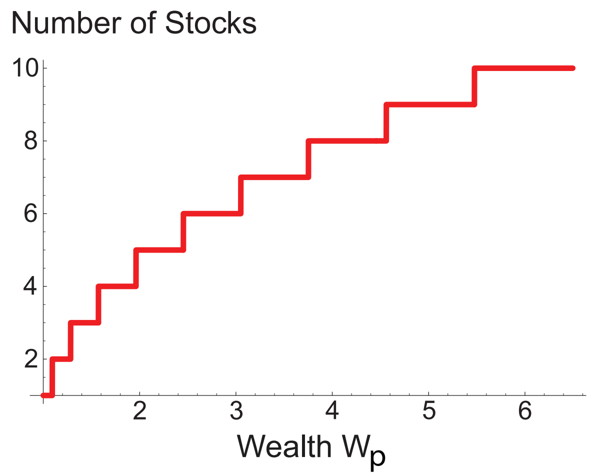

Figure 4: Number of stocks held against W_p for the following parameters: for $i = 1, 2, ..., 10$, $\eta_i = 1.2 - (1.2 - 1)(i - 1)/10$, $\varphi_i = 0.25 - (0.25 - 0.1)(i - 1)/10$, $\bar{\omega}_i = 1$, $W_r = 2$, $\underline{C} = 1$, $\lambda = 0.1$, and $A = 0.75$.

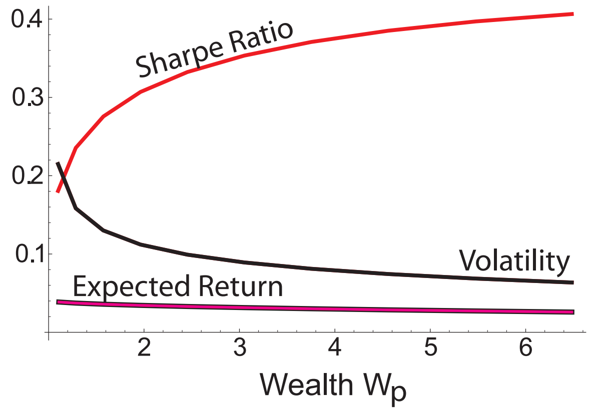

Figure 5: Stock portfolio expected return, volatility, and Sharpe ratio against W_p for the following parameters: for $i = 1, 2, ..., 10$, $\eta_i = 1.2 - (1.2 - 1)(i - 1)/10$, $\varphi_i = 0.25 - (0.25 - 0.1)(i - 1)/10$, $\bar{\omega}_i = 1$, $W_r = 2$, $\underline{C} = 1$, $\lambda = 0.1$, and $A = 0.75$.

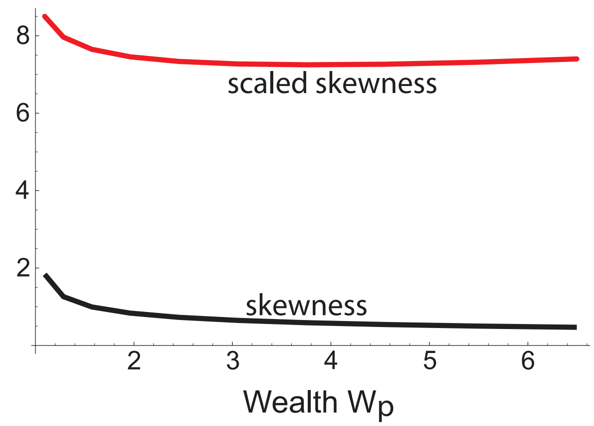

Figure 6: Stock portfolio return skewness and volatility scaled skewness against wealth W_p for parameters: for $i = 1, 2, ..., 10$, $\eta_i = 1.2 - (1.2 - 1)/10$, $\varphi_i = 0.25 - (0.25 - 0.1)/10$, $\bar{\omega}_i = 1$, $W_r = 2$, $\underline{C} = 1$, $\lambda = 0.1$, and $A = 0.75$.

less diversified portfolio is lower than that of a more diversified one. Figure 6 shows that the skewness of the portfolio return is also higher for a less diversified portfolio. All these patterns are consistent with the findings in Mitton and Vorkink (2007). Figure 6 also plots the volatility scaled skewness (skewness/volatility) against the initial wealth level. This figure suggests that the scaled skewness is nonmonotonic in the number of the stocks held. Specifically, it first decreases as the poor diversifies more and then increases. The decreasing pattern is one of the main conclusions of Mitton and Vorkink (2007). Our model suggests that the investors in the Mitton and Vorkink (2007) may have relatively low wealth. Although in our model skewness is irrelevant for stock selection, the implied skewness pattern as shown in Figure 6 may appear to indicate the preference of investors for skewness.

5. Conclusions

We show that it can be exactly the need for risk reduction that causes investors to underdiversify and thus offer a new explanation for the well-documented underdiversification puzzle. In contrast to the existing literature, this explanation does not require any types of trading costs, behavioral biases, or special preferences. The only important assumptions for this explanation are that investors do not borrow or short-sell and they save more than what is necessary for survival. Consistent with empirical findings, the model implies that (1) less wealthy and less risk averse investors underdiversify more; (2) a less diversified stock portfolio has a higher expected return, a higher volatility, and under some reasonable conditions also a higher skewness; and (3) idiosyncratic risks can be priced. In addition, somewhat surprisingly the choice of which stocks to hold only depends on expected returns but not on any higher moments. In particular, Sharpe ratio and skewness are irrelevant for stock selection.

Our model provides a possible explanation for the underdiversification puzzle that complements the existing theories and yields some empirically testable predictions that are already supported by some empirical studies. However, a more rigorous test of the importance of various theories would be interesting and helpful to resolve the puzzle.

Appendix

In this appendix, we provide the proofs for our main results.

PROOF OF THEOREM 1: When $W_0 = \underline{C}$, obviously, the investor can only invest in the risk-free asset. Now suppose his wealth increases to $W_0 = \underline{C} + \varepsilon$, where $\varepsilon > 0$ is small. Since he cannot borrow or short sell, the most he can invest in the stocks is ε. If he invests the entire amount ε in stock j, then

$$W_1 = \underline{C} + \varepsilon(\tilde{z}_j + 1),$$

and the investor's marginal utility from investing only in Stock j at $\varepsilon = 0$ is

$$\frac{\partial E[u(W_1)]}{\partial \varepsilon}|_{\varepsilon=0} = E[u'(\underline{C})(\tilde{z}_j + 1)] = u'(\underline{C})(\mu_j + 1). \tag{29}$$

Therefore when ε is small enough, since $\infty > u'(\underline{C}) > 0$, the marginal utility from investing in the stock with the highest expected return is strictly the highest. Thus, if wealth is slightly above the minimum level $\underline{C}$, the investor invests $\underline{C}$ in the risk-free asset and the rest in the stock with the highest expected return, i.e., Stock 1. Higher moments, such as variance, skewness, and kurtosis, do not matter for this choice.

On the other hand, if multiple stocks have exactly the same expected returns, then the marginal utilities from investing in these stocks at $\varepsilon = 0$ are the same across these stocks. Therefore it is optimal to add all these stocks simultaneously when the initial wealth is slightly above the minimum level $\underline{C}$.

Suppose now given initial wealth W_0, the investor optimally invests $\theta_0 \geq \underline{C}$ in the risk-free asset, $\theta_i > 0$ in Stock i for $1 \leq i \leq k < n$, and 0 in the rest of the stocks. In addition, suppose the constraints $\theta_0 + \sum_{i=1}^{k} \theta_i \leq W_0$ and $\theta_0 \geq \underline{C}$ are still binding.

The end-of-period wealth is $W_1 = \theta_0 + \sum_{i=1}^{k} \theta_i(\tilde{z}_i + 1)$. The Lagrangian is[18]

$$V(W_0, \underline{C}) = \max_{\theta} E\left[u\left(\theta_0 + \sum_{i=1}^{k} \theta_i(\tilde{z}_i + 1)\right)\right] + \nu\left(W_0 - \theta_0 - \sum_{i=1}^{k} \theta_i\right) + \kappa(\theta_0 - \underline{C}),$$

where ν and κ are Lagrangian multipliers. From the first order conditions we have that the marginal utility of investing in Stock i for $1 \leq i \leq k$ is

$$\frac{\partial E\left[u\left(W_1\right)\right]}{\partial \theta_i} = E[u'(W_1)(\tilde{z}_i + 1)] = \nu, \tag{30}$$

and the marginal utility of investing in the risk-free asset is

$$\frac{\partial E\left[u\left(W_1\right)\right]}{\partial \theta_0} = E[u'(W_1)] = \nu - \kappa. \tag{31}$$

By the Envelop theorem, we have

$$\nu = \frac{\partial V(W_0, \underline{C})}{\partial W_0} > 0$$

$$\kappa = -\frac{\partial V(W_0, \underline{C})}{\partial \underline{C}} > 0$$

since the constraints are still binding. Therefore at the initial wealth level W_0, the marginal utilities of investing in the first k stocks are equal, strictly positive, and strictly greater than the marginal utilities of investing in the risk-free asset and the other stocks (because it is optimal for the investor not to hold any of the other stocks). Since

$$\frac{\partial E\left[u'\left(\theta_0 + \sum_{i=1}^{k} \theta_i(\tilde{z}_i + 1)\right)(\tilde{z}_i + 1)\right]}{\partial \theta_i} = E[u''(W_1)(\tilde{z}_i + 1)^2] < 0,$$

the marginal utility of investing in a stock decreases as the investment in the stock increases. Thus as the initial wealth increases from W_0, the investor will increase the investment in all these k stocks, i.e.,

$$\frac{\partial \theta_i}{\partial W_0} > 0, \quad i = 1, 2, ..., k. \tag{32}$$

[18]The short sale constraint is not binding for these k stocks.

The marginal utility of investing in a new stock j at W_0 is

$$E[u'(W_1)(\tilde{z}_j + 1)] = E[u'(W_1)](\mu_j + 1). \tag{33}$$

Therefore if it becomes optimal to add another stock to the portfolio, the investor will add the stock with the next highest expected return, i.e., Stock $k+1$, since $\infty > u'(W_1) > 0$. Again, higher moments are irrelevant for this choice.[19]

Next we derive the critical wealth level $\hat{W}_{k+1}$ at which it is optimal to add Stock $k+1$. By (30) and (33) with $j = k+1$, we must have that at this critical level, the marginal utilities of investing in these $k+1$ stocks are exactly the same, i.e., for $i = 1, 2, ..., k,$, since $\theta_0 = \underline{C}$, we have

$$E\left[u'\left(\underline{C} + \sum_{i=1}^{k} \theta_i(\tilde{z}_i + 1)\right)(\tilde{z}_i + 1)\right] = E\left[u'\left(\underline{C} + \sum_{i=1}^{k} \theta_i(\tilde{z}_i + 1)\right)\right](\mu_{k+1} + 1), \tag{34}$$

which can be simplified to

$$E\left[u'\left(\underline{C} + \sum_{i=1}^{k} \theta_i(\tilde{z}_i + 1)\right)(\tilde{z}_i - \mu_{k+1})\right] = 0, i = 1, 2, ..., k. \tag{35}$$

Therefore we have $\hat{W}_{k+1} = \underline{C} + \sum_{i=1}^{k} \theta_i$ where θ_i's are the solution to (35). By (32), we have that as long as the constraint is binding, the critical wealth level $\hat{W}_i$ at which it is optimal to add Stock i strictly increases with i.

For Part 4, Since the expected end-of-period wealth is equal to

$$W_0 + \sum_{i=1}^{k} \theta_i \mu_i$$

and the end-of-period wealth variance is equal to

$$\sum_{i=1}^{k} \theta_i^2 \sigma_i^2,$$

[19]If returns are correlated, then since the marginal utility $E[u'(W_1)(\tilde{z}_j + 1)] = E[u'(W_1)](\mu_j + 1) + \mathrm{Cov}(u'(W_1), \tilde{z}_j)$, only the expected return and the covariance matter. Therefore conditional on covariance, the choice of stocks is still independent of higher moments (e.g., variance, skewness).

both of which, by inequality (32), increase as W_0 increases.

If the investor has a CRRA utility, i.e.,

$$u(W) = \frac{W^{1-\gamma}}{1-\gamma},$$

then equation (35) is equivalent to

$$E\left[u'\left(1+\sum_{i=1}^{k} w_i \tilde{z}_i\right)(\tilde{z}_i - \mu_{k+1})\right] = 0, i = 1, 2, ..., k. \tag{36}$$

where $w_i \equiv \theta_i / W_0$ represent the fraction of the initial wealth W_0 invested in stock i. And the constraints (3) become

$$\sum_{i=1}^{k} w_i \leq 1 - \frac{C}{W_0} \quad \text{and} \quad w_i \geq 0, i = 1, 2, ..., n.$$

Similar argument to that for (32) shows that

$$\frac{\partial w_i}{\partial W_0} \geq 0, \quad i = 1, 2, ..., k. \tag{37}$$

Since the expected return and the variance of the optimal portfolio are respectively

$$\sum_{i=1}^{k} w_i \mu_i$$

and

$$\sum_{i=1}^{k} w_i^2 \sigma_i^2,$$

both of them increase as W_0 increases by inequality (37). Since k is any integer number between 1 and n, this completes the proof for Parts 1-4.

Now we show Part 5. Because of the no borrowing constraint, if the w_i's that solve (36) when $k < n$ are such that $\sum_{i=1}^{k} w_i > 1$, then the investor never holds more than k stocks, no matter how wealthy he is. To show that this could happen, suppose we have

$$E\left[u'\left(1 + w_1 \tilde{z}_1\right)(\tilde{z}_1 - \mu_2)\right] > 0 \tag{38}$$

for all $w_1 \in [0, 1]$, which can hold if μ_2 is low enough and μ_1 is high enough. Then it is never optimal for the investor to hold more than one stock. This is because the marginal utility of investing in Stock 1 is strictly greater than that of investing in Stock 2 (the stock with the next highest expected return) for all feasible w_1. □

PROOF OF PROPOSITION 1: Suppose $W_0 \in (\hat{W}_i, \hat{W}_{i+1}]$ and thus the poor holds the i stocks with the highest expected returns. The mth root of the mth moment of the poor's stock portfolio return is

$$\xi_m = \sum_{j=1}^{i} w_j^m \left(\frac{M_{mj}}{\check{p}_j} \right)^m, \tag{39}$$

where the portfolio weight

$$w_j = \frac{\check{\delta}_j}{\sum_{j=1}^{i} \check{\delta}_j},$$

and $M_{mj}$is the mth moment of the payoff of the jth stock. Using (24) and (25), we have

$$\xi_m = \frac{k \left(\sum_{j=1}^{i} C_{mj} (\mu_j - k)^m \right)^{1/m}}{(\lambda + k) \sum_{j=1}^{i} a_j (\mu_j - k)}, \tag{40}$$

where $C_{mj} = M_{mj} \eta_j^m / \varphi_j^{2m}$. Since as wealth decreases, the poor becomes less diversified and k increases, we show that the expected return, volatility and skewness all increase with k. It is straightforward to show that ξ_m is strictly increasing in k if

$$f_m(k) = \sum_{j=1}^{i} a_j \mu_j \sum_{j=1}^{i} C_{mj} (\mu_j - k)^{m-1} - \sum_{j=1}^{i} a_j \sum_{j=1}^{i} C_{mj} \mu_j (\mu_j - k)^{m-1} \leq 0.$$

First, consider the expected return. Since $M_{1j} = \eta_j$, we have $C_{1j} = \eta_j^2 / \varphi_j^2 = a_j(\lambda + \mu_j)$. Since

$$\frac{\sum_{j=1}^{i} a_j \mu_j}{\sum_{j=1}^{i} a_j} = \frac{\sum_{j=1}^{i} C_{1j}}{\sum_{j=1}^{i} C_{1j} / (\lambda + \mu_j)} - \lambda \leq \frac{\sum_{j=1}^{i} C_{1j} (\lambda + \mu_j)}{\sum_{j=1}^{i} C_{1j}} - \lambda = \frac{\sum_{j=1}^{i} C_{1j} \mu_j}{\sum_{j=1}^{i} C_{1j}},$$

where the inequality follows from Cauchy's inequality, we have

$$f_1(k) = \sum_{j=1}^{i} a_j \mu_j \sum_{j=1}^{i} C_{1j} - \sum_{j=1}^{i} a_j \sum_{j=1}^{i} C_{1j} \mu_j \leq 0.$$

Therefore the expected return increases with k.

Next consider the volatility. In this case, we have $C_{2j} = \eta_j^2/\varphi_j^4 \times M_{2j} = \eta_j^2/\varphi_j^2 = C_{1j}$. Since $k < \mu_i$, which is the smallest one among the μ_j's for $1 \le j \le i$, and

$$\frac{\sum_{j=1}^i C_{2j}\mu_j}{\sum_{j=1}^i C_{2j}}$$

is a weighted average of the μ_j's, we must have

$$\frac{\sum_{j=1}^i C_{2j}\mu_j}{\sum_{j=1}^i C_{2j}} > k. \tag{41}$$

In addition, by Cauchy's inequality, we have

$$\frac{\sum_{j=1}^i C_{2j}\mu_j^2}{\sum_{j=1}^i C_{2j}\mu_j} \ge \frac{\sum_{j=1}^i C_{2j}\mu_j}{\sum_{j=1}^i C_{2j}} \ge \frac{\sum_{j=1}^i a_j\mu_j}{\sum_{j=1}^i a_j}. \tag{42}$$

Therefore

$$\begin{aligned}
&\frac{f_2(k)}{\sum_{j=1}^i a_j \sum_{j=1}^i C_{2j}} \\
= \; & \frac{\sum_{j=1}^i a_j\mu_j \sum_{j=1}^i C_{2j}(\mu_j - k) - \sum_{j=1}^i a_j \sum_{j=1}^i C_{2j}\mu_j(\mu_j - k)}{\sum_{j=1}^i a_j \sum_{j=1}^i C_{2j}} \\
= \; & \left(\frac{\sum_{j=1}^i a_j\mu_j}{\sum_{j=1}^i a_j} - \frac{\sum_{j=1}^i C_{2j}\mu_j^2}{\sum_{j=1}^i C_{2j}\mu_j}\right)\frac{\sum_{j=1}^i C_{2j}\mu_j}{\sum_{j=1}^i C_{2j}} + \left(\frac{\sum_{j=1}^i C_{2j}\mu_j}{\sum_{j=1}^i C_{2j}} - \frac{\sum_{j=1}^i a_j\mu_j}{\sum_{j=1}^i a_j}\right)k \\
< \; & \left(\frac{\sum_{j=1}^i a_j\mu_j}{\sum_{j=1}^i a_j} - \frac{\sum_{j=1}^i C_{2j}\mu_j^2}{\sum_{j=1}^i C_{2j}\mu_j}\right)k + \left(\frac{\sum_{j=1}^i C_{2j}\mu_j}{\sum_{j=1}^i C_{2j}} - \frac{\sum_{j=1}^i a_j\mu_j}{\sum_{j=1}^i a_j}\right)k \\
\le \; & 0.
\end{aligned}$$

where the inequalities follow from (41) and (42). Therefore as the poor becomes less diversified, the volatility of the optimal stock portfolio increases.

For the skewness, let

$$\begin{aligned}
h(k) &= \sum_{j=1}^i C_{3j} \sum_{j=1}^i C_{3j}(\mu_j - k)^2\mu_j - \sum_{j=1}^i C_{3j}\mu_j \sum_{j=1}^i C_{3j}(\mu_j - k)^2 \\
&= -2\left(\sum_{j=1}^i C_{3j} \sum_{j=1}^i C_{3j}\mu_j^2 - \left(\sum_{j=1}^i C_{3j}\mu_j\right)^2\right)k \\
&\quad + \sum_{j=1}^i C_{3j} \sum_{j=1}^i C_{3j}\mu_j^3 - \sum_{j=1}^i C_{3j}\mu_j \sum_{j=1}^i C_{3j}\mu_j^2.
\end{aligned}$$

It is easy to verify that

$$h'(k) = -2\left(\sum_{j=1}^{i} C_{3j} \sum_{j=1}^{i} C_{3j}\mu_j^2 - \left(\sum_{j=1}^{i} C_{3j}\mu_j\right)^2\right) \leq 0,$$

by Cauchy's inequality. In addition,

$$h(k^*) = 0,$$

where

$$\begin{aligned} k^* &= \frac{\sum_{j=1}^{i} C_{3j} \sum_{j=1}^{i} C_{3j}\mu_j^3 - \sum_{j=1}^{i} C_{3j}\mu_j \sum_{j=1}^{i} C_{3j}\mu_j^2}{2\left(\sum_{j=1}^{i} C_{3j} \sum_{j=1}^{i} C_{3j}\mu_j^2 - \left(\sum_{j=1}^{i} C_{3j}\mu_j\right)^2\right)} \\ &= \frac{\frac{\sum_{j=1}^{i} C_{3j}\mu_j^3}{\sum_{j=1}^{i} C_{3j}\mu_j^2} - \frac{\sum_{j=1}^{i} C_{3j}\mu_j}{\sum_{j=1}^{i} C_{3j}}}{2\left(\frac{\sum_{j=1}^{i} C_{3j}\mu_j^2}{\sum_{j=1}^{i} C_{3j}\mu_j} - \frac{\sum_{j=1}^{i} C_{3j}\mu_j}{\sum_{j=1}^{i} C_{3j}}\right)} \frac{\sum_{j=1}^{i} C_{3j}\mu_j^2}{\sum_{j=1}^{i} C_{3j}\mu_j} \\ &\geq \frac{\sum_{j=1}^{i} C_{3j}\mu_j^2}{\sum_{j=1}^{i} C_{3j}\mu_j} \qquad (43) \\ &> \mu_i, \end{aligned}$$

where the first inequality follows from (27) and the second inequality follows because (43) is a weighted average of the μ_j's and μ_i is smaller than any of the μ_j's. Therefore, for all $k \leq \mu_i$, we have

$$\frac{\sum_{j=1}^{i} C_{3j}(\mu_j - k)^2\mu_j}{\sum_{j=1}^{i} C_{3j}(\mu_j - k)^2} \geq \frac{\sum_{j=1}^{i} C_{3j}\mu_j}{\sum_{j=1}^{i} C_{3j}}.$$

Finally, by (26), we have

$$\frac{\sum_{j=1}^{i} C_{3j}(\mu_j - k)^2\mu_j}{\sum_{j=1}^{i} C_{3j}(\mu_j - k)^2} \geq \frac{\sum_{j=1}^{i} a_j\mu_j}{\sum_{j=1}^{i} a_j},$$

which implies that $f_3(k) \leq 0$ and thus a less diversified stock portfolio has a higher skewness.

The Sharpe ratio of the stock portfolio is

$$\frac{\sum_{j=1}^{i}(C_{1j} - \frac{\lambda+k}{k}a_j)(\mu_j - k)}{\sqrt{\sum_{j=1}^{i} C_{1j}(\mu_j - k)^2}} = \frac{\sum_{j=1}^{i}(C_{1j} - a_j)(\mu_j - k) - \sum_{j=1}^{i} \frac{\lambda}{k}a_j(\mu_j - k)}{\sqrt{\sum_{j=1}^{i} C_{1j}(\mu_j - k)^2}}.$$

So if λ is small enough and

$$g(k) \equiv \frac{\sum_{j=1}^{i}(C_{1j} - a_j)(\mu_j - k)}{\sqrt{\sum_{j=1}^{i} C_{1j}(\mu_j - k)^2}}$$

is strictly decreasing in k, then the Sharpe ratio will also be strictly decreasing in k and thus the claim holds. It can be easily verified that condition (28) directly implies that $g'(k) < 0$. □

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